

Mathematica 11.3 Integration Test Results

Test results for the 886 problems in "1.3.2 Algebraic functions.m"

Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} + x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps) :

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{1+x^3}}\right]}{3 \sqrt{3}} +$$
$$\frac{2 \times 2^{1/3} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}}$$

Result (type 4, 148 leaves) :

$$\frac{4 i \sqrt{2} \sqrt{\frac{i (1+x)}{3 i+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{i+2 i 2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i+\sqrt{3}}\right]}{\left(1+2 \times 2^{2/3}-i \sqrt{3}\right) \sqrt{1+x^3}}$$

Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} - x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 160 leaves, 4 steps) :

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} (1-2^{1/3} x)}{\sqrt{1-x^3}}\right]}{3 \sqrt{3}} -$$
$$\frac{2 \times 2^{1/3} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3}}$$

Result (type 4, 148 leaves) :

$$-\frac{4 \pm \sqrt{2} \sqrt{-\frac{i (-1+x)}{3 i+\sqrt{3}}} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{i+2 \pm 2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}}+2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i+\sqrt{3}}\right]}{\left(1+2 \times 2^{2/3}-\pm \sqrt{3}\right) \sqrt{1-x^3}}$$

Problem 3: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3}-x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 163 leaves, 4 steps):

$$\begin{aligned} & -\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3} (1-2^{1/3} x)}{\sqrt{-1+x^3}}\right]}{3 \sqrt{3}} - \\ & \left(2 \times 2^{1/3} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]\right) / \\ & \left(3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}\right) \end{aligned}$$

Result (type 4, 146 leaves):

$$\begin{aligned} & -\left(4 \pm \sqrt{2} \sqrt{-\frac{i (-1+x)}{3 i+\sqrt{3}}} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{i+2 \pm 2^{2/3}+\sqrt{3}}, \right.\right. \\ & \left.\left. \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}}+2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i+\sqrt{3}}\right]\right) / \left(\left(1+2 \times 2^{2/3}-\pm \sqrt{3}\right) \sqrt{-1+x^3}\right) \end{aligned}$$

Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3}+x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1-x^3}}\right]}{3 \sqrt{3}} +$$

$$\left(2 \times 2^{1/3} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]\right)/$$

$$\left(3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}\right)$$

Result (type 4, 150 leaves):

$$\left(4 \pm \sqrt{2} \sqrt{\frac{i (1+x)}{3 \pm \sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{i+2 \pm 2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}}-2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \pm \sqrt{3}}\right]\right)/\left(\left(1+2 \times 2^{2/3}-i \sqrt{3}\right) \sqrt{-1-x^3}\right)$$

Problem 5: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a+b x^3}} dx$$

Optimal (type 4, 280 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3}+2^{1/3} b^{1/3} x)}{\sqrt{a+b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} +$$

$$\left(2 \times 2^{1/3} \sqrt{2+\sqrt{3}} (a^{1/3}+b^{1/3} x)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3}+b^{1/3} x}{(1+\sqrt{3}) a^{1/3}+b^{1/3} x}\right], -7-4\sqrt{3}\right]\right)/$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{\frac{a^{1/3} (a^{1/3}+b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 164 leaves):

$$\begin{aligned}
& - \left(\left(2 \frac{i}{\sqrt{\left(1 + (-1)^{1/3} \right) a^{1/3}}} \sqrt{\frac{a^{1/3} + b^{1/3} x}{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3} \right) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \Big/ \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{1/3} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 288 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} - \left(2 \times 2^{1/3} \sqrt{2 + \sqrt{3}} (a^{1/3} - b^{1/3} x) \right. \\
& \left. \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} - b^{1/3} x \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 - 4\sqrt{3} \right] \right) \Big/ \\
& \left(3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} - b^{1/3} x \right)^2}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Result (type 4, 166 leaves):

$$\begin{aligned}
& \left(2 \frac{i}{\sqrt{\left(1 + (-1)^{1/3} \right) a^{1/3}}} \sqrt{\frac{a^{1/3} - b^{1/3} x}{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3} \right) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \Big/ \\
& \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{1/3} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 297 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} - \left(2 \times 2^{1/3} \sqrt{2 - \sqrt{3}} (a^{1/3} - b^{1/3} x) \right. \\
& \quad \left. \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3}] \right) / \\
& \quad \left(3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Result (type 4, 167 leaves) :

$$\begin{aligned}
& \left(2 \frac{i}{\sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \quad \left. \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \\
& \quad \left(((-1)^{1/3} + 2^{2/3}) b^{1/3} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Problem 8: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 293 leaves, 4 steps) :

$$\begin{aligned}
& \frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} + \left(2 \times 2^{1/3} \sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \right. \\
& \quad \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3}] \right) / \\
& \quad \left(3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Result (type 4, 167 leaves) :

$$- \left(\left(2 \frac{i}{\sqrt{\left(1 + (-1)^{1/3} \right) a^{1/3}}} \sqrt{\frac{a^{1/3} + b^{1/3} x}{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}}} \text{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \right. \right. \\
 \left. \left. \text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3} \right) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{1/3} \sqrt{-a - b x^3} \right)$$

Problem 9: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 4, 249 leaves, 4 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{3} \sqrt{c} (c + 2 d x)}{\sqrt{c^3 + 4 d^3 x^3}} \right]}{3 \sqrt{3} c^{3/2} d} + \left(2 \times 2^{1/3} \sqrt{2 + \sqrt{3}} (c + 2^{2/3} d x) \right. \\
 \left. \sqrt{\frac{c^2 - 2^{2/3} c d x + 2 \times 2^{1/3} d^2 x^2}{\left((1 + \sqrt{3}) c + 2^{2/3} d x \right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) c + 2^{2/3} d x}{(1 + \sqrt{3}) c + 2^{2/3} d x} \right], -7 - 4 \sqrt{3} \right] \right) / \\
 \left(3 \times 3^{1/4} c d \sqrt{\frac{c (c + 2^{2/3} d x)}{\left((1 + \sqrt{3}) c + 2^{2/3} d x \right)^2}} \sqrt{c^3 + 4 d^3 x^3} \right)$$

Result (type 4, 169 leaves):

$$- \left(\left(\frac{i 2^{5/6} \sqrt{\frac{2^{1/3} c + 2 d x}{\left(1 + (-1)^{1/3} \right) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}{c^2}} \text{EllipticPi}\left[\frac{i 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \right. \right. \right. \\
 \left. \left. \left. \text{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{\left(1 + (-1)^{1/3} \right) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] \right) / \left((2 + (-2)^{1/3}) d \sqrt{c^3 + 4 d^3 x^3} \right)$$

Problem 10: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1 + \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 146 leaves, 4 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{3 (3+2 \sqrt{3})}} + \\
& \left(\sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right] \right) / \\
& \left(3^{3/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)
\end{aligned}$$

Result (type 4, 136 leaves) :

$$\begin{aligned}
& - \left(\left(4 \sqrt{2} \sqrt{\frac{i (1+x)}{3 i + \sqrt{3}}} \sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i + (1+2 i) \sqrt{3}}, \right. \right. \right. \\
& \left. \left. \left. \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}}-2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \right) / \left((3 i + (1+2 i) \sqrt{3}) \sqrt{1+x^3} \right)
\end{aligned}$$

Problem 11: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}-x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 164 leaves, 4 steps) :

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{1-x^3}}\right]}{\sqrt{3 (3+2 \sqrt{3})}} - \\
& \left(\sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right] \right) / \\
& \left(3^{3/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right)
\end{aligned}$$

Result (type 4, 136 leaves) :

$$\left(4 \sqrt{2} \sqrt{-\frac{\frac{i}{2} (-1+x)}{3 i + \sqrt{3}}} \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i + (1+2 i) \sqrt{3}}, \right. \right.$$

$$\left. \left. \text{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3}} + 2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \right) \Big/ \left((3 i + (1+2 i) \sqrt{3}) \sqrt{1-x^3} \right)$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}-x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 167 leaves, 4 steps):

$$\begin{aligned} & -\frac{\text{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{-1+x^3}}\right]}{\sqrt{3 (3+2 \sqrt{3})}} - \\ & \left(\sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right] \right) \Big/ \\ & \left(3^{3/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right) \end{aligned}$$

Result (type 4, 134 leaves):

$$\left(4 \sqrt{2} \sqrt{-\frac{\frac{i}{2} (-1+x)}{3 i + \sqrt{3}}} \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i + (1+2 i) \sqrt{3}}, \right. \right.$$

$$\left. \left. \text{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3}} + 2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \right) \Big/ \left((3 i + (1+2 i) \sqrt{3}) \sqrt{-1+x^3} \right)$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}+x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 157 leaves, 4 steps):

$$\begin{aligned}
& \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{3 (3+2 \sqrt{3})}} + \\
& \left(\sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right] \right) / \\
& \left(3^{3/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)
\end{aligned}$$

Result (type 4, 138 leaves) :

$$\begin{aligned}
& - \left(\left(4 \sqrt{2} \sqrt{\frac{i (1+x)}{3 i + \sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i + (1+2 i) \sqrt{3}}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}}-2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \right) / \left((3 i + (1+2 i) \sqrt{3}) \sqrt{-1-x^3} \right)
\end{aligned}$$

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 331 leaves, 8 steps) :

$$\begin{aligned}
& \frac{\left(1+x\right) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right]}{\sqrt{26} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}} + \\
& \left(2 \sqrt{26+15 \sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(3^{1/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}\right) + \\
& \left(4 \times 3^{1/4} (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticPi}\left[97-56 \sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}\right)
\end{aligned}$$

Result (type 4, 128 leaves):

$$-\left(\left(4 \sqrt{2} \sqrt{\frac{\frac{i}{2} (1+x)}{3 \frac{i}{2} + \sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{7 \frac{i}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3}-2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}\right]\right)\right)$$

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 382 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh} \left[\frac{\sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{2 \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}} \right]}{2 \sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}} - \\
& \left(2 \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right] \right) / \\
& \left(3^{1/4} (4+\sqrt{3}) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right) + \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi} \left[\frac{1}{169} (553+304\sqrt{3}), \right. \right. \\
& \left. \left. -\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right] \right) / \left(13 \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right)
\end{aligned}$$

Result (type 4, 128 leaves):

$$- \left(\left(4 \sqrt{2} \sqrt{\frac{\frac{i}{2}(-1+x)}{-3\frac{i}{2}+\sqrt{3}}} \sqrt{1+x+x^2} \right. \right. \\
\left. \left. \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{5\frac{i}{2}+\sqrt{3}}, \operatorname{ArcSin} \left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}}-2\frac{i}{2}x}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3\frac{i}{2}+\sqrt{3}} \right] \right) / \left(\left(5\frac{i}{2}+\sqrt{3} \right) \sqrt{1-x^3} \right)
\right)$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 376 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\left(1-x\right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{7} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2 \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right]}{2 \sqrt{7} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{-1+x^3}} - \\
& \left(2 \sqrt{62-35 \sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]\right) / \\
& \left(13 \times 3^{1/4} \sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}} \sqrt{-1+x^3}\right) + \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{EllipticPi}\left[\frac{1}{169} \left(553+304 \sqrt{3}\right),\right.\right. \\
& \left.\left.-\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]\right) / \left(13 \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{-1+x^3}\right)
\end{aligned}$$

Result (type 4, 126 leaves) :

$$- \left(\left(4 \sqrt{2} \sqrt{\frac{\frac{i}{2} (-1+x)}{-3 \frac{i}{2} + \sqrt{3}}} \sqrt{1+x+x^2} \right. \right. \\
\left. \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{5 \frac{i}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3}} - 2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2} + \sqrt{3}}\right]\right) / \left(\left(5 \frac{i}{2} + \sqrt{3} \right) \sqrt{-1+x^3} \right)
\right)$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 342 leaves, 8 steps) :

$$\begin{aligned}
& \frac{(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right]}{\sqrt{26} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}} + \\
& \frac{2 (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}} + \\
& \left(4 \times 3^{1/4} (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticPi}\left[97-56 \sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{-1-x^3}\right)
\end{aligned}$$

Result (type 4, 130 leaves) :

$$-\left(\left(4 \sqrt{2} \sqrt{\frac{\frac{i}{2} (1+x)}{3 \frac{i}{2}+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{7 \frac{i}{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}}-2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{i}{2}+\sqrt{3}}\right]\right)\right) / \\
\left(\left(7 \frac{i}{2}+\sqrt{3}\right) \sqrt{-1-x^3}\right)$$

Problem 18: Unable to integrate problem.

$$\int \frac{1}{(c+d x) (-c^3+d^3 x^3)^{1/3}} dx$$

Optimal (type 3, 139 leaves, 1 step) :

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2^{1/3} (c-d x)}{(-c^3+d^3 x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3} c d} + \frac{\operatorname{Log}\left[(c-d x) (c+d x)^2\right]}{4 \times 2^{1/3} c d} - \frac{3 \operatorname{Log}\left[d (c-d x)+2^{2/3} d (-c^3+d^3 x^3)^{1/3}\right]}{4 \times 2^{1/3} c d}$$

Result (type 8, 27 leaves) :

$$\int \frac{1}{(c+d x) (-c^3+d^3 x^3)^{1/3}} dx$$

Problem 19: Unable to integrate problem.

$$\int \frac{1}{(c + d x) (2 c^3 + d^3 x^3)^{1/3}} dx$$

Optimal (type 3, 186 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 d x}{(2 c^3+d^3 x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \sqrt{3} c d}-\frac{\sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2 (2 c+d x)}{(2 c^3+d^3 x^3)^{1/3}}}{\sqrt{3}}\right]}{2 c d}-\frac{\text{Log}[c+d x]}{2 c d}+\frac{\text{Log}\left[-d x+\left(2 c^3+d^3 x^3\right)^{1/3}\right]}{4 c d}+\frac{3 \text{Log}\left[d \left(2 c+d x\right)-d \left(2 c^3+d^3 x^3\right)^{1/3}\right]}{4 c d}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(c + d x) (2 c^3 + d^3 x^3)^{1/3}} dx$$

Problem 20: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3}-2 x}{(2^{2/3}+x) \sqrt{1+x^3}} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \text{ArcTan}\left[\frac{\sqrt{3} \left(1+2^{1/3} x\right)}{\sqrt{1+x^3}}\right]}{\sqrt{3}}$$

Result (type 4, 326 leaves):

$$\begin{aligned} & -\left(\left(4 \times 2^{1/6} \sqrt{\frac{\frac{\dot{x}}{x} (1+x)}{3 \dot{x}+\sqrt{3}}}\right.\right. \\ & \left.\left.\left(\sqrt{-\frac{\dot{x}}{x}+\sqrt{3}}+2 \frac{\dot{x}}{x} x\right)\left(6 \frac{\dot{x}}{x}+3 \frac{\dot{x}}{x} 2^{1/3}-2 \sqrt{3}+2^{1/3} \sqrt{3}+\left(-3 \frac{\dot{x}}{x} 2^{1/3}+4 \sqrt{3}+2^{1/3} \sqrt{3}\right) x\right)\right. \\ & \left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\dot{x}}{x}+\sqrt{3}}-2 \frac{\dot{x}}{x} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \dot{x}+\sqrt{3}}\right]-6 \frac{\dot{x}}{x} \sqrt{3} \sqrt{\frac{\dot{x}}{x}+\sqrt{3}}-2 \frac{\dot{x}}{x} x\right.\right. \\ & \left.\left.\sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2 \sqrt{3}}{\dot{x}+2 \frac{\dot{x}}{x} 2^{2/3}+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{\dot{x}}{x}+\sqrt{3}}-2 \frac{\dot{x}}{x} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \dot{x}+\sqrt{3}}\right]\right)\right)/ \\ & \left(\sqrt{3} \left(1+2 \times 2^{2/3}-\frac{\dot{x}}{x} \sqrt{3}\right) \sqrt{\frac{\dot{x}}{x}+\sqrt{3}}-2 \frac{\dot{x}}{x} x \sqrt{1+x^3}\right)\end{aligned}$$

Problem 21: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} + 2x}{(2^{2/3} - x) \sqrt{1-x^3}} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$-\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} (1-2^{1/3} x)}{\sqrt{1-x^3}}\right]}{\sqrt{3}}$$

Result (type 4, 327 leaves):

$$\begin{aligned} & - \left(\left(4 \times 2^{1/6} \sqrt{-\frac{i (-1+x)}{3 i + \sqrt{3}}} \right. \right. \\ & \left. \left. \left(\sqrt{-i + \sqrt{3}} - 2 i x \right) \left(-6 i - 3 i 2^{1/3} + 2 \sqrt{3} - 2^{1/3} \sqrt{3} + \left(-3 i 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3} \right) x \right) \right. \\ & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3}} + 2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}\right] + 6 i \sqrt{3} \sqrt{i + \sqrt{3}} + 2 i x \right. \\ & \left. \left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{i + 2 i 2^{2/3} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3}} + 2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}\right] \right) \right) / \\ & \left(\sqrt{3} \left(1 + 2 \times 2^{2/3} - i \sqrt{3} \right) \sqrt{i + \sqrt{3}} + 2 i x \sqrt{1-x^3} \right) \end{aligned}$$

Problem 22: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} + 2x}{(2^{2/3} - x) \sqrt{-1+x^3}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$-\frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} (1-2^{1/3} x)}{\sqrt{-1+x^3}}\right]}{\sqrt{3}}$$

Result (type 4, 325 leaves):

$$\begin{aligned}
& - \left(\left(4 \times 2^{1/6} \sqrt{-\frac{\frac{i}{2}(-1+x)}{3\frac{i}{2}+\sqrt{3}}} \right. \right. \\
& \left. \left(\sqrt{-\frac{i}{2}+\sqrt{3}} - 2\frac{i}{2}x \right) \left(-6\frac{i}{2} - 3\frac{i}{2}2^{1/3} + 2\sqrt{3} - 2^{1/3}\sqrt{3} + (-3\frac{i}{2}2^{1/3} + 4\sqrt{3} + 2^{1/3}\sqrt{3})x \right) \right. \\
& \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}} + 2\frac{i}{2}x}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}} \right] + 6\frac{i}{2}\sqrt{3} \sqrt{\frac{i}{2}+\sqrt{3}} + 2\frac{i}{2}x \right. \right. \\
& \left. \left. \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{\frac{i}{2}+2\frac{i}{2}2^{2/3}+\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}} + 2\frac{i}{2}x}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}} \right] \right) \right) / \\
& \left. \left(\sqrt{3} \left(1 + 2 \times 2^{2/3} - \frac{i}{2}\sqrt{3} \right) \sqrt{\frac{i}{2}+\sqrt{3}} + 2\frac{i}{2}x \sqrt{-1+x^3} \right) \right)
\end{aligned}$$

Problem 23: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x)\sqrt{-1-x^3}} dx$$

Optimal (type 3, 39 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \text{ArcTanh} \left[\frac{\sqrt{3} (1+2^{1/3}x)}{\sqrt{-1-x^3}} \right]}{\sqrt{3}}$$

Result (type 4, 328 leaves):

$$\begin{aligned}
& - \left(\left(4 \times 2^{1/6} \sqrt{\frac{\frac{i}{2}(1+x)}{3\frac{i}{2}+\sqrt{3}}} \right. \right. \\
& \left. \left(\sqrt{-\frac{i}{2}+\sqrt{3}} + 2\frac{i}{2}x \right) \left(6\frac{i}{2} + 3\frac{i}{2}2^{1/3} - 2\sqrt{3} + 2^{1/3}\sqrt{3} + (-3\frac{i}{2}2^{1/3} + 4\sqrt{3} + 2^{1/3}\sqrt{3})x \right) \right. \\
& \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}} - 2\frac{i}{2}x}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}} \right] - 6\frac{i}{2}\sqrt{3} \sqrt{\frac{i}{2}+\sqrt{3}} - 2\frac{i}{2}x \right. \right. \\
& \left. \left. \sqrt{1-x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{\frac{i}{2}+2\frac{i}{2}2^{2/3}+\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}} - 2\frac{i}{2}x}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}} \right] \right) \right) / \\
& \left. \left(\sqrt{3} \left(1 + 2 \times 2^{2/3} - \frac{i}{2}\sqrt{3} \right) \sqrt{\frac{i}{2}+\sqrt{3}} - 2\frac{i}{2}x \sqrt{-1+x^3} \right) \right)
\end{aligned}$$

Problem 24: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} - 2 b^{1/3} x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 63 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \text{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{a+b x^3}}\right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 325 leaves):

$$\begin{aligned} & \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(2 \times 3^{1/4} ((-1)^{1/3} a^{1/3} - b^{1/3} x) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \right.\right. \\ & \quad \left.\left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}] \right) \middle/ \left(\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right)\right. \\ & \quad \left.\left. \frac{1}{(-1)^{1/3} + 2^{2/3}} 3 (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi}\right.\right. \\ & \quad \left.\left. \frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}]\right) \middle/ (\sqrt{3} b^{1/3} \sqrt{a + b x^3})\right) \end{aligned}$$

Problem 25: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} + 2 b^{1/3} x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 65 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \text{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a-b x^3}}\right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 336 leaves):

$$\begin{aligned}
& \frac{1}{b^{1/3} \sqrt{a - b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(- \left(\left(2 \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \text{EllipticF} \right. \right. \right. \\
& \left. \left. \left. \left(\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right) \right) / \left(\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \right) + \\
& \frac{1}{(-1)^{1/3} + 2^{2/3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{3 + \frac{3 b^{1/3} x}{a^{1/3}} + \frac{3 b^{2/3} x^2}{a^{2/3}}} \\
& \text{EllipticPi} \left[\frac{\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \text{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] \right]
\end{aligned}$$

Problem 26: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} + 2 b^{1/3} x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 66 leaves, 2 steps) :

$$\begin{aligned}
& \frac{2 \times 2^{2/3} \text{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}} \right]}{\sqrt{3} a^{1/6} b^{1/3}}
\end{aligned}$$

Result (type 4, 390 leaves) :

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left(2 \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \right. \right. \\
& \quad \left. \left. \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] - \right. \\
& \quad \left. (-1)^{1/3} 2^{2/3} \sqrt{3} \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) / \\
& \quad \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Problem 27: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} - 2 b^{1/3} x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 66 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 375 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{3^{1/4}} 2 \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] + \right. \right. \\
& \quad \left. \left. \left. (-1)^{1/3} 2^{2/3} \sqrt{3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) / \right. \\
& \quad \left. \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right) \right)
\end{aligned}$$

Problem 28: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{c - 2 d x}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 3, 49 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} \sqrt{c} (c+2 d x)}{\sqrt{c^3+4 d^3 x^3}} \right]}{\sqrt{3} \sqrt{c} d}$$

Result (type 4, 373 leaves):

$$\begin{aligned}
& \left(2^{1/6} \sqrt{\frac{2^{1/3} c + 2 d x}{(1 + (-1)^{1/3}) c}} \right. \\
& \left. - 2 \sqrt{\frac{(-2)^{1/3} c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \left((-1)^{1/3} (2 + (-2)^{1/3}) c - 2 ((-1)^{1/3} + 2^{2/3}) d x \right) \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}}\right], (-1)^{1/3}] - \right. \\
& \left. (-1)^{1/3} 2^{2/3} \sqrt{3} (1 + (-1)^{1/3}) c \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}{c^2}} \right. \\
& \left. \text{EllipticPi}\left[\frac{\pm 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}}\right], (-1)^{1/3}]\right] \right) / \\
& \left((2 + (-2)^{1/3}) d \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 + 4 d^3 x^3} \right)
\end{aligned}$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{2 + 3 x}{(2^{2/3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 158 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (2 - 3 \times 2^{2/3}) \text{ArcTan}\left[\frac{\sqrt{3} (1 + 2^{1/3} x)}{\sqrt{1 + x^3}}\right]}{3 \sqrt{3}} + \\
& \left(2 (3 + 2 \times 2^{1/3}) \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4 \sqrt{3}] \right) / \\
& \left(3 \times 3^{1/4} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3} \right)
\end{aligned}$$

Result (type 4, 336 leaves) :

$$\begin{aligned} & \left(2 \times 2^{1/6} \sqrt{\frac{i (1+x)}{3 i + \sqrt{3}}} \right. \\ & \left(3 \sqrt{-i + \sqrt{3} + 2 i x} \left(-6 - 3 \times 2^{1/3} - 2 i \sqrt{3} + i 2^{1/3} \sqrt{3} + \left(3 \times 2^{1/3} + 4 i \sqrt{3} + i 2^{1/3} \sqrt{3} \right) x \right) \right. \\ & \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] - 4 \sqrt{3} \left(-3 + 2^{1/3} \right) \sqrt{i + \sqrt{3} - 2 i x} \right. \\ & \left. \left. \sqrt{1 - x + x^2} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{i + 2 i 2^{2/3} + \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \right) \right) / \\ & \left(\sqrt{3} \left(i + 2 i 2^{2/3} + \sqrt{3} \right) \sqrt{i + \sqrt{3} - 2 i x} \sqrt{1 + x^3} \right) \end{aligned}$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2 + 3 x}{(2^{2/3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 173 leaves, 4 steps) :

$$\begin{aligned} & -\frac{2 (2 + 3 \times 2^{2/3}) \text{ArcTan} \left[\frac{\sqrt{3} (1 - 2^{1/3} x)}{\sqrt{1 - x^3}} \right]}{3 \sqrt{3}} + \\ & \left(2 (3 - 2 \times 2^{1/3}) \sqrt{2 + \sqrt{3}} (1 - x) \sqrt{\frac{1 + x + x^2}{(1 + \sqrt{3} - x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1 - \sqrt{3} - x}{1 + \sqrt{3} - x} \right], -7 - 4 \sqrt{3} \right] \right) / \\ & \left(3 \times 3^{1/4} \sqrt{\frac{1 - x}{(1 + \sqrt{3} - x)^2}} \sqrt{1 - x^3} \right) \end{aligned}$$

Result (type 4, 335 leaves) :

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{-\frac{\frac{i}{3} (-1+x)}{3 i + \sqrt{3}}} \right. \\
& \left(-3 i \sqrt{-i + \sqrt{3}} - 2 i x \right) \left(-6 i - 3 i 2^{1/3} + 2 \sqrt{3} - 2^{1/3} \sqrt{3} + \left(-3 i 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3} \right) x \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} + 2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] + 4 \sqrt{3} \left(3 + 2^{1/3} \right) \sqrt{i + \sqrt{3}} + 2 i x \\
& \sqrt{1 + x + x^2} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{i + 2 i 2^{2/3} + \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} + 2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \left. \right) / \\
& \left(\sqrt{3} \left(i + 2 i 2^{2/3} + \sqrt{3} \right) \sqrt{i + \sqrt{3}} + 2 i x \right) \sqrt{1 - x^3}
\end{aligned}$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 176 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (2 + 3 \times 2^{2/3}) \text{ArcTanh} \left[\frac{\sqrt{3} (1 - 2^{1/3} x)}{\sqrt{-1+x^3}} \right]}{3 \sqrt{3}} + \\
& \left(2 (3 - 2 \times 2^{1/3}) \sqrt{2 - \sqrt{3}} (1 - x) \sqrt{\frac{1 + x + x^2}{(1 - \sqrt{3} - x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1 + \sqrt{3} - x}{1 - \sqrt{3} - x} \right], -7 + 4 \sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} \sqrt{-\frac{1 - x}{(1 - \sqrt{3} - x)^2}} \sqrt{-1 + x^3} \right)
\end{aligned}$$

Result (type 4, 333 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{-\frac{\frac{i}{3} (-1+x)}{3 i + \sqrt{3}}} \right. \\
& \left(-3 i \sqrt{-i + \sqrt{3}} - 2 i x \right) \left(-6 i - 3 i 2^{1/3} + 2 \sqrt{3} - 2^{1/3} \sqrt{3} + \left(-3 i 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3} \right) x \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} + 2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] + 4 \sqrt{3} \left(3 + 2^{1/3} \right) \sqrt{i + \sqrt{3}} + 2 i x \\
& \sqrt{1 + x + x^2} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{i + 2 i 2^{2/3} + \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} + 2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \left. \right) / \\
& \left(\sqrt{3} \left(i + 2 i 2^{2/3} + \sqrt{3} \right) \sqrt{i + \sqrt{3}} + 2 i x \right) \sqrt{-1 + x^3}
\end{aligned}$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 169 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (2 - 3 \times 2^{2/3}) \text{ArcTanh} \left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1-x^3}} \right]}{3 \sqrt{3}} + \\
& \left(2 (3 + 2 \times 2^{1/3}) \sqrt{2 - \sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7 + 4 \sqrt{3} \right] \right. \\
& \left. \left. 3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right) \right)
\end{aligned}$$

Result (type 4, 338 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{\frac{i(1+x)}{3i + \sqrt{3}}} \right. \\
& \left(3 \sqrt{-i + \sqrt{3}} + 2ix \right) \left(-6 - 3 \times 2^{1/3} - 2i\sqrt{3} + i2^{1/3}\sqrt{3} + \left(3 \times 2^{1/3} + 4i\sqrt{3} + i2^{1/3}\sqrt{3} \right)x \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2ix}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}} \right] - 4\sqrt{3} \left(-3 + 2^{1/3} \right) \sqrt{i + \sqrt{3}} - 2ix \\
& \sqrt{1 - x + x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{i + 2i2^{2/3} + \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2ix}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}} \right] \Bigg) / \\
& \left(\sqrt{3} \left(i + 2i2^{2/3} + \sqrt{3} \right) \sqrt{i + \sqrt{3}} - 2ix \right) \sqrt{-1 - x^3}
\end{aligned}$$

Problem 33: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{e + fx}{(2^{2/3} + x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 159 leaves, 4 steps):

$$\begin{aligned}
& \frac{2(e - 2^{2/3}f) \text{ArcTan} \left[\frac{\sqrt{3}(1+2^{1/3}x)}{\sqrt{1+x^3}} \right]}{3\sqrt{3}} + \\
& \left(2\sqrt{2+\sqrt{3}} (2^{1/3}e+f)(1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x} \right], -7-4\sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)
\end{aligned}$$

Result (type 4, 340 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{\frac{i(1+x)}{3i + \sqrt{3}}} \right. \\
& \left(f \sqrt{-i + \sqrt{3}} + 2ix \right) \left(-6 - 3 \times 2^{1/3} - 2i\sqrt{3} + i2^{1/3}\sqrt{3} + \left(3 \times 2^{1/3} + 4i\sqrt{3} + i2^{1/3}\sqrt{3} \right)x \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2ix}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}} \right] - 2\sqrt{3} \left(2^{1/3}e - 2f \right) \sqrt{i + \sqrt{3} - 2ix} \\
& \left. \sqrt{1 - x + x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{i + 2ix2^{2/3} + \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2ix}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}} \right] \right) / \\
& \left(\sqrt{3} \left(i + 2ix2^{2/3} + \sqrt{3} \right) \sqrt{i + \sqrt{3} - 2ix} \sqrt{1 - x^3} \right)
\end{aligned}$$

Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + fx}{(2^{2/3} - x)\sqrt{1 - x^3}} dx$$

Optimal (type 4, 175 leaves, 4 steps):

$$\begin{aligned}
& -\frac{2(e + 2^{2/3}f) \text{ArcTan} \left[\frac{\sqrt{3}(1 - 2^{1/3}x)}{\sqrt{1 - x^3}} \right]}{3\sqrt{3}} - \\
& \left(2\sqrt{2 + \sqrt{3}} (2^{1/3}e - f) (1 - x) \sqrt{\frac{1 + x + x^2}{(1 + \sqrt{3} - x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1 - \sqrt{3} - x}{1 + \sqrt{3} - x} \right], -7 - 4\sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} \sqrt{\frac{1 - x}{(1 + \sqrt{3} - x)^2}} \sqrt{1 - x^3} \right)
\end{aligned}$$

Result (type 4, 340 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{-\frac{\frac{i}{2}(-1+x)}{3\frac{i}{2}+\sqrt{3}}} \right. \\
& \left(-\frac{i}{2} f \sqrt{-\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x} \left(-6\frac{i}{2}-3\frac{i}{2}2^{1/3}+2\sqrt{3}-2^{1/3}\sqrt{3} + \left(-3\frac{i}{2}2^{1/3}+4\sqrt{3}+2^{1/3}\sqrt{3} \right) x \right) \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}}+2\frac{i}{2}x}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}} \right] + 2\sqrt{3} \left(2^{1/3}e+2f \right) \sqrt{\frac{i}{2}+\sqrt{3}+2\frac{i}{2}x} \right. \\
& \left. \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{\frac{i}{2}+2\frac{i}{2}2^{2/3}+\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}}+2\frac{i}{2}x}{\sqrt{2}3^{1/4}} \right], \frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}} \right] \right) / \\
& \left(\sqrt{3} \left(\frac{i}{2}+2\frac{i}{2}2^{2/3}+\sqrt{3} \right) \sqrt{\frac{i}{2}+\sqrt{3}+2\frac{i}{2}x} \sqrt{1-x^3} \right)
\end{aligned}$$

Problem 35: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{(2^{2/3}-x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 178 leaves, 4 steps):

$$\begin{aligned}
& -\frac{2(e+2^{2/3}f) \text{ArcTanh} \left[\frac{\sqrt{3}(1-2^{1/3}x)}{\sqrt{-1+x^3}} \right]}{3\sqrt{3}} - \\
& \left(2\sqrt{2-\sqrt{3}} (2^{1/3}e-f) (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x} \right], -7+4\sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right)
\end{aligned}$$

Result (type 4, 338 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{-\frac{\frac{i}{2}(-1+x)}{3i + \sqrt{3}}} \right. \\
& \left(-\frac{i}{2} f \sqrt{-\frac{i}{2} + \sqrt{3}} - 2 \frac{i}{2} x \right) \left(-6 \frac{i}{2} - 3 \frac{i}{2} 2^{1/3} + 2 \sqrt{3} - 2^{1/3} \sqrt{3} + \left(-3 \frac{i}{2} 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3} \right) x \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2} + \sqrt{3}} + 2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}} \right] + 2 \sqrt{3} \left(2^{1/3} e + 2 f \right) \sqrt{\frac{i}{2} + \sqrt{3}} + 2 \frac{i}{2} x \\
& \left. \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{\frac{i}{2} + 2 \frac{i}{2} 2^{2/3} + \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{i}{2} + \sqrt{3}} + 2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}} \right] \right) / \\
& \left(\sqrt{3} \left(\frac{i}{2} + 2 \frac{i}{2} 2^{2/3} + \sqrt{3} \right) \sqrt{\frac{i}{2} + \sqrt{3}} + 2 \frac{i}{2} x \right) \sqrt{-1+x^3}
\end{aligned}$$

Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{e + f x}{(2^{2/3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 170 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (e - 2^{2/3} f) \text{ArcTanh} \left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1-x^3}} \right]}{3 \sqrt{3}} + \\
& \left(2 \sqrt{2-\sqrt{3}} (2^{1/3} e + f) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4 \sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)
\end{aligned}$$

Result (type 4, 342 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{\frac{i(1+x)}{3i + \sqrt{3}}} \right. \\
& \left(f \sqrt{-i + \sqrt{3}} + 2ix \right) \left(-6 - 3 \times 2^{1/3} - 2i\sqrt{3} + i2^{1/3}\sqrt{3} + \left(3 \times 2^{1/3} + 4i\sqrt{3} + i2^{1/3}\sqrt{3} \right)x \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2ix}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}} \right] - 2\sqrt{3} \left(2^{1/3}e - 2f \right) \sqrt{i + \sqrt{3}} - 2ix \\
& \left. \sqrt{1 - x + x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{i + 2ix 2^{2/3} + \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2ix}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}} \right] \right) / \\
& \left(\sqrt{3} \left(i + 2ix 2^{2/3} + \sqrt{3} \right) \sqrt{i + \sqrt{3}} - 2ix \sqrt{-1 - x^3} \right)
\end{aligned}$$

Problem 37: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + fx}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a + bx^3}} dx$$

Optimal (type 4, 316 leaves, 4 steps):

$$\begin{aligned}
& \frac{2(b^{1/3}e - 2^{2/3}a^{1/3}f) \text{ArcTan} \left[\frac{\sqrt{3}a^{1/6}(a^{1/3} + 2^{1/3}b^{1/3}x)}{\sqrt{a+bx^3}} \right]}{3\sqrt{3}\sqrt{a}b^{2/3}} + \\
& \left(2\sqrt{2+\sqrt{3}}(2^{1/3}b^{1/3}e + a^{1/3}f)(a^{1/3} + b^{1/3}x) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3})a^{1/3} + b^{1/3}x}{(1+\sqrt{3})a^{1/3} + b^{1/3}x} \right], -7 - 4\sqrt{3} \right] \right) /
\end{aligned}$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3}(a^{1/3} + b^{1/3}x)}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \sqrt{a+bx^3} \right)$$

Result (type 4, 336 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left(- \left(\left(3^{1/4} f \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] \right) \right) \right) \Big/ \left(\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) + \\
& \frac{1}{(-1)^{1/3} + 2^{2/3}} (-1)^{1/3} (1 + (-1)^{1/3}) (-b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi}[\\
& \left. \left. \left. \left. \frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) \Big/ \left(\sqrt{3} b^{2/3} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 38: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 324 leaves, 4 steps) :

$$\begin{aligned}
& - \frac{2 (b^{1/3} e + 2^{2/3} a^{1/3} f) \text{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{2/3}} - \\
& \left(2 \sqrt{2 + \sqrt{3}} (2^{1/3} b^{1/3} e - a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 - 4 \sqrt{3}] \right) \Big/ \\
& \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Result (type 4, 399 leaves) :

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left(- \left((-1)^{1/3} + 2^{2/3} \right) f \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \right. \\
& \quad \left. \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] + \right. \\
& \quad \left. \frac{1}{\sqrt{3}} (-1)^{1/3} (1 + (-1)^{1/3}) (b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \quad \left. \operatorname{EllipticPi}\left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \\
& \quad \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 39: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 333 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 (b^{1/3} e + 2^{2/3} a^{1/3} f) \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{2/3}} - \\
& \left(2 \sqrt{2 - \sqrt{3}} (2^{1/3} b^{1/3} e - a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \right. \\
& \quad \left. \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3}] \right) /
\end{aligned}$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3} \right)$$

Result (type 4, 400 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left(- \left((-1)^{1/3} + 2^{2/3} \right) f \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \right. \\
& \quad \left. \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] + \right. \\
& \quad \left. \frac{1}{\sqrt{3}} (-1)^{1/3} (1 + (-1)^{1/3}) (b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \quad \left. \operatorname{EllipticPi}\left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \\
& \quad \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 329 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (b^{1/3} e - 2^{2/3} a^{1/3} f) \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{2/3}} + \\
& \left(2 \sqrt{2 - \sqrt{3}} (2^{1/3} b^{1/3} e + a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \right. \\
& \quad \left. \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3}] \right) /
\end{aligned}$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3} \right)$$

Result (type 4, 387 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{3^{1/4}} \left((-1)^{1/3} + 2^{2/3} \right) f \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \right. \right. \\
& \quad \left. \left. - \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] + \frac{1}{\sqrt{3}} \right. \right. \\
& \quad \left. \left. (-1)^{1/3} \left(1 + (-1)^{1/3} \right) (-b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) / \\
& \quad \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 4, 265 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (d e - c f) \operatorname{ArcTan}\left[\frac{\sqrt{3} \sqrt{c} (c + 2 d x)}{\sqrt{c^3 + 4 d^3 x^3}} \right]}{3 \sqrt{3} c^{3/2} d^2} + \\
& \left(2^{1/3} \sqrt{2 + \sqrt{3}} (2 d e + c f) (c + 2^{2/3} d x) \sqrt{\frac{c^2 - 2^{2/3} c d x + 2 \times 2^{1/3} d^2 x^2}{((1 + \sqrt{3}) c + 2^{2/3} d x)^2}} \right. \\
& \quad \left. \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) c + 2^{2/3} d x}{(1 + \sqrt{3}) c + 2^{2/3} d x} \right], -7 - 4 \sqrt{3}] \right) / \\
& \quad \left(3 \times 3^{1/4} c d^2 \sqrt{\frac{c (c + 2^{2/3} d x)}{((1 + \sqrt{3}) c + 2^{2/3} d x)^2}} \sqrt{c^3 + 4 d^3 x^3} \right)
\end{aligned}$$

Result (type 4, 380 leaves):

$$\begin{aligned}
& \left(2^{1/6} \sqrt{\frac{2^{1/3} c + 2 d x}{(1 + (-1)^{1/3}) c}} \right. \\
& \left. - f \sqrt{\frac{(-2)^{1/3} c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \left((-1)^{1/3} (2 + (-2)^{1/3}) c - 2 ((-1)^{1/3} + 2^{2/3}) d x \right) \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}}\right], (-1)^{1/3}] + \frac{1}{\sqrt{3}} \right. \\
& \left. (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) (-d e + c f) \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}{c^2}} \right. \\
& \left. \text{EllipticPi}\left[\frac{\pm 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}}\right], (-1)^{1/3}\right] \right) / \\
& \left((2 + (-2)^{1/3}) d^2 \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 + 4 d^3 x^3} \right)
\end{aligned}$$

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 \times 2^{2/3} \text{ArcTan}\left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{1+x^3}}\right]}{3 \sqrt{3}} + \\
& \left(2 \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4 \sqrt{3}] \right) / \\
& \left(3 \times 3^{1/4} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3} \right)
\end{aligned}$$

Result (type 4, 207 leaves):

$$\frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} \right.$$

$$\left((-1)^{1/3}-x \right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3}x}{1+(-1)^{1/3}}} \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] +$$

$$\left. \frac{1}{(-1)^{1/3}+2^{2/3}} i 2^{2/3} \sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3}+2^{2/3}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] \right)$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3}-x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 160 leaves, 4 steps) :

$$\begin{aligned} & -\frac{2 \times 2^{2/3} \text{ArcTan}\left[\frac{\sqrt{3} (1-2^{1/3}x)}{\sqrt{1-x^3}}\right]}{3 \sqrt{3}} + \\ & \left(2 \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}] \right) / \\ & \left(3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right) \end{aligned}$$

Result (type 4, 209 leaves) :

$$\frac{1}{\sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} \right.$$

$$\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] +$$

$$\left. \frac{1}{(-1)^{1/3} + 2^{2/3}} i 2^{2/3} \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right)$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3}-x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 163 leaves, 4 steps) :

$$\begin{aligned} & \frac{2 \times 2^{2/3} \text{ArcTanh}\left[\frac{\sqrt{3} (1-2^{1/3}x)}{\sqrt{-1+x^3}}\right]}{3 \sqrt{3}} + \\ & \left(2 \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}] \right) / \\ & \left(3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right) \end{aligned}$$

Result (type 4, 207 leaves) :

$$\frac{1}{\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} \right.$$

$$\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] +$$

$$\left. \frac{1}{(-1)^{1/3} + 2^{2/3}} i 2^{2/3} \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right)$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps) :

$$\begin{aligned} & \frac{2 \times 2^{2/3} \text{ArcTanh}\left[\frac{\sqrt{3} (1+2^{1/3}x)}{\sqrt{-1-x^3}}\right]}{3 \sqrt{3}} + \\ & \left(2 \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}] \right) / \\ & \left(3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right) \end{aligned}$$

Result (type 4, 209 leaves) :

$$\begin{aligned}
& \frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} \right. \\
& \left((-1)^{1/3}-x \right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] + \\
& \left. \frac{1}{(-1)^{1/3}+2^{2/3}} i 2^{2/3} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3}+2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] \right)
\end{aligned}$$

Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a+b x^3}} dx$$

Optimal (type 4, 275 leaves, 4 steps) :

$$\begin{aligned}
& \frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{a+b x^3}} \right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \\
& \left(2 \sqrt{2+\sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3} + b^{1/3} x}{(1+\sqrt{3}) a^{1/3} + b^{1/3} x} \right], \right. \\
& \left. -7 - 4\sqrt{3} \right] \Bigg) \Bigg/ \left(3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1+\sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a+b x^3} \right)
\end{aligned}$$

Result (type 4, 324 leaves) :

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. - \left(3^{1/4} ((-1)^{1/3} a^{1/3} - b^{1/3} x) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], \right. \right. \\
& \left. \left. (-1)^{1/3}\right] \right) \Bigg/ \left(\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) + \frac{1}{(-1)^{1/3} + 2^{2/3}} \\
& (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right) \Bigg/ \left(\sqrt{3} b^{2/3} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 283 leaves, 4 steps) :

$$\begin{aligned}
& \frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}}\right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \\
& \left(2 \sqrt{2 + \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}\right], \right. \\
& \left. \left. - 7 - 4 \sqrt{3}\right] \right) \Bigg/ \left(3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Result (type 4, 388 leaves) :

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \right. \\
& \quad \left. \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] - \right. \\
& \quad \left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \quad \left. \operatorname{EllipticPi}\left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \\
& \quad \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 292 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}} \right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \\
& \quad \left(2 \sqrt{2 - \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], \right. \\
& \quad \left. - 7 + 4 \sqrt{3} \right] \right) / \left(3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Result (type 4, 389 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \right. \\
& \quad \left. \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] - \right. \\
& \quad \left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \quad \left. \operatorname{EllipticPi}\left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \\
& \quad \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 288 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \\
& \left(2 \sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], \right. \\
& \quad \left. - 7 + 4 \sqrt{3} \right] \right) / \left(3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Result (type 4, 375 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{3^{1/4}} \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \right. \right. \\
& \left. \left. \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] + \right. \\
& \left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \\
& \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 4, 246 leaves, 4 steps) :

$$\begin{aligned}
& -\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} \sqrt{c} (c+2 d x)}{\sqrt{c^3+4 d^3 x^3}} \right]}{3 \sqrt{3} \sqrt{c} d^2} + \left(2^{1/3} \sqrt{2+\sqrt{3}} (c+2^{2/3} d x) \right. \\
& \left. \sqrt{\frac{c^2 - 2^{2/3} c d x + 2 \times 2^{1/3} d^2 x^2}{\left((1+\sqrt{3}) c + 2^{2/3} d x \right)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) c + 2^{2/3} d x}{(1+\sqrt{3}) c + 2^{2/3} d x} \right], -7 - 4 \sqrt{3}] \right) / \\
& \left(3 \times 3^{1/4} d^2 \sqrt{\frac{c (c+2^{2/3} d x)}{\left((1+\sqrt{3}) c + 2^{2/3} d x \right)^2}} \sqrt{c^3 + 4 d^3 x^3} \right)
\end{aligned}$$

Result (type 4, 372 leaves) :

$$\begin{aligned}
& \left(2^{1/6} \sqrt{\frac{2^{1/3} c + 2 d x}{(1 + (-1)^{1/3}) c}} \right. \\
& \left. - \sqrt{\frac{(-2)^{1/3} c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \left((-1)^{1/3} (2 + (-2)^{1/3}) c - 2 ((-1)^{1/3} + 2^{2/3}) d x \right) \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}}\right], (-1)^{1/3}] + \frac{1}{\sqrt{3}} \right. \\
& \left. (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) c \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}{c^2}} \right. \\
& \left. \text{EllipticPi}\left[\frac{\pm 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}}\right], (-1)^{1/3}]\right] \right) / \\
& \left((2 + (-2)^{1/3}) d^2 \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 + 4 d^3 x^3} \right)
\end{aligned}$$

Problem 51: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+x}{(2-x) \sqrt{1+x^3}} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{2}{3} \text{ArcTanh}\left[\frac{(1+x)^2}{3 \sqrt{1+x^3}}\right]$$

Result (type 4, 265 leaves):

$$\begin{aligned} & \left(2\sqrt{6} \sqrt{-\frac{\frac{i}{2}(1+x)}{-3\frac{i}{2}+\sqrt{3}}} \left(-\frac{i}{2}\sqrt{\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x} \left(-\frac{i}{2}-\sqrt{3} + \left(-\frac{i}{2}+\sqrt{3} \right) x \right) \right. \right. \\ & \quad \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}+2\frac{i}{2}x}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3\frac{i}{2}+\sqrt{3}} \right] + 2\sqrt{3} \sqrt{-\frac{i}{2}+\sqrt{3}+2\frac{i}{2}x} \\ & \quad \left. \sqrt{1-x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}+2\frac{i}{2}x}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3\frac{i}{2}+\sqrt{3}} \right] \right) \Bigg) / \\ & \quad \left(\left(3\frac{i}{2}+\sqrt{3} \right) \sqrt{-\frac{i}{2}+\sqrt{3}+2\frac{i}{2}x} \sqrt{1+x^3} \right) \end{aligned}$$

Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}} dx$$

Optimal (type 3, 27 leaves, 2 steps):

$$-\frac{2}{3} \text{ArcTanh} \left[\frac{(1-x)^2}{3\sqrt{1-x^3}} \right]$$

Result (type 4, 262 leaves):

$$\begin{aligned} & - \left(\left(2\sqrt{6} \sqrt{-\frac{\frac{i}{2}(-1+x)}{-3\frac{i}{2}+\sqrt{3}}} \left(\sqrt{\frac{i}{2}+\sqrt{3}+2\frac{i}{2}x} \left(-1+\frac{i}{2}\sqrt{3}+x+\frac{i}{2}\sqrt{3}x \right) \right. \right. \right. \\ & \quad \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3\frac{i}{2}+\sqrt{3}} \right] + 2\sqrt{3} \sqrt{-\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x} \\ & \quad \left. \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3\frac{i}{2}+\sqrt{3}} \right] \right) \Bigg) / \\ & \quad \left(\left(3\frac{i}{2}+\sqrt{3} \right) \sqrt{-\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x} \sqrt{1-x^3} \right) \end{aligned}$$

Problem 53: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$-\frac{2}{3} \operatorname{ArcTan}\left[\frac{(1-x)^2}{3 \sqrt{-1+x^3}}\right]$$

Result (type 4, 260 leaves):

$$\begin{aligned} & -\left(\left(2 \sqrt{6} \sqrt{\frac{\frac{i}{2}(-1+x)}{-3 \frac{i}{2} + \sqrt{3}}} \left(\sqrt{\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2}x} (-1 + \frac{i}{2}\sqrt{3} + x + \frac{i}{2}\sqrt{3}x) \right. \right. \right. \\ & \quad \left. \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2}x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2} + \sqrt{3}}\right] + 2 \sqrt{3} \sqrt{-\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2}x} \right. \right. \\ & \quad \left. \left. \left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2}x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2} + \sqrt{3}}\right]\right)\right) \right. \\ & \quad \left. \left(\left(3 \frac{i}{2} + \sqrt{3} \right) \sqrt{-\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2}x} \sqrt{-1+x^3} \right) \right) \end{aligned}$$

Problem 54: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+x}{(2-x) \sqrt{-1-x^3}} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\frac{2}{3} \operatorname{ArcTan}\left[\frac{(1+x)^2}{3 \sqrt{-1-x^3}}\right]$$

Result (type 4, 267 leaves):

$$\begin{aligned} & \left(2 \sqrt{6} \sqrt{-\frac{\frac{i}{2}(1+x)}{-3 \frac{i}{2} + \sqrt{3}}} \left(-\frac{i}{2} \sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2}x} (-\frac{i}{2} - \sqrt{3} + (-\frac{i}{2} + \sqrt{3})x) \right. \right. \\ & \quad \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2}x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2} + \sqrt{3}}\right] + 2 \sqrt{3} \sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2}x} \right. \right. \\ & \quad \left. \left. \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2}x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2} + \sqrt{3}}\right]\right)\right) \right. \\ & \quad \left. \left(\left(3 \frac{i}{2} + \sqrt{3} \right) \sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2}x} \sqrt{-1+x^3} \right) \right) \end{aligned}$$

Problem 55: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} + b^{1/3} x}{(2 a^{1/3} - b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 50 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{a + b x^3}} \right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 407 leaves):

$$\begin{aligned} & \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2 \sqrt{2}} 3^{1/4} \left((\frac{i}{2} + \sqrt{3}) a^{1/3} - (-\frac{i}{2} + \sqrt{3}) b^{1/3} x \right) \sqrt{\frac{\frac{i}{2} + \sqrt{3} - \frac{2 i b^{1/3} x}{a^{1/3}}}{a^{1/3}}} \right. \right. \\ & \quad \left. \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{-2 \frac{i}{2} a^{1/3} + (\frac{i}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{i}{2} \sqrt{3}) \right] \right] + \right. \\ & \quad \left. 3 \frac{i}{2} a^{1/3} \sqrt{\frac{-2 \frac{i}{2} a^{1/3} + (\frac{i}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\ & \quad \left. \left. \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{-2 \frac{i}{2} a^{1/3} + (\frac{i}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{i}{2} \sqrt{3}) \right] \right] \right) / \\ & \quad \left((-2 + (-1)^{1/3}) b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right) \end{aligned}$$

Problem 56: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} - b^{1/3} x}{(2 a^{1/3} + b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 52 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{a - b x^3}} \right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 370 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left(\left(-2 + (-1)^{1/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \right. \right. \\
& \quad \left. \left. \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] + \right. \\
& \quad \left. \left((-1)^{1/3} \sqrt{3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) / \\
& \quad \left(\left(-2 + (-1)^{1/3} \right) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 57: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} - b^{1/3} x}{(2 a^{1/3} + b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 53 leaves, 2 steps):

$$\begin{aligned}
& - \frac{2 \operatorname{ArcTan}\left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a + b x^3}} \right]}{3 a^{1/6} b^{1/3}}
\end{aligned}$$

Result (type 4, 371 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left((-2 + (-1)^{1/3}) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \right. \right. \\
& \quad \left. \left. \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] + \right. \\
& \quad \left. \left. (-1)^{1/3} \sqrt{3} \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right] \right) / \\
& \quad \left((-2 + (-1)^{1/3}) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Problem 58: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} + b^{1/3} x}{(2 a^{1/3} - b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 53 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a - b x^3}} \right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 410 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} 3^{1/4} \left((\frac{i}{2} + \sqrt{3}) a^{1/3} - (-\frac{i}{2} + \sqrt{3}) b^{1/3} x \right) \sqrt{\frac{i}{2} + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \right. \right. \\
& \quad \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{-2i a^{1/3} + (\frac{i}{2} + \sqrt{3}) b^{1/3} x}{(-3\frac{i}{2} + \sqrt{3}) a^{1/3}} \right], \frac{1}{2} (1 + \frac{i}{2} \sqrt{3}) \right] + \right. \right. \\
& \quad \left. \left. 3\frac{i}{2} a^{1/3} \sqrt{\frac{-2i a^{1/3} + (\frac{i}{2} + \sqrt{3}) b^{1/3} x}{(-3\frac{i}{2} + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \right. \\
& \quad \left. \left. \text{EllipticPi} \left[\frac{2\sqrt{3}}{3\frac{i}{2} + \sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (\frac{i}{2} + \sqrt{3}) b^{1/3} x}{(-3\frac{i}{2} + \sqrt{3}) a^{1/3}}}, \frac{1}{2} (1 + \frac{i}{2} \sqrt{3}) \right] \right] \right) \right) / \\
& \quad \left((-2 + (-1)^{1/3}) b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 59: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{c - 2 dx}{(c + d x) \sqrt{c^3 - 8 d^3 x^3}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTanh} \left[\frac{(c-2d x)^2}{3\sqrt{c} \sqrt{c^3-8d^3x^3}} \right]}{3\sqrt{c} d}$$

Result (type 4, 295 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{\frac{c - 2 dx}{(1 + (-1)^{1/3}) c}} \right. \right. \\
& \left. \left. \left((-2 + (-1)^{1/3}) ((-1)^{1/3} c + 2 dx) \sqrt{\frac{(-1)^{1/3} (c + 2 (-1)^{1/3} dx)}{(1 + (-1)^{1/3}) c}} \operatorname{EllipticF}[\right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}}\right], (-1)^{1/3}\right] + (-1)^{1/3} \sqrt{3} (1 + (-1)^{1/3}) c \sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \right. \right. \\
& \left. \left. \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}}\right], (-1)^{1/3}\right]\right) \right) \\
& \left. \left((-2 + (-1)^{1/3}) d \sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 - 8 d^3 x^3}\right)\right)
\end{aligned}$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 - x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 139 leaves, 4 steps):

$$\begin{aligned}
& \frac{2}{9} (e + 2 f) \operatorname{ArcTanh}\left[\frac{(1+x)^2}{3 \sqrt{1+x^3}}\right] + \\
& \left(2 \sqrt{2 + \sqrt{3}} (e - f) (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4 \sqrt{3}\right] \right) / \\
& \left(3 \times 3^{1/4} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3} \right)
\end{aligned}$$

Result (type 4, 273 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{\frac{i(1+x)}{-3i+\sqrt{3}}}{-\frac{3i}{-3i+\sqrt{3}}}} \left(-3i f \sqrt{\frac{i+\sqrt{3}-2ix}{\sqrt{2}3^{1/4}}} (-i-\sqrt{3} + (-i+\sqrt{3})x) \right. \right. \\ \left. \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}] + 2\sqrt{3} (e+2f) \sqrt{\frac{-i+\sqrt{3}+2ix}{\sqrt{2}3^{1/4}}} \right. \right. \\ \left. \left. \sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] \right) \right) / \\ \left((3i+\sqrt{3}) \sqrt{-i+\sqrt{3}+2ix} \sqrt{1+x^3} \right)$$

Problem 61: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{(2+x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 153 leaves, 4 steps):

$$\left(-\frac{2}{9} (e-2f) \text{ArcTanh}\left[\frac{(1-x)^2}{3\sqrt{1-x^3}}\right] - \right. \\ \left. \left(2\sqrt{2+\sqrt{3}} (e+f) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}] \right) \right) / \\ \left(3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right)$$

Result (type 4, 271 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{\frac{i(-1+x)}{-3i+\sqrt{3}}}{-\frac{3i}{-3i+\sqrt{3}}}} \left(3f \sqrt{\frac{i+\sqrt{3}+2ix}{\sqrt{2}3^{1/4}}} (-1+i\sqrt{3}+x+i\sqrt{3}x) \right. \right. \\ \left. \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}] - 2\sqrt{3} (e-2f) \sqrt{\frac{-i+\sqrt{3}-2ix}{\sqrt{2}3^{1/4}}} \right. \right. \\ \left. \left. \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-3i+\sqrt{3}}\right] \right) \right) / \\ \left((3i+\sqrt{3}) \sqrt{-i+\sqrt{3}-2ix} \sqrt{1-x^3} \right)$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 + x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps) :

$$\begin{aligned} & -\frac{2}{9} (e - 2f) \operatorname{ArcTan}\left[\frac{(1-x)^2}{3 \sqrt{-1+x^3}}\right] - \\ & \left(2 \sqrt{2-\sqrt{3}} (e+f) (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]\right) / \\ & \left(3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}\right) \end{aligned}$$

Result (type 4, 269 leaves) :

$$\begin{aligned} & \left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{\frac{i}{2}(-1+x)}{-3 \frac{i}{2}+\sqrt{3}}} \left(3 f \sqrt{\frac{i}{2}+\sqrt{3}+2 \frac{i}{2} x} (-1+\frac{i}{2} \sqrt{3}+x+\frac{i}{2} \sqrt{3} x)\right.\right. \\ & \left.\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}}-2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2}+\sqrt{3}}\right]-2 \sqrt{3} (e-2 f) \sqrt{-\frac{i}{2}+\sqrt{3}-2 \frac{i}{2} x}\right.\right. \\ & \left.\left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}}-2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2}+\sqrt{3}}\right]\right)\right) / \\ & \left(\left(3 \frac{i}{2}+\sqrt{3}\right) \sqrt{-\frac{i}{2}+\sqrt{3}-2 \frac{i}{2} x} \sqrt{-1+x^3}\right) \end{aligned}$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 - x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 150 leaves, 4 steps) :

$$\frac{2}{9} (e + 2 f) \operatorname{ArcTan}\left[\frac{(1+x)^2}{3 \sqrt{-1-x^3}}\right] +$$

$$\left(2 \sqrt{2-\sqrt{3}} (e-f) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]\right)/$$

$$\left(3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}\right)$$

Result (type 4, 275 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{-\frac{\frac{i}{2} (1+x)}{-3 \frac{i}{2}+\sqrt{3}}} \left(-3 \frac{i}{2} f \sqrt{\frac{i}{2}+\sqrt{3}-2 \frac{i}{2} x} \left(-\frac{i}{2}-\sqrt{3}+\left(-\frac{i}{2}+\sqrt{3}\right) x\right)\right.\right.$$

$$\left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}+2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2}+\sqrt{3}}\right]+2 \sqrt{3} (e+2 f) \sqrt{-\frac{i}{2}+\sqrt{3}+2 \frac{i}{2} x}\right.$$

$$\left.\left.\sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}+2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2}+\sqrt{3}}\right]\right)\right)/$$

$$\left(\left(3 \frac{i}{2}+\sqrt{3}\right) \sqrt{-\frac{i}{2}+\sqrt{3}+2 \frac{i}{2} x} \sqrt{-1-x^3}\right)$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{(2 a^{1/3}-b^{1/3} x) \sqrt{a+b x^3}} dx$$

Optimal (type 4, 297 leaves, 4 steps):

$$\frac{2 \left(b^{1/3} e+2 a^{1/3} f\right) \operatorname{ArcTanh}\left[\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{3 a^{1/6} \sqrt{a+b x^3}}\right]}{9 \sqrt{a} b^{2/3}} +$$

$$\left(2 \sqrt{2+\sqrt{3}} \left(b^{1/3} e-a^{1/3} f\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right)\right/$$

$$\left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 419 leaves) :

$$\begin{aligned}
 & \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} 3^{1/4} f \left((\pm i + \sqrt{3}) a^{1/3} - (-\pm i + \sqrt{3}) b^{1/3} x \right) \sqrt{\frac{\pm i + \sqrt{3} - \frac{2\pm i b^{1/3} x}{a^{1/3}}}{a^{1/3}}} \right. \right. \\
 & \quad \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2\pm i a^{1/3} + (\pm i + \sqrt{3}) b^{1/3} x}{(-3\pm i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \pm i \sqrt{3}) \right] + \right. \right. \\
 & \quad \left. \left. \pm i (b^{1/3} e + 2 a^{1/3} f) \sqrt{\frac{-2\pm i a^{1/3} + (\pm i + \sqrt{3}) b^{1/3} x}{(-3\pm i + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \right. \\
 & \quad \left. \left. \text{EllipticPi} \left[\frac{2\sqrt{3}}{3\pm i + \sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2\pm i a^{1/3} + (\pm i + \sqrt{3}) b^{1/3} x}{(-3\pm i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \pm i \sqrt{3}) \right] \right) \right) / \\
 & \quad \left((-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right)
 \end{aligned}$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 a^{1/3} + b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 304 leaves, 4 steps) :

$$\begin{aligned}
 & - \frac{2 (b^{1/3} e - 2 a^{1/3} f) \text{ArcTanh} \left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{a - b x^3}} \right]}{9 \sqrt{a} b^{2/3}} - \\
 & \quad \left(2 \sqrt{2 + \sqrt{3}} (b^{1/3} e + a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \right. \\
 & \quad \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 - 4\sqrt{3} \right] \right) / \\
 & \quad \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3} \right)
 \end{aligned}$$

Result (type 4, 447 leaves) :

$$\begin{aligned}
& \frac{1}{\left(-2 + (-1)^{1/3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \sqrt{a - b x^3}}} 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \\
& \left(-\frac{1}{2} \text{i} f \sqrt{\frac{(-\text{i} + \sqrt{3}) a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3 \text{i} + \sqrt{3}) a^{1/3}}} \left((-3 \text{i} + \sqrt{3}) a^{1/3} - (3 \text{i} + \sqrt{3}) b^{1/3} x \right) \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{-\frac{\text{i} (2 a^{1/3} + (1 - \text{i} \sqrt{3}) b^{1/3} x)}{(-3 \text{i} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \text{i} \sqrt{3})] - \\
& \text{i} (b^{1/3} e - 2 a^{1/3} f) \sqrt{-\frac{\text{i} (2 a^{1/3} + (1 - \text{i} \sqrt{3}) b^{1/3} x)}{(-3 \text{i} + \sqrt{3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \\
& \left. \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \text{i} + \sqrt{3}}, \text{ArcSin}\left[\sqrt{-\frac{\text{i} (2 a^{1/3} + (1 - \text{i} \sqrt{3}) b^{1/3} x)}{(-3 \text{i} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \text{i} \sqrt{3}) \right] \right)
\end{aligned}$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 a^{1/3} + b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 313 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (b^{1/3} e - 2 a^{1/3} f) \text{ArcTan}\left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a + b x^3}} \right]}{9 \sqrt{a} b^{2/3}} - \\
& \left(2 \sqrt{2 - \sqrt{3}} (b^{1/3} e + a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3}] \right) / \\
& \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2} \sqrt{-a + b x^3}} \right)
\end{aligned}$$

Result (type 4, 448 leaves):

$$\begin{aligned}
& \frac{1}{\left(-2 + (-1)^{1/3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \sqrt{-a + b x^3}}} 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \\
& \left(-\frac{1}{2} \operatorname{EllipticF}\left[\sqrt{\frac{(-\frac{i}{2} + \sqrt{3}) a^{1/3} + (\frac{i}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \right] \left((-3 \frac{i}{2} + \sqrt{3}) a^{1/3} - (3 \frac{i}{2} + \sqrt{3}) b^{1/3} x \right) \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} (2 a^{1/3} + (1 - \frac{i}{2} \sqrt{3}) b^{1/3} x)}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{i}{2} \sqrt{3}) \right] - \right. \\
& \left. \frac{i}{2} (b^{1/3} e - 2 a^{1/3} f) \sqrt{-\frac{\frac{i}{2} (2 a^{1/3} + (1 - \frac{i}{2} \sqrt{3}) b^{1/3} x)}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} (2 a^{1/3} + (1 - \frac{i}{2} \sqrt{3}) b^{1/3} x)}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{i}{2} \sqrt{3}) \right] \right)
\end{aligned}$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 a^{1/3} - b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 310 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (b^{1/3} e + 2 a^{1/3} f) \operatorname{ArcTan}\left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a - b x^3}} \right]}{9 \sqrt{a} b^{2/3}} + \\
& \left(2 \sqrt{2 - \sqrt{3}} (b^{1/3} e - a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2} \sqrt{-a - b x^3}} \right)
\end{aligned}$$

Result (type 4, 422 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} 3^{1/4} f \left((\pm i + \sqrt{3}) a^{1/3} - (-\pm i + \sqrt{3}) b^{1/3} x \right) \sqrt{\pm i + \sqrt{3} - \frac{2 \pm b^{1/3} x}{a^{1/3}}} \right. \right. \\
& \quad \left. \left. \text{EllipticF}[\text{ArcSin}\left[\frac{-2 \pm a^{1/3} + (\pm i + \sqrt{3}) b^{1/3} x}{(-3 \pm i + \sqrt{3}) a^{1/3}} \right], \frac{1}{2} (1 + \pm i \sqrt{3})] + \right. \\
& \quad \left. \pm (b^{1/3} e + 2 a^{1/3} f) \sqrt{\frac{-2 \pm a^{1/3} + (\pm i + \sqrt{3}) b^{1/3} x}{(-3 \pm i + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \quad \left. \text{EllipticPi}\left[\frac{2\sqrt{3}}{3 \pm i + \sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{-2 \pm a^{1/3} + (\pm i + \sqrt{3}) b^{1/3} x}{(-3 \pm i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \pm i \sqrt{3}) \right] \right) / \\
& \quad \left((-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{c^3 - 8 d^3 x^3}} dx$$

Optimal (type 4, 221 leaves, 4 steps):

$$\begin{aligned}
& -\frac{2 (d e - c f) \text{ArcTanh}\left[\frac{(c-2 d x)^2}{3 \sqrt{c} \sqrt{c^3-8 d^3 x^3}}\right]}{9 c^{3/2} d^2} - \\
& \left(\sqrt{2+\sqrt{3}} (2 d e + c f) (c - 2 d x) \sqrt{\frac{c^2 + 2 c d x + 4 d^2 x^2}{((1 + \sqrt{3}) c - 2 d x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) c - 2 d x}{(1 + \sqrt{3}) c - 2 d x}\right], \right. \\
& \quad \left. -7 - 4 \sqrt{3}\right] \Bigg) / \left(3 \times 3^{1/4} c d^2 \sqrt{\frac{c (c - 2 d x)}{((1 + \sqrt{3}) c - 2 d x)^2}} \sqrt{c^3 - 8 d^3 x^3} \right)
\end{aligned}$$

Result (type 4, 384 leaves):

$$\begin{aligned}
& - \left(\left(\frac{\sqrt{\frac{c - 2 dx}{(1 + (-1)^{1/3}) c}}}{\sqrt{\frac{(-\frac{1}{2} + \sqrt{3}) c + 2 (\frac{1}{2} + \sqrt{3}) dx}{(-3 \frac{1}{2} + \sqrt{3}) c}}} \right) \left(\frac{\left(-3 \frac{1}{2} + \sqrt{3} \right) c - 2 (3 \frac{1}{2} + \sqrt{3}) dx}{\left(-3 \frac{1}{2} + \sqrt{3} \right) c} \right) \right. \\
& \quad \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{2} \sqrt{\frac{\frac{1}{2} c + \frac{1}{2} d x + \sqrt{3} d x}{3 \frac{1}{2} c - \sqrt{3} c}}\right], \frac{1}{2} (1 + \frac{1}{2} \sqrt{3})] + \right. \\
& \quad \left. 4 \sqrt{2} (d e - c f) \sqrt{\frac{\frac{1}{2} c + \frac{1}{2} d x + \sqrt{3} d x}{3 \frac{1}{2} c - \sqrt{3} c}} \sqrt{\frac{c^2 + 2 c d x + 4 d^2 x^2}{c^2}} \right. \\
& \quad \left. \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}, \text{ArcSin}\left[\sqrt{2} \sqrt{\frac{\frac{1}{2} c + \frac{1}{2} d x + \sqrt{3} d x}{3 \frac{1}{2} c - \sqrt{3} c}}\right], \frac{1}{2} (1 + \frac{1}{2} \sqrt{3})\right]\right) / \\
& \quad \left(2 \left(-2 + (-1)^{1/3} \right) d^2 \sqrt{\frac{c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 - 8 d^3 x^3} \right)
\end{aligned}$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2-x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 129 leaves, 4 steps) :

$$\begin{aligned}
& \frac{4}{9} \text{ArcTanh}\left[\frac{(1+x)^2}{3 \sqrt{1+x^3}}\right] - \\
& \left(2 \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right] \right) / \\
& \quad \left(3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)
\end{aligned}$$

Result (type 4, 193 leaves) :

$$\begin{aligned} & \frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \\ & \left(\frac{1}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} \left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \right. \\ & \left. \frac{1}{-2+(-1)^{1/3}} 2 \pm \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) \end{aligned}$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2+x)\sqrt{1-x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps) :

$$\begin{aligned} & \frac{4}{9} \operatorname{ArcTanh}\left[\frac{(1-x)^2}{3\sqrt{1-x^3}}\right] - \\ & \left(2\sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right] \right) / \\ & \left(3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right) \end{aligned}$$

Result (type 4, 195 leaves) :

$$\begin{aligned} & \frac{1}{\sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \\ & \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \right. \\ & \left. \frac{1}{-2+(-1)^{1/3}} 2 \pm \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) \end{aligned}$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2+x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 148 leaves, 4 steps):

$$\begin{aligned} & \frac{4}{9} \operatorname{ArcTan}\left[\frac{(1-x)^2}{3 \sqrt{-1+x^3}}\right] - \\ & \left(2 \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]\right) / \\ & \left(3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}\right) \end{aligned}$$

Result (type 4, 193 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \\ & \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}}\left((-1)^{1/3}+x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]+ \right. \\ & \left.\frac{1}{-2+(-1)^{1/3}} 2 i \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]\right) \end{aligned}$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2-x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 140 leaves, 4 steps):

$$\frac{4}{9} \operatorname{ArcTan}\left[\frac{(1+x)^2}{3 \sqrt{-1-x^3}}\right] - \left(2 \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]\right) / \left(3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}\right)$$

Result (type 4, 195 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} - \left(\frac{1}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} ((-1)^{1/3}-x) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \frac{1}{-2+(-1)^{1/3}} 2 \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3\pm\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2a^{1/3} - b^{1/3}x) \sqrt{a+b x^3}} dx$$

Optimal (type 4, 260 leaves, 4 steps):

$$\frac{4 \operatorname{ArcTanh}\left[\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{3 a^{1/6} \sqrt{a+b x^3}}\right]}{9 a^{1/6} b^{2/3}} - \left(2 \sqrt{2+\sqrt{3}} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4\sqrt{3}\right]\right) / \left(3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 407 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left(-\sqrt{2} \cdot 3^{1/4} \left((\text{i} + \sqrt{3}) a^{1/3} - (-\text{i} + \sqrt{3}) b^{1/3} x \right) \sqrt{\frac{\text{i} + \sqrt{3}}{a^{1/3}} - \frac{2 \text{i} b^{1/3} x}{a^{1/3}}} \right. \\
& \quad \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{-2 \text{i} a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3 \text{i} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \text{i} \sqrt{3})] + \right. \\
& \quad \left. 8 \text{i} a^{1/3} \sqrt{\frac{-2 \text{i} a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3 \text{i} + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \quad \left. \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \text{i} + \sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{-2 \text{i} a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3 \text{i} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \text{i} \sqrt{3}) \right] \right) / \\
& \quad \left(2 \left(-2 + (-1)^{1/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2 a^{1/3} + b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 268 leaves, 4 steps):

$$\begin{aligned}
& \frac{4 \text{ArcTanh}\left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{a - b x^3}} \right]}{9 a^{1/6} b^{2/3}} - \\
& \left(2 \sqrt{2 + \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], \right. \\
& \quad \left. -7 - 4 \sqrt{3}] \right) / \left(3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Result (type 4, 371 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\left(-2 + (-1)^{1/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \right. \right. \\
& \left. \left. \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] + \right. \\
& \left. \frac{1}{\sqrt{3}} 2 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \\
& \left((-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2 a^{1/3} + b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 277 leaves, 4 steps) :

$$\begin{aligned}
& \frac{4 \operatorname{ArcTan}\left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a + b x^3}} \right]}{9 a^{1/6} b^{2/3}} - \left(2 \sqrt{2 - \sqrt{3}} (a^{1/3} - b^{1/3} x) \right. \\
& \left. \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3}] \right) / \\
& \left(3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Result (type 4, 372 leaves) :

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) \left(\left(-2 + (-1)^{1/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \right. \\
& \left. \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3}] + \right. \\
& \left. \frac{1}{\sqrt{3}} 2 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \\
& \left((-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 76: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2 a^{1/3} - b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 273 leaves, 4 steps):

$$\begin{aligned}
& \frac{4 \operatorname{ArcTan}\left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a - b x^3}} \right]}{9 a^{1/6} b^{2/3}} - \left(2 \sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \right. \\
& \left. \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3}] \right) / \\
& \left(3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Result (type 4, 410 leaves):

$$\begin{aligned}
& \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\sqrt{2} \cdot 3^{1/4} \left((\text{i} + \sqrt{3}) a^{1/3} - (-\text{i} + \sqrt{3}) b^{1/3} x \right) \sqrt{\frac{\text{i} + \sqrt{3} - \frac{2 \text{i} b^{1/3} x}{a^{1/3}}}{\text{i} + \sqrt{3}}} \right. \\
& \quad \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{-2 \text{i} a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3 \text{i} + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2} (1 + \text{i} \sqrt{3})] + \right. \\
& \quad \left. 8 \text{i} a^{1/3} \sqrt{\frac{-2 \text{i} a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3 \text{i} + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \quad \left. \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \text{i} + \sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{-2 \text{i} a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3 \text{i} + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2} (1 + \text{i} \sqrt{3})\right]\right) / \\
& \quad \left(2 \left(-2 + (-1)^{1/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 77: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(c + d x) \sqrt{c^3 - 8 d^3 x^3}} dx$$

Optimal (type 4, 202 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTanh}\left[\frac{(c-2 d x)^2}{3 \sqrt{c} \sqrt{c^3-8 d^3 x^3}}\right]}{9 \sqrt{c} d^2} - \\
& \left(\sqrt{2+\sqrt{3}} (c-2 d x) \sqrt{\frac{c^2+2 c d x+4 d^2 x^2}{\left((1+\sqrt{3}) c-2 d x\right)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{(1-\sqrt{3}) c-2 d x}{(1+\sqrt{3}) c-2 d x}\right], \right. \\
& \quad \left. -7-4 \sqrt{3}\right) \right) / \left(3 \times 3^{1/4} d^2 \sqrt{\frac{c (c-2 d x)}{\left((1+\sqrt{3}) c-2 d x\right)^2}} \sqrt{c^3-8 d^3 x^3} \right)
\end{aligned}$$

Result (type 4, 295 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{c - 2 dx}{(1 + (-1)^{1/3}) c}} \right) \left((-2 + (-1)^{1/3}) \left((-1)^{1/3} c + 2 dx \right) \right. \\
& \left. \sqrt{\frac{(-1)^{1/3} (c + 2 (-1)^{1/3} dx)}{(1 + (-1)^{1/3}) c}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \right], (-1)^{1/3}] + \right. \\
& \left. \frac{1}{\sqrt{3}} 2 (-1)^{1/3} (1 + (-1)^{1/3}) c \sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \sqrt{\frac{c^2 + 2 c dx + 4 d^2 x^2}{c^2}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i + \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \right], (-1)^{1/3} \right] \right) / \\
& \left((-2 + (-1)^{1/3}) d^2 \sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 - 8 d^3 x^3} \right)
\end{aligned}$$

Problem 78: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{-3+2 \sqrt{3}}}$$

Result (type 4, 267 leaves):

$$\begin{aligned}
& -\left(2 \sqrt{6} \sqrt{\frac{i (1+x)}{3 i + \sqrt{3}}} \left(\sqrt{-i + \sqrt{3} + 2 i x} \left((1 + 2 i) - i \sqrt{3} + ((-2 - i) + \sqrt{3}) x \right) \right. \right. \\
& \left. \left. \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}] + 4 i \sqrt{i + \sqrt{3} - 2 i x} \sqrt{1 - x + x^2} \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{-3 + (2 + i) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \right) / \\
& \left((-3 + (2 + i) \sqrt{3}) \sqrt{i + \sqrt{3} - 2 i x} \sqrt{1 + x^3} \right)
\end{aligned}$$

Problem 79: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} (1-x)}{\sqrt{1-x^3}}\right]}{\sqrt{-3+2 \sqrt{3}}}$$

Result (type 4, 269 leaves):

$$\begin{aligned} & \left(2 \sqrt{6} \sqrt{\frac{\frac{i}{2} (-1+x)}{-3 i+\sqrt{3}}} \left(\sqrt{i+\sqrt{3}+2 i x} \left((2+i)-\sqrt{3}+\left((1+2 i)-i \sqrt{3}\right) x\right)\right.\right. \\ & \quad \left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i+\sqrt{3}}\right]+4 \sqrt{-i+\sqrt{3}-2 i x} \sqrt{1+x+x^2}\right.\right. \\ & \quad \left.\left.\operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 i+(1+2 i) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i+\sqrt{3}}\right]\right)\right)/ \\ & \quad \left.\left.\left((-3 i+(1+2 i) \sqrt{3}) \sqrt{-i+\sqrt{3}-2 i x} \sqrt{1-x^3}\right)\right.\right) \end{aligned}$$

Problem 80: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-3+2 \sqrt{3}} (1-x)}{\sqrt{-1+x^3}}\right]}{\sqrt{-3+2 \sqrt{3}}}$$

Result (type 4, 267 leaves):

$$\begin{aligned}
& \left(2 \sqrt{6} \sqrt{\frac{i (-1+x)}{-3 i + \sqrt{3}}} \left(\sqrt{i + \sqrt{3} + 2 i x} \left((2+i) - \sqrt{3} + ((1+2 i) - i \sqrt{3}) x \right) \right. \right. \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}] + 4 \sqrt{-i + \sqrt{3} - 2 i x} \sqrt{1+x+x^2} \\
& \quad \left. \left. \text{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 i + (1+2 i) \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}\right]\right) \right) / \\
& \quad \left((-3 i + (1+2 i) \sqrt{3}) \sqrt{-i + \sqrt{3} - 2 i x} \sqrt{-1+x^3} \right)
\end{aligned}$$

Problem 81: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$-\frac{2 \text{ArcTan}\left[\frac{\sqrt{-3+2 \sqrt{3}} (1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{-3+2 \sqrt{3}}}$$

Result (type 4, 269 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{6} \sqrt{\frac{i (1+x)}{3 i + \sqrt{3}}} \left(\sqrt{-i + \sqrt{3} + 2 i x} \left((1+2 i) - i \sqrt{3} + ((-2-i) + \sqrt{3}) x \right) \right. \right. \right. \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}] + 4 i \sqrt{i + \sqrt{3} - 2 i x} \sqrt{1-x+x^2} \\
& \quad \left. \left. \left. \text{EllipticPi}\left[\frac{2 i \sqrt{3}}{-3 + (2+i) \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}\right]\right) \right) / \\
& \quad \left((-3 + (2+i) \sqrt{3}) \sqrt{i + \sqrt{3} - 2 i x} \sqrt{-1-x^3} \right)
\end{aligned}$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 69 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} \sqrt[6]{a^{1/3}+b^{1/3} x}}{\sqrt{a+b x^3}}\right]}{\sqrt{-3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 322 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{a+b x^3}} 2 \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}} \\ & \left(-\left(\left(\left(-1\right)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\left(-1\right)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\right],\right. \\ & \left.\left.\left(-1\right)^{1/3}\right]\right) \Big/ \left(3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\right)+ \\ & \left(4\left(-1\right)^{5/6}\left(1+\left(-1\right)^{1/3}\right) a^{1/3} \sqrt{1-\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}}\right) \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 i+\left(1+2 i\right) \sqrt{3}},\right. \\ & \left.\left.\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\right],\left(-1\right)^{1/3}\right]\right) \Big/ \left(\left(-3 i+\left(1+2 i\right) \sqrt{3}\right) b^{1/3}\right) \end{aligned}$$

Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x}{\left(\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x\right) \sqrt{a-b x^3}} dx$$

Optimal (type 3, 71 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} \sqrt[6]{a^{1/3}-b^{1/3} x}}{\sqrt{a-b x^3}}\right]}{\sqrt{-3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 446 leaves):

$$\begin{aligned}
& \frac{1}{\left(-3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}\right) b^{1/3}} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1+(-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \\
& \left(\sqrt{\frac{\left(-\frac{i}{2} + \sqrt{3}\right) a^{1/3} + \left(\frac{i}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{i}{2} + \sqrt{3}\right) a^{1/3}}} \left(\left(-3 + (2 + \frac{i}{2}) \sqrt{3}\right) a^{1/3} + \left(-3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}\right) b^{1/3} x \right) \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3} + (1 - \frac{i}{2} \sqrt{3}) b^{1/3} x\right)}{\left(-3 \frac{i}{2} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{i}{2} \sqrt{3}\right)] + \\
& 4 \sqrt{3} a^{1/3} \sqrt{-\frac{2 \frac{i}{2} a^{1/3} + \left(\frac{i}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{i}{2} + \sqrt{3}\right) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \\
& \left. \text{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}}, \text{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3} + (1 - \frac{i}{2} \sqrt{3}) b^{1/3} x\right)}{\left(-3 \frac{i}{2} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{i}{2} \sqrt{3}\right) \right] \right)
\end{aligned}$$

Problem 84: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x\right) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} \left(a^{1/3}-b^{1/3} x\right)}{\sqrt{-a+b x^3}}\right]}{\sqrt{-3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 447 leaves):

$$\begin{aligned}
& \frac{1}{\left(-3 \frac{1}{x} + (1+2 \frac{1}{x}) \sqrt{3}\right) b^{1/3}} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1+(-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \\
& \left(\sqrt{\frac{\left(-\frac{1}{x} + \sqrt{3}\right) a^{1/3} + \left(\frac{1}{x} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{1}{x} + \sqrt{3}\right) a^{1/3}}} \left(\left(-3 + (2+\frac{1}{x}) \sqrt{3}\right) a^{1/3} + \left(-3 \frac{1}{x} + (1+2 \frac{1}{x}) \sqrt{3}\right) b^{1/3} x \right) \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{-\frac{\frac{1}{x} \left(2 a^{1/3} + (1-\frac{1}{x} \sqrt{3}) b^{1/3} x\right)}{\left(-3 \frac{1}{x} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{x} \sqrt{3}\right)] + \\
& 4 \sqrt{3} a^{1/3} \sqrt{-\frac{2 \frac{1}{x} a^{1/3} + \left(\frac{1}{x} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{1}{x} + \sqrt{3}\right) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \\
& \left. \text{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 \frac{1}{x} + (1+2 \frac{1}{x}) \sqrt{3}}, \text{ArcSin}\left[\sqrt{-\frac{\frac{1}{x} \left(2 a^{1/3} + (1-\frac{1}{x} \sqrt{3}) b^{1/3} x\right)}{\left(-3 \frac{1}{x} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{x} \sqrt{3}\right) \right] \right)
\end{aligned}$$

Problem 85: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1+\sqrt{3}) a^{1/3} + b^{1/3} x}{\left((1-\sqrt{3}) a^{1/3} + b^{1/3} x\right) \sqrt{-a-b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$-\frac{2 \text{ArcTan}\left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} (a^{1/3}+b^{1/3} x)}{\sqrt{-a-b x^3}}\right]}{\sqrt{-3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 325 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{-a - b x^3}} 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \\
& - \left(\left(\left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], \right. \right. \\
& \left. \left. (-1)^{1/3} \right] \right) \Big/ \left(3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) + \right. \\
& \left(4 (-1)^{5/6} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 i + (1 + 2 i) \sqrt{3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \Big/ \left((-3 i + (1 + 2 i) \sqrt{3}) b^{1/3} \right)
\end{aligned}$$

Problem 86: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{\left(1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 73 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(1+\left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{a+b x^3}} \right]}{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}
\end{aligned}$$

Result (type 6, 1527 leaves):

$$\begin{aligned}
& \left(32 \left(26 - 15 \sqrt{3}\right) a^2 x \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) \Big/ \\
& \left(\left(-5 + 3 \sqrt{3}\right) \left(2 \left(-5 + 3 \sqrt{3}\right) a - b x^3\right) \sqrt{a + b x^3} \right. \\
& \left(8 \left(-5 + 3 \sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \\
& 3 b x^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \\
& \left. \left. \left. \left(5 - 3 \sqrt{3}\right) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \Big) -
\end{aligned}$$

$$\begin{aligned}
& \left(32 \sqrt{3} \left(26 - 15 \sqrt{3} \right) a^2 \times \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) / \\
& \left(\left(-5 + 3 \sqrt{3} \right) \left(2 \left(-5 + 3 \sqrt{3} \right) a - b x^3 \right) \sqrt{a + b x^3} \right. \\
& \left. \left(8 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) - \\
& \left(60 \left(26 - 15 \sqrt{3} \right) a^2 \left(\frac{b}{a} \right)^{1/3} x^2 \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) / \\
& \left(\left(-5 + 3 \sqrt{3} \right) \left(2 \left(-5 + 3 \sqrt{3} \right) a - b x^3 \right) \sqrt{a + b x^3} \right. \\
& \left. \left(10 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) + \\
& \left(20 \sqrt{3} \left(26 - 15 \sqrt{3} \right) a^2 \left(\frac{b}{a} \right)^{1/3} x^2 \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) / \\
& \left(\left(-5 + 3 \sqrt{3} \right) \left(2 \left(-5 + 3 \sqrt{3} \right) a - b x^3 \right) \sqrt{a + b x^3} \right. \\
& \left. \left(10 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) - \\
& \left(16 \left(26 - 15 \sqrt{3} \right) a^2 \left(\frac{b}{a} \right)^{2/3} x^3 \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) / \\
& \left(\sqrt{3} \left(-5 + 3 \sqrt{3} \right) \left(2 \left(-5 + 3 \sqrt{3} \right) a - b x^3 \right) \sqrt{a + b x^3} \right. \\
& \left. \left(4 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. b x^3 \left(\text{AppellF1} \left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) -
\end{aligned}$$

$$\begin{aligned} & \left(7 \left(26 - 15\sqrt{3} \right) a b x^4 \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) / \\ & \left(\left(-5 + 3\sqrt{3} \right) \left(2 \left(-5 + 3\sqrt{3} \right) a - b x^3 \right) \sqrt{a + b x^3} \right. \\ & \left(14 \left(-5 + 3\sqrt{3} \right) a \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \\ & 3 b x^3 \left(\text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \\ & \left. \left. \left. \left(5 - 3\sqrt{3} \right) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \end{aligned}$$

Problem 87: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x}{\left(1 - \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 75 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(1 - \left(\frac{b}{a} \right)^{1/3} x \right)}{\sqrt{a-bx^3}} \right]}{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(\frac{b}{a} \right)^{1/3}}$$

Result (type 6, 835 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(-5 + 3 \sqrt{3} \right) \sqrt{a - b x^3} \left(2 \left(-5 + 3 \sqrt{3} \right) a + b x^3 \right)} \\
& \left(26 - 15 \sqrt{3} \right) a x \left(- \left(\left(96 \left(-1 + \sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right. \right. \\
& \left. \left. \left(8 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) + \\
& x \left(- \left(\left(60 \left(-3 + \sqrt{3} \right) a \left(\frac{b}{a} \right)^{1/3} \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right. \right. \\
& \left. \left. \left(a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] - 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{b x^3}{a}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) + \\
& x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a} \right)^{2/3} \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right. \right. \\
& \left. \left. \left(a \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] - b x^3 \left(\text{AppellF1} \left[2, \frac{1}{2}, 2, 3, \frac{b x^3}{a}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[2, \frac{3}{2}, 1, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) + \\
& \left(21 b x \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \Big/ \\
& \left(14 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] - \right. \\
& \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \Big)
\end{aligned}$$

Problem 88: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - \left(\frac{b}{a} \right)^{1/3} x}{\left(1 - \sqrt{3} - \left(\frac{b}{a} \right)^{1/3} x \right) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(1-\left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{-a+b x^3}}\right]}{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 836 leaves):

$$\begin{aligned} & \frac{1}{3 \left(-5+3 \sqrt{3}\right) \sqrt{-a+b x^3} \left(2 \left(-5+3 \sqrt{3}\right) a+b x^3\right)} \\ & \left(26-15 \sqrt{3}\right) a x \left(-\left(\left(96 \left(-1+\sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]\right)\right.\right. \\ & \left.\left.\left(8 \left(-5+3 \sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]-\right.\right. \\ & \left.\left.3 b x^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]+\right.\right. \\ & \left.\left.\left(5-3 \sqrt{3}\right) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]\right)\right)+ \\ & x \left(-\left(\left(60 \left(-3+\sqrt{3}\right) a \left(\frac{b}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]\right)\right.\right. \\ & \left.\left.a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]-3 b x^3 \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{b x^3}{a}, \right.\right.\right. \\ & \left.\left.\left.\frac{b x^3}{10 a-6 \sqrt{3} a}\right]+\left(5-3 \sqrt{3}\right) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]\right)\right)+ \\ & x \left(-\left(\left(16 \sqrt{3} a \left(\frac{b}{a}\right)^{2/3} \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]\right)\right.\right. \\ & \left.\left.a \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]-b x^3 \left(\operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, \frac{b x^3}{a}, \right.\right.\right. \\ & \left.\left.\left.\frac{b x^3}{10 a-6 \sqrt{3} a}\right]+\left(5-3 \sqrt{3}\right) \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]\right)\right)+ \\ & \left(21 b x \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]\right)\right)/ \\ & \left(14 \left(-5+3 \sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]-\right. \\ & \left.3 b x^3 \left(\operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]+\right.\right. \\ & \left.\left.\left(5-3 \sqrt{3}\right) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a-6 \sqrt{3} a}\right]\right)\right)\right)\right)$$

Problem 89: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{\left(1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(1+\left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{-a-b x^3}}\right]}{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 1545 leaves):

$$\begin{aligned} & \left(32 \left(26 - 15 \sqrt{3}\right) a^2 x \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a}\right]\right) / \\ & \left(\left(-5 + 3 \sqrt{3}\right) \sqrt{-a - b x^3} \left(2 \left(-5 + 3 \sqrt{3}\right) a - b x^3\right)\right. \\ & \left(8 \left(-5 + 3 \sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a}\right] + \right. \\ & 3 b x^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a}\right] + \right. \\ & \left.\left.\left(5 - 3 \sqrt{3}\right) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a}\right]\right)\right) - \\ & \left(32 \sqrt{3} \left(26 - 15 \sqrt{3}\right) a^2 x \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a}\right]\right) / \\ & \left(\left(-5 + 3 \sqrt{3}\right) \sqrt{-a - b x^3} \left(2 \left(-5 + 3 \sqrt{3}\right) a - b x^3\right)\right. \\ & \left(8 \left(-5 + 3 \sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a}\right] + \right. \\ & 3 b x^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a}\right] + \right. \\ & \left.\left.\left(5 - 3 \sqrt{3}\right) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a}\right]\right)\right) - \\ & \left(60 \left(26 - 15 \sqrt{3}\right) a^2 \left(\frac{b}{a}\right)^{1/3} x^2 \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a}\right]\right) / \\ & \left(\left(-5 + 3 \sqrt{3}\right) \sqrt{-a - b x^3} \left(2 \left(-5 + 3 \sqrt{3}\right) a - b x^3\right)\right. \\ & \left(10 \left(-5 + 3 \sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a}\right] + \right. \end{aligned}$$

$$\begin{aligned}
& 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \\
& \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \Bigg) + \\
& \left(20 \sqrt{3} \left(26 - 15 \sqrt{3} \right) a^2 \left(\frac{b}{a} \right)^{1/3} x^2 \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) / \\
& \left(\left(-5 + 3 \sqrt{3} \right) \sqrt{-a - b x^3} \left(2 \left(-5 + 3 \sqrt{3} \right) a - b x^3 \right) \right. \\
& \left. \left(10 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right) - \\
& \left(16 \left(26 - 15 \sqrt{3} \right) a^2 \left(\frac{b}{a} \right)^{2/3} x^3 \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) / \\
& \left(\sqrt{3} \left(-5 + 3 \sqrt{3} \right) \sqrt{-a - b x^3} \left(2 \left(-5 + 3 \sqrt{3} \right) a - b x^3 \right) \right. \\
& \left. \left(4 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. b x^3 \left(\text{AppellF1} \left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right) - \\
& \left(7 \left(26 - 15 \sqrt{3} \right) a b x^4 \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) / \\
& \left(\left(-5 + 3 \sqrt{3} \right) \sqrt{-a - b x^3} \left(2 \left(-5 + 3 \sqrt{3} \right) a - b x^3 \right) \right. \\
& \left. \left(14 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right)
\end{aligned}$$

Problem 90: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{3+2 \sqrt{3}}}$$

Result (type 4, 269 leaves):

$$\begin{aligned} & \left(2 \sqrt{6} \sqrt{\frac{\frac{i (1+x)}{3 i + \sqrt{3}}}{3 i + \sqrt{3}}} \left(\sqrt{-i + \sqrt{3} + 2 i x} \left((-2 - i) - \sqrt{3} + ((1 + 2 i) + i \sqrt{3}) x\right)\right.\right. \\ & \quad \left.\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}\right] + 4 \sqrt{i + \sqrt{3} - 2 i x} \sqrt{1 - x + x^2}\right.\right. \\ & \quad \left.\left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i + (1 + 2 i) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}\right]\right)\right) / \\ & \quad \left((3 i + (1 + 2 i) \sqrt{3}) \sqrt{i + \sqrt{3} - 2 i x} \sqrt{1 + x^3}\right) \end{aligned}$$

Problem 91: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{1-x^3}}\right]}{\sqrt{3+2 \sqrt{3}}}$$

Result (type 4, 267 leaves):

$$\left(2 \sqrt{6} \sqrt{\frac{i (-1+x)}{-3 i + \sqrt{3}}} \left(\sqrt{i + \sqrt{3} + 2 i x} \left((1+2 i) + i \sqrt{3} + ((2+i) + \sqrt{3}) x \right) \right. \right.$$

$$\text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}] - 4 i \sqrt{-i + \sqrt{3} - 2 i x}$$

$$\left. \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2 i \sqrt{3}}{3 + (2+i) \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}\right] \right) /$$

$$\left. \left((3 + (2+i) \sqrt{3}) \sqrt{-i + \sqrt{3} - 2 i x} \sqrt{1-x^3} \right) \right)$$

Problem 92: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{1+x^3}}\right]}{\sqrt{3+2 \sqrt{3}}}$$

Result (type 4, 265 leaves):

$$\left(2 \sqrt{6} \sqrt{\frac{i (-1+x)}{-3 i + \sqrt{3}}} \left(\sqrt{i + \sqrt{3} + 2 i x} \left((1+2 i) + i \sqrt{3} + ((2+i) + \sqrt{3}) x \right) \right. \right.$$

$$\text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}] - 4 i \sqrt{-i + \sqrt{3} - 2 i x}$$

$$\left. \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2 i \sqrt{3}}{3 + (2+i) \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}\right] \right) /$$

$$\left. \left((3 + (2+i) \sqrt{3}) \sqrt{-i + \sqrt{3} - 2 i x} \sqrt{-1+x^3} \right) \right)$$

Problem 93: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}}}{\sqrt{-1-x^3}}(1+x)\right]}{\sqrt{3+2 \sqrt{3}}}$$

Result (type 4, 271 leaves) :

$$\begin{aligned} & \left(2 \sqrt{6} \sqrt{\frac{\frac{i}{3}(1+x)}{3 i+\sqrt{3}}} \left(\sqrt{-i+\sqrt{3}+2 i x} \left((-2-i)-\sqrt{3}+\left((1+2 i)+i \sqrt{3}\right) x\right)\right.\right. \\ & \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i+\sqrt{3}}\right]+4 \sqrt{i+\sqrt{3}-2 i x} \sqrt{1-x+x^2} \\ & \left.\left.\operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i+(1+2 i) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i+\sqrt{3}}\right]\right)\right) / \\ & \left(\left(3 i+(1+2 i) \sqrt{3}\right) \sqrt{i+\sqrt{3}-2 i x} \sqrt{-1-x^3}\right) \end{aligned}$$

Problem 94: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1-\sqrt{3}) a^{1/3}+b^{1/3} x}{\left((1+\sqrt{3}) a^{1/3}+b^{1/3} x\right) \sqrt{a+b x^3}} dx$$

Optimal (type 3, 69 leaves, 2 steps) :

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} a^{1/6} \left(a^{1/3}+b^{1/3} x\right)}{\sqrt{a+b x^3}}\right]}{\sqrt{3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 320 leaves) :

$$\begin{aligned}
& \frac{1}{\sqrt{a+b x^3}} 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \\
& - \left(\left(\left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], \right. \right. \\
& \left. \left. (-1)^{1/3} \right] \right) \Big/ \left(3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) + \\
& \left(4 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{3 + (2 + i) \sqrt{3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \Big/ \left((3 + (2 + i) \sqrt{3}) b^{1/3} \right)
\end{aligned}$$

Problem 95: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 71 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} a^{1/6} (a^{1/3} - b^{1/3} x)}{\sqrt{a - b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 329 leaves):

$$\begin{aligned}
& \frac{1}{b^{1/3} \sqrt{a - b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \right. \\
& \left. \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right) / \left(\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) - \\
& \frac{1}{3 + (2 + \frac{1}{2}) \sqrt{3}} 4 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \\
& \operatorname{EllipticPi} \left[\frac{2 \pm \sqrt{3}}{3 + (2 + \frac{1}{2}) \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right]
\end{aligned}$$

Problem 96: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps) :

$$\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{3+2 \sqrt{3}} a^{1/6} (a^{1/3} - b^{1/3} x)}{\sqrt{-a + b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 330 leaves) :

$$\begin{aligned}
& \frac{1}{b^{1/3} \sqrt{-a + b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \right. \\
& \left. \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right) / \left(\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) - \\
& \frac{1}{3 + (2 + \frac{1}{2}) \sqrt{3}} 4 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \\
& \operatorname{EllipticPi} \left[\frac{2 \pm \sqrt{3}}{3 + (2 + \frac{1}{2}) \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right]
\end{aligned}$$

Problem 97: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps) :

$$-\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{3+2 \sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 323 leaves) :

$$\begin{aligned}
& \frac{1}{\sqrt{-a - b x^3}} 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \\
& - \left(\left(\left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], \right. \right. \\
& \left. \left. (-1)^{1/3} \right] \right) \Big/ \left(3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right) + \right. \\
& \left. \left(4 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{3 + (2 + i) \sqrt{3}}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \Big/ \left((3 + (2 + i) \sqrt{3}) b^{1/3} \right) \right)
\end{aligned}$$

Problem 98: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{\left(1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 73 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(1 + \left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{a+b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}
\end{aligned}$$

Result (type 6, 866 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(5 + 3 \sqrt{3}\right) \sqrt{a + b x^3} \left(2 \left(5 + 3 \sqrt{3}\right) a + b x^3\right)} \\
& \left(26 + 15 \sqrt{3}\right) a x \left(- \left(\left(96 \left(1 + \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right. \right. \\
& \left.\left. \left(8 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \right. \right. \\
& \left. \left. \left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) + \\
& x \left(\left(60 \left(3 + \sqrt{3}\right) a \left(\frac{b}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right. \\
& \left.\left(10 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \right. \right. \\
& \left. \left. \left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) + \\
& x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a}\right)^{2/3} \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right. \\
& \left.\left(4 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] - b \right. \right. \\
& x^3 \left(\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \right. \\
& \left. \left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) + \\
& \left(21 b x \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right) \\
& \left(14 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] - \right. \\
& \left. 3 b x^3 \left(\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \right. \\
& \left. \left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right)
\end{aligned}$$

Problem 99: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x}{\left(1 + \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 75 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan} \left[\frac{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(1-\left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{a-b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 835 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(5 + 3 \sqrt{3}\right) \sqrt{a - b x^3} \left(2 \left(5 + 3 \sqrt{3}\right) a - b x^3\right)} \\
& \left(26 + 15 \sqrt{3}\right) a x \left(- \left(\left(96 \left(1 + \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right. \right. \\
& \left(8 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \\
& 3 b x^3 \left(\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \\
& \left.\left.\left.\left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right)\right) + \\
& x \left(- \left(\left(60 \left(3 + \sqrt{3}\right) a \left(\frac{b}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right. \right. \\
& a \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + 3 b x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{b x^3}{a}, \right. \right. \\
& \left.\left.\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) + \\
& x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a}\right)^{2/3} \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right. \right. \\
& a \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + b x^3 \left(\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, \frac{b x^3}{a}, \right. \right. \\
& \left.\left.\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) - \\
& \left(21 b x \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right) / \\
& \left(14 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \\
& 3 b x^3 \left(\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \\
& \left.\left.\left.\left.\left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right)\right)\right)
\end{aligned}$$

Problem 100: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x}{\left(1 + \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(1-\left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{-a+b x^3}}\right]}{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 836 leaves) :

$$\begin{aligned} & \frac{1}{3 \left(5+3 \sqrt{3}\right) \left(2 \left(5+3 \sqrt{3}\right) a-b x^3\right) \sqrt{-a+b x^3}} \\ & \left(26+15 \sqrt{3}\right) a x \left(-\left(\left(96 \left(1+\sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]\right)\right.\right. \\ & \left.\left.\left(8 \left(5+3 \sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]+\right.\right. \\ & 3 b x^3 \left(\operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]+\right. \\ & \left.\left.\left.\left(5+3 \sqrt{3}\right) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]\right)\right)+ \\ & x \left(-\left(\left(60 \left(3+\sqrt{3}\right) a \left(\frac{b}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]\right)\right.\right. \\ & a \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]+3 b x^3 \left(\operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{b x^3}{a}, \right.\right. \\ & \left.\left.\frac{b x^3}{10 a+6 \sqrt{3} a}\right]+\left(5+3 \sqrt{3}\right) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]\right)\right)+ \\ & x \left(-\left(\left(16 \sqrt{3} a \left(\frac{b}{a}\right)^{2/3} \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]\right)\right.\right. \\ & a \operatorname{AppellF1}\left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]+b x^3 \left(\operatorname{AppellF1}\left[2, \frac{1}{2}, 2, 3, \frac{b x^3}{a}, \right.\right. \\ & \left.\left.\frac{b x^3}{10 a+6 \sqrt{3} a}\right]+\left(5+3 \sqrt{3}\right) \operatorname{AppellF1}\left[2, \frac{3}{2}, 1, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]\right)\right)- \\ & \left(21 b x \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]\right)\right)/ \\ & \left(14 \left(5+3 \sqrt{3}\right) a \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]+\right. \\ & 3 b x^3 \left(\operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]+\right. \\ & \left.\left.\left.\left(5+3 \sqrt{3}\right) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a+6 \sqrt{3} a}\right]\right)\right)\right)\right)$$

Problem 101: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{\left(1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(1+\left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{-a-b x^3}}\right]}{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 869 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(5 + 3 \sqrt{3}\right) \sqrt{-a - b x^3} \left(2 \left(5 + 3 \sqrt{3}\right) a + b x^3\right)} \\
& \left(26 + 15 \sqrt{3}\right) a x \left(- \left(\left(96 \left(1 + \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) / \\
& \left(8 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] - \right. \\
& \left. 3 b x^3 \left(\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \right. \\
& \left. \left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) + \\
& x \left(\left(60 \left(3 + \sqrt{3}\right) a \left(\frac{b}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) / \\
& \left(10 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] - \right. \\
& \left. 3 b x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \right. \\
& \left. \left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) + \\
& x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a}\right)^{2/3} \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) / \\
& \left(4 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] - b \right. \\
& \left. x^3 \left(\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \right. \\
& \left. \left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right) + \\
& \left(21 b x \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right) / \\
& \left(14 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] - \right. \\
& \left. 3 b x^3 \left(\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \right. \right. \\
& \left. \left.\left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right]\right)\right)
\end{aligned}$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+x}{(1+\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps) :

$$\begin{aligned}
 & -\frac{\text{ArcTan}\left[\frac{\sqrt{-3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{3+2 \sqrt{3}}} + \\
 & \left(\sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
 & \left(3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}\right)
 \end{aligned}$$

Result (type 4, 269 leaves) :

$$\begin{aligned}
 & \left(2 \sqrt{6} \sqrt{\frac{\frac{i (1+x)}{3 i+ \sqrt{3}}}{}} \left(\sqrt{-\frac{i}{3}+\sqrt{3}+2 i x} \left((-2-\frac{i}{3})-\sqrt{3}+\left((1+2 i)+i \sqrt{3}\right) x\right)\right.\right. \\
 & \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{i}{3}+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i+\sqrt{3}}\right]+2 \sqrt{\frac{i}{3}+\sqrt{3}-2 i x} \sqrt{1-x+x^2} \\
 & \left.\left.\text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i+(1+2 i) \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{i}{3}+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i+\sqrt{3}}\right]\right)\right) / \\
 & \left(\left(3 \frac{i}{3}+(1+2 i) \sqrt{3}\right) \sqrt{\frac{i}{3}+\sqrt{3}-2 i x} \sqrt{1+x^3}\right)
 \end{aligned}$$

Problem 103: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+x}{(1-\sqrt{3}+x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps) :

$$\begin{aligned}
 & -\frac{\text{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{-3+2 \sqrt{3}}} + \\
 & \left(\sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
 & \left(3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}\right)
 \end{aligned}$$

Result (type 4, 267 leaves) :

$$\begin{aligned}
 & - \left(\left(2 \sqrt{6} \sqrt{\frac{\frac{i}{2} (1+x)}{3 \frac{i}{2} + \sqrt{3}}} \left(\sqrt{-\frac{i}{2} + \sqrt{3}} + 2 \frac{i}{2} x \right) \left((1+2 \frac{i}{2}) - \frac{i}{2} \sqrt{3} + \left((-2 - \frac{i}{2}) + \sqrt{3} \right) x \right) \right. \right. \\
 & \quad \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2} + \sqrt{3}} - 2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}} \right] + 2 \frac{i}{2} \sqrt{\frac{i}{2} + \sqrt{3}} - 2 \frac{i}{2} x \sqrt{1-x+x^2} \\
 & \quad \left. \text{EllipticPi} \left[\frac{2 \frac{i}{2} \sqrt{3}}{-3 + (2 + \frac{i}{2}) \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{i}{2} + \sqrt{3}} - 2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}} \right] \right) / \\
 & \quad \left. \left(\left(-3 + (2 + \frac{i}{2}) \sqrt{3} \right) \sqrt{\frac{i}{2} + \sqrt{3}} - 2 \frac{i}{2} x \sqrt{1+x^3} \right) \right)
 \end{aligned}$$

Problem 104: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(1 + \sqrt{3} + x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 173 leaves, 4 steps) :

$$\begin{aligned}
 & \frac{(e - f - \sqrt{3} f) \text{ArcTan} \left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}} \right]}{\sqrt{3 (3+2 \sqrt{3})}} + \left(\sqrt{2+\sqrt{3}} (e - (1-\sqrt{3}) f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \right. \\
 & \quad \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x} \right], -7-4 \sqrt{3} \right] \right) / \left(3^{3/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)
 \end{aligned}$$

Result (type 4, 291 leaves) :

$$\begin{aligned}
& \left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{i(1+x)}{3i + \sqrt{3}}} \right. \\
& \left(3f \sqrt{-i + \sqrt{3} + 2ix} \left((-2 - i) - \sqrt{3} + ((1+2i) + i\sqrt{3})x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}}] + 2 \left(-\sqrt{3}e + (3 + \sqrt{3})f \right) \sqrt{i + \sqrt{3} - 2ix} \sqrt{1-x+x^2} \right. \\
& \left. \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i + (1+2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}} \right] \right) / \\
& \left((3i + (1+2i)\sqrt{3}) \sqrt{i + \sqrt{3} - 2ix} \sqrt{1+x^3} \right)
\end{aligned}$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + fx}{(1 + \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 187 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(e + f + \sqrt{3}f) \text{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}} \right]}{\sqrt{3(3+2\sqrt{3})}} - \left(\sqrt{2+\sqrt{3}} (e + (1-\sqrt{3})f) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3}] \right) / \left(3^{3/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3} \right)
\end{aligned}$$

Result (type 4, 291 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{2}{3}} \sqrt{-\frac{i(-1+x)}{3i + \sqrt{3}}} \left(-3i f \sqrt{-i + \sqrt{3} - 2ix} \left(-i \left((2+i) + \sqrt{3} \right) + \left((2-i) + \sqrt{3} \right)x \right) \right. \right. \\
& \left. \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} + 2ix}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}}] + 2 \left(\sqrt{3}e + (3 + \sqrt{3})f \right) \sqrt{i + \sqrt{3} + 2ix} \right. \\
& \left. \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i + (1+2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} + 2ix}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3i + \sqrt{3}} \right] \right) / \\
& \left((3i + (1+2i)\sqrt{3}) \sqrt{i + \sqrt{3} + 2ix} \sqrt{1+x^3} \right)
\end{aligned}$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(1 + \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 190 leaves, 4 steps) :

$$-\frac{\left(e + f + \sqrt{3} f\right) \operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{-1+x^3}}\right]}{\sqrt{3 (3+2 \sqrt{3})}} - \left(\sqrt{2-\sqrt{3}} \left(e + (1-\sqrt{3}) f\right) (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]\right) / \left(3^{3/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}\right)$$

Result (type 4, 289 leaves) :

$$\begin{aligned} & \left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{\frac{i}{2} (-1+x)}{3 \frac{i}{2} + \sqrt{3}}} \left(-3 \frac{i}{2} f \sqrt{-\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \left(-\frac{i}{2} \left((2+\frac{i}{2}) + \sqrt{3}\right) + \left((2-\frac{i}{2}) + \sqrt{3}\right) x\right)\right.\right. \\ & \left.\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3}} + 2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}\right] + 2 \left(\sqrt{3} e + (3 + \sqrt{3}) f\right) \sqrt{\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x}\right.\right. \\ & \left.\left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3}} + 2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}\right]\right)\right) / \\ & \left(\left(3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}\right) \sqrt{\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x} \sqrt{-1+x^3}\right) \end{aligned}$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(1 + \sqrt{3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 183 leaves, 4 steps) :

$$\begin{aligned} & \frac{\left(e - (1+\sqrt{3}) f\right) \operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{3 (3+2 \sqrt{3})}} + \left(\sqrt{2-\sqrt{3}} \left(e - (1-\sqrt{3}) f\right) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]\right) / \left(3^{3/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}\right) \end{aligned}$$

Result (type 4, 293 leaves) :

$$\begin{aligned}
 & \left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{\frac{i}{2} (1+x)}{3 \frac{i}{2} + \sqrt{3}}} \right. \\
 & \left(3 f \sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x} \left((-2 - \frac{i}{2}) - \sqrt{3} + \left((1 + 2 \frac{i}{2}) + \frac{i}{2} \sqrt{3} \right) x \right) \text{EllipticF}[\text{ArcSin}\left[\right. \right. \\
 & \left. \left. \frac{\sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}] + 2 \left(-\sqrt{3} e + (3 + \sqrt{3}) f \right) \sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \sqrt{1-x+x^2} \right. \\
 & \left. \left. \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}} \right] \right] \right) / \\
 & \left((3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}) \sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \sqrt{-1-x^3} \right)
 \end{aligned}$$

Problem 108: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{\left((1 - \sqrt{3}) a^{1/3} + b^{1/3} x \right) \sqrt{a + b x^3}} dx$$

Optimal (type 4, 332 leaves, 4 steps) :

$$\begin{aligned}
 & - \frac{\left(b^{1/3} e - (1 - \sqrt{3}) a^{1/3} f \right) \text{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{a+b x^3}} \right]}{\sqrt{3 (-3+2 \sqrt{3})} \sqrt{a} b^{2/3}} - \\
 & \left(\sqrt{2+\sqrt{3}} \left(b^{1/3} e - (1 + \sqrt{3}) a^{1/3} f \right) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \right. \\
 & \left. \text{EllipticF}[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3}] \right) / \\
 & \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right)
 \end{aligned}$$

Result (type 4, 438 leaves) :

$$\begin{aligned}
& - \left(\left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \left. \left. - \frac{1}{2 \sqrt{2}} i \sqrt{3}^{1/4} F \left(\left((-2 - i) + \sqrt{3} \right) a^{1/3} + \left((1 + 2 i) - i \sqrt{3} \right) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2 i b^{1/3} x}{a^{1/3}}} \right. \right. \\
& \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2 i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3 i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i \sqrt{3}) \right] + \right. \right. \\
& \left. \left. i \left(b^{1/3} e + (-1 + \sqrt{3}) a^{1/3} f \right) \sqrt{\frac{-2 i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3 i + \sqrt{3}) a^{1/3}}} \right. \right. \\
& \left. \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{-3 i + (1 + 2 i) \sqrt{3}}, \right. \right. \\
& \left. \left. \text{ArcSin} \left[\sqrt{\frac{-2 i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3 i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i \sqrt{3}) \right] \right] \right) / \\
& \left((3 - (2 - i) \sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 109: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x \right) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 336 leaves, 4 steps) :

$$\begin{aligned}
& \frac{\left(b^{1/3} e + (1 - \sqrt{3}) a^{1/3} f\right) \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3} x)}{\sqrt{a-b x^3}}\right]}{\sqrt{3 (-3+2 \sqrt{3})} \sqrt{a} b^{2/3}} + \\
& \left(\sqrt{2+\sqrt{3}} \left(b^{1/3} e + (1 + \sqrt{3}) a^{1/3} f\right) (a^{1/3}-b^{1/3} x) \sqrt{\frac{a^{2/3}+a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left((1+\sqrt{3}) a^{1/3}-b^{1/3} x\right)^2}}\right. \\
& \left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) a^{1/3}-b^{1/3} x}{(1+\sqrt{3}) a^{1/3}-b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3}-b^{1/3} x)}{\left((1+\sqrt{3}) a^{1/3}-b^{1/3} x\right)^2}} \sqrt{a-b x^3}\right)
\end{aligned}$$

Result (type 4, 466 leaves):

$$\begin{aligned}
& -\frac{1}{\left(3-\left(2-\frac{i}{2}\right) \sqrt{3}\right) b^{2/3} \sqrt{\frac{a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}} 4 \sqrt{\frac{a^{1/3}-b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}} \\
& \left(\frac{1}{2} f\left(\frac{i}{2}\left(-3+\left(2+\frac{i}{2}\right) \sqrt{3}\right) a^{1/3}+\left(3-\left(2-\frac{i}{2}\right) \sqrt{3}\right) b^{1/3} x\right) \sqrt{\frac{\left(-\frac{i}{2}+\sqrt{3}\right) a^{1/3}+\left(\frac{i}{2}+\sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right. \\
& \left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3}+\left(1-\frac{i}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2}\left(1+\frac{i}{2} \sqrt{3}\right)\right]-\right. \\
& \left.\frac{i}{2}\left(b^{1/3} e-\left(-1+\sqrt{3}\right) a^{1/3} f\right) \sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3}+\left(1-\frac{i}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right. \\
& \left.\sqrt{1+\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 \frac{i}{2}+\left(1+2 \frac{i}{2}\right) \sqrt{3}},\right.\right. \\
& \left.\left.\operatorname{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3}+\left(1-\frac{i}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2}\left(1+\frac{i}{2} \sqrt{3}\right)\right]\right)
\end{aligned}$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x \right) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 345 leaves, 4 steps) :

$$\begin{aligned} & \frac{\left(b^{1/3} e + (1 - \sqrt{3}) a^{1/3} f \right) \operatorname{ArcTan} \left[\frac{\sqrt{-3 + 2 \sqrt{3}} a^{1/6} (a^{1/3} - b^{1/3} x)}{\sqrt{-a + b x^3}} \right]}{\sqrt{3 (-3 + 2 \sqrt{3})} \sqrt{a} b^{2/3}} + \\ & \left(\sqrt{2 - \sqrt{3}} \left(b^{1/3} e + (1 + \sqrt{3}) a^{1/3} f \right) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \right. \\ & \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3} \right] \right) / \\ & \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3} \right) \end{aligned}$$

Result (type 4, 467 leaves) :

$$\begin{aligned}
& - \frac{1}{\left(3 - (2 - \frac{1}{x}) \sqrt{3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}} 4 \sqrt[4]{\frac{a^{1/3} - b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(\frac{1}{2} f \left(\frac{1}{x} \left(-3 + (2 + \frac{1}{x}) \sqrt{3} \right) a^{1/3} + \left(3 - (2 - \frac{1}{x}) \sqrt{3} \right) b^{1/3} x \right) \sqrt{\frac{\left(-\frac{1}{x} + \sqrt{3} \right) a^{1/3} + \left(\frac{1}{x} + \sqrt{3} \right) b^{1/3} x}{\left(-3 \frac{1}{x} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{\frac{1}{x} \left(2 a^{1/3} + \left(1 - \frac{1}{x} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{x} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{x} \sqrt{3} \right) \right] - \right. \\
& \left. \frac{1}{x} \left(b^{1/3} e - \left(-1 + \sqrt{3} \right) a^{1/3} f \right) \sqrt{-\frac{\frac{1}{x} \left(2 a^{1/3} + \left(1 - \frac{1}{x} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{x} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{-3 \frac{1}{x} + (1 + 2 \frac{1}{x}) \sqrt{3}}, \right. \right. \\
& \left. \left. \text{ArcSin} \left[\sqrt{-\frac{\frac{1}{x} \left(2 a^{1/3} + \left(1 - \frac{1}{x} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{x} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{x} \sqrt{3} \right) \right] \right)
\end{aligned}$$

Problem 111: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 345 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\left(b^{1/3} e - (1 - \sqrt{3}) a^{1/3} f \right) \operatorname{ArcTan} \left[\frac{\sqrt{-3 + 2\sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{\sqrt{3 (-3 + 2\sqrt{3})} \sqrt{a} b^{2/3}} - \\
& \left(\sqrt{2 - \sqrt{3}} (b^{1/3} e - (1 + \sqrt{3}) a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \right. \\
& \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 + 4\sqrt{3} \right] \right) / \\
& \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Result (type 4, 441 leaves):

$$\begin{aligned}
& - \left(\left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \left. \left. - \frac{1}{2\sqrt{2}} \text{i} 3^{1/4} f (((-2 - \text{i}) + \sqrt{3}) a^{1/3} + ((1 + 2\text{i}) - \text{i}\sqrt{3}) b^{1/3} x) \sqrt{\text{i} + \sqrt{3} - \frac{2\text{i} b^{1/3} x}{a^{1/3}}} \right. \right. \\
& \left. \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{-2\text{i} a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3\text{i} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \text{i}\sqrt{3}) \right] + \right. \right. \\
& \left. \left. \text{i} (b^{1/3} e + (-1 + \sqrt{3}) a^{1/3} f) \sqrt{\frac{-2\text{i} a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3\text{i} + \sqrt{3}) a^{1/3}}} \right. \right. \\
& \left. \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{-3\text{i} + (1 + 2\text{i})\sqrt{3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\sqrt{\frac{-2\text{i} a^{1/3} + (\text{i} + \sqrt{3}) b^{1/3} x}{(-3\text{i} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \text{i}\sqrt{3}) \right] \right) \right) / \\
& \left((3 - (2 - \text{i})\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1 + \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 136 leaves, 4 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right]}{3^{3/4}}+\frac{\sqrt{2} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{3^{3/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 209 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right. \\ & \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \frac{1}{3+(2+i) \sqrt{3}} \right. \\ & \quad \left. 2 i \left(1+\sqrt{3}\right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{3+(2+i) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) \end{aligned}$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1 + \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 152 leaves, 4 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{1-x^3}}\right]}{3^{3/4}}+\frac{\sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{3^{3/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 232 leaves):

$$\begin{aligned}
& \left(2^{\frac{1}{3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} \right)^{\frac{1}{3}} \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \right. \\
& \left. \left(3^{\frac{1}{3}} + (1+2^{\frac{1}{3}}) \sqrt{3} + (3 + (2+\frac{1}{3}) \sqrt{3}) x \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right. \\
& \left. + 2(1+\sqrt{3}) \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2^{\frac{1}{3}} \sqrt{3}}{3 + (2+\frac{1}{3}) \sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right] \right) / \\
& \left((3 + (2+\frac{1}{3}) \sqrt{3}) \sqrt{1-x^3} \right)
\end{aligned}$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1+\sqrt{3}-x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 164 leaves, 4 steps):

$$\begin{aligned}
& -\frac{\sqrt{2} \text{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right]}{3^{3/4}} + \\
& \left(2 \sqrt{\frac{7}{6} - \frac{2}{\sqrt{3}}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}] \right) / \\
& \left(3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right)
\end{aligned}$$

Result (type 4, 230 leaves):

$$\begin{aligned}
& \left(2^{\frac{1}{3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} \right)^{\frac{1}{3}} \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \right. \\
& \quad \left. \left(3^{\frac{1}{3}} + (1+2^{\frac{1}{3}}) \sqrt{3} + (3 + (2+\frac{1}{3}) \sqrt{3}) x \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right. \\
& \quad \left. 2 \left(1+\sqrt{3}\right) \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2^{\frac{1}{3}} \sqrt{3}}{3+(2+\frac{1}{3}) \sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) / \\
& \quad \left((3 + (2+\frac{1}{3}) \sqrt{3}) \sqrt{-1+x^3} \right)
\end{aligned}$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1+\sqrt{3}+x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$\begin{aligned}
& -\frac{\sqrt{2} \text{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{-1-x^3}}\right]}{3^{3/4}} + \\
& \left(2 \sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}] \right) / \\
& \quad \left(3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)
\end{aligned}$$

Result (type 4, 211 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} \left((-1)^{1/3} - x \right) \right.$$

$$\left. \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \frac{1}{3+(2+\text{i})\sqrt{3}} \right.$$

$$\left. 2\text{i} \left(1+\sqrt{3}\right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\text{i}\sqrt{3}}{3+(2+\text{i})\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1-\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 147 leaves, 4 steps):

$$\begin{aligned} & -\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right]}{3^{3/4}} + \\ & \left(2 \sqrt{\frac{7}{6} + \frac{2}{\sqrt{3}}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}] \right) / \\ & \left(3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) \end{aligned}$$

Result (type 4, 225 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3}x}{1+(-1)^{1/3}}} \left(3 - (2+\frac{i}{2})\sqrt{3} + (-3\frac{i}{2} + (1+2\frac{i}{2})\sqrt{3})x \right) \right. \right. \\
& \left. \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] - 2(-1+\sqrt{3})\sqrt{1-x+x^2} \right. \right. \\
& \left. \left. \text{EllipticPi}\left[\frac{2\sqrt{3}}{-3\frac{i}{2} + (1+2\frac{i}{2})\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] \right) / \\
& \left((-3\frac{i}{2} + (1+2\frac{i}{2})\sqrt{3})\sqrt{1+x^3} \right)
\end{aligned}$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)\sqrt{a+b x^3}} dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$\begin{aligned}
& -\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} \left(a^{1/3}+b^{1/3} x\right)}{\sqrt{a+b x^3}}\right]}{3^{3/4} a^{1/6} b^{2/3}} + \left(2 \sqrt{\frac{7}{6} + \frac{2}{\sqrt{3}}} (a^{1/3} + b^{1/3} x) \right. \\
& \left. \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4\sqrt{3}] \right) / \\
& \left(3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3} \right)
\end{aligned}$$

Result (type 4, 427 leaves):

$$\begin{aligned}
& - \left(\left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \left. \left. - \frac{1}{2\sqrt{2}} i \sqrt{3}^{1/4} \left(\left((-2 - i) + \sqrt{3} \right) a^{1/3} + \left((1 + 2i) - i\sqrt{3} \right) b^{1/3} x \right) \sqrt{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}} \right. \right. \\
& \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] + \right. \right. \\
& \left. \left. i \left(-1 + \sqrt{3} \right) a^{1/3} \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\right. \right. \right. \\
& \left. \left. \left. \frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] \right] \right) \right) \\
& \left((3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x \right) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\sqrt{2} \text{ArcTanh} \left[\frac{\sqrt{-3+2\sqrt{3}} a^{1/6} (a^{1/3} - b^{1/3} x)}{\sqrt{a - b x^3}} \right]}{3^{3/4} a^{1/6} b^{2/3}} + \left(2 \sqrt{\frac{7}{6} + \frac{2}{\sqrt{3}}} (a^{1/3} - b^{1/3} x) \right. \\
& \left. \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 - 4\sqrt{3} \right] \right) \\
& \left(3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Result (type 4, 454 leaves):

$$\begin{aligned}
& - \frac{1}{\left(3 - (2 - \frac{1}{2}) \sqrt{3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}} 4 \sqrt{\frac{a^{1/3} - b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(\frac{1}{2} \left(\frac{1}{2} \left(-3 + (2 + \frac{1}{2}) \sqrt{3} \right) a^{1/3} + \left(3 - (2 - \frac{1}{2}) \sqrt{3} \right) b^{1/3} x \right) \sqrt{\frac{\left(-\frac{1}{2} + \sqrt{3} \right) a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] + \\
& \left. \frac{\frac{1}{2} \left(-1 + \sqrt{3} \right) a^{1/3} \sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\right. \right. \\
& \left. \left. \frac{2 \sqrt{3}}{-3 \frac{1}{2} + \left(1 + 2 \frac{1}{2} \right) \sqrt{3}}, \text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] \right]
\end{aligned}$$

Problem 119: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(\left(1 - \sqrt{3} \right) a^{1/3} - b^{1/3} x \right) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 282 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\sqrt{2} \text{ArcTan} \left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} \left(a^{1/3} - b^{1/3} x \right)}{\sqrt{-a+b x^3}} \right]}{3^{3/4} a^{1/6} b^{2/3}} + \\
& \left(\sqrt{2} \left(a^{1/3} - b^{1/3} x \right) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 - \sqrt{3} \right) a^{1/3} - b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(1 + \sqrt{3} \right) a^{1/3} - b^{1/3} x}{\left(1 - \sqrt{3} \right) a^{1/3} - b^{1/3} x} \right], \right. \right. \\
& \left. \left. - 7 + 4 \sqrt{3} \right] \right) / \left(3^{3/4} b^{2/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3} - b^{1/3} x \right)}{\left(\left(1 - \sqrt{3} \right) a^{1/3} - b^{1/3} x \right)^2}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Result (type 4, 455 leaves):

$$\begin{aligned}
& - \frac{1}{\left(3 - (2 - \frac{1}{x}) \sqrt{3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}} 4 \sqrt[4]{\frac{a^{1/3} - b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(\frac{1}{2} \left(\frac{1}{x} \left(-3 + (2 + \frac{1}{x}) \sqrt{3} \right) a^{1/3} + \left(3 - (2 - \frac{1}{x}) \sqrt{3} \right) b^{1/3} x \right) \sqrt{\frac{\left(-\frac{1}{x} + \sqrt{3} \right) a^{1/3} + \left(\frac{1}{x} + \sqrt{3} \right) b^{1/3} x}{\left(-3 \frac{1}{x} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{\frac{1}{x} \left(2 a^{1/3} + \left(1 - \frac{1}{x} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{x} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{x} \sqrt{3} \right) \right] + \right. \\
& \left. \frac{\frac{1}{x} \left(-1 + \sqrt{3} \right) a^{1/3} \sqrt{-\frac{\frac{1}{x} \left(2 a^{1/3} + \left(1 - \frac{1}{x} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{x} + \sqrt{3} \right) a^{1/3}}}}{\sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\right. \right. \\
& \left. \left. \frac{2 \sqrt{3}}{-3 \frac{1}{x} + \left(1 + 2 \frac{1}{x} \right) \sqrt{3}}, \text{ArcSin} \left[\sqrt{-\frac{\frac{1}{x} \left(2 a^{1/3} + \left(1 - \frac{1}{x} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{x} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{x} \sqrt{3} \right) \right] \right]
\end{aligned}$$

Problem 120: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\sqrt{2} \text{Arctan} \left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} \left(a^{1/3} + b^{1/3} x \right)}{\sqrt{-a-b x^3}} \right]}{3^{3/4} a^{1/6} b^{2/3}} + \\
& \left(\sqrt{2} \left(a^{1/3} + b^{1/3} x \right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(1 + \sqrt{3} \right) a^{1/3} + b^{1/3} x}{\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x} \right], \right. \right. \\
& \left. \left. -7 + 4 \sqrt{3} \right] \right) / \left(3^{3/4} b^{2/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Result (type 4, 430 leaves):

$$\begin{aligned}
& - \left(\left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \right. \\
& \left. \left. - \frac{1}{2 \sqrt{2}} \right) \frac{1}{2} \sqrt{3^{1/4}} \left(\left((-2 - \frac{1}{2}) + \sqrt{3} \right) a^{1/3} + \left((1 + 2 \frac{1}{2}) - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right) \sqrt{\frac{\frac{1}{2} + \sqrt{3} - \frac{2 \frac{1}{2} b^{1/3} x}{a^{1/3}}}{a^{1/3}}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + (\frac{1}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{1}{2} \sqrt{3}) \right] + \right. \\
& \left. \frac{\frac{1}{2} (-1 + \sqrt{3}) a^{1/3}}{2 \sqrt{3}} \sqrt{\frac{-2 \frac{1}{2} a^{1/3} + (\frac{1}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\right. \right. \\
& \left. \left. \frac{2 \sqrt{3}}{-3 \frac{1}{2} + (1 + 2 \frac{1}{2}) \sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + (\frac{1}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{1}{2} \sqrt{3}) \right] \right] \right) / \\
& \left((3 - (2 - \frac{1}{2}) \sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 121: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} + x}{(c + d x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 319 leaves, 6 steps):

$$\begin{aligned}
& - \frac{\left(c - (1 + \sqrt{3}) d \right) (1 + x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{ArcTan} \left[\frac{\sqrt{c^2+c d+d^2}}{\sqrt{c-d} \sqrt{d}} \sqrt{\frac{1+x}{\frac{(1+\sqrt{3}+x)^2}{1-x+x^2}}} \right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2}} - \\
& \left(4 \times 3^{1/4} \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \text{EllipticPi} \left[\frac{(c - (1 + \sqrt{3}) d)^2}{(c - (1 - \sqrt{3}) d)^2}, \right. \right. \\
& \left. \left. - \text{ArcSin} \left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}, -7 - 4 \sqrt{3} \right] \right] \right) / \left((c - (1 - \sqrt{3}) d) \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3} \right)
\end{aligned}$$

Result (type 4, 214 leaves) :

$$\begin{aligned} & \frac{1}{d \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} \left((-1)^{1/3} - x \right) \right. \\ & \left. \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \frac{1}{c+(-1)^{1/3}d} \right. \\ & \left. \pm \left(c - (1 + \sqrt{3}) d \right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c+(-1)^{1/3}d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right) \right) \end{aligned}$$

Problem 122: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}-x}{(c+d x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 331 leaves, 6 steps) :

$$\begin{aligned} & \left(c + d + \sqrt{3} d \right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2-c d+d^2}}{\sqrt{d} \sqrt{c+d}} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\right] \\ & - \frac{\sqrt{d} \sqrt{c+d} \sqrt{c^2-c d+d^2}}{\sqrt{1-x} \sqrt{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3} + \\ & \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{(c+d+\sqrt{3} d)^2}{(c+d-\sqrt{3} d)^2}, \right. \right. \\ & \left. \left. -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3} \right] \right) / \left((c+d-\sqrt{3} d) \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3} \right) \end{aligned}$$

Result (type 4, 235 leaves) :

$$\begin{aligned}
& \frac{1}{3d\sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} \right. \\
& 3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] + \\
& \frac{1}{c - (-1)^{1/3}d} (-1)^{1/3} (1+(-1)^{1/3}) (\sqrt{3} c + (3+\sqrt{3}) d) \sqrt{1+x+x^2} \\
& \left. \operatorname{EllipticPi}\left[\frac{\frac{i\sqrt{3}d}{-c+(-1)^{1/3}d}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}}{} \right] \right)
\end{aligned}$$

Problem 123: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}-x}{(c+d x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 327 leaves, 6 steps):

$$\begin{aligned}
& \left(c + d + \sqrt{3} d \right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{\sqrt{d} \sqrt{c+d} \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}} \right] \\
& - \frac{\sqrt{d} \sqrt{c+d} \sqrt{c^2 - c d + d^2} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3}}{+} \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{(c+d+\sqrt{3}d)^2}{(c+d-\sqrt{3}d)^2}, \right. \right. \\
& \left. \left. -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right] \right) / \left((c+d-\sqrt{3}d) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right)
\end{aligned}$$

Result (type 4, 233 leaves):

$$\begin{aligned}
& \frac{1}{3 d \sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} \right. \\
& 3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] + \\
& \frac{1}{c - (-1)^{1/3} d} (-1)^{1/3} (1+(-1)^{1/3}) (\sqrt{3} c + (3+\sqrt{3}) d) \sqrt{1+x+x^2} \\
& \left. \operatorname{EllipticPi}\left[\frac{\frac{i \sqrt{3} d}{-c + (-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}}{} \right] \right)
\end{aligned}$$

Problem 124: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}+x}{(c+d x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 323 leaves, 6 steps):

$$\begin{aligned}
& \left(c - (1+\sqrt{3}) d \right) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{c^2+c d+d^2}{\sqrt{c-d} \sqrt{d}}}}{\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}} \right] - \\
& - \frac{\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2}}{\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}} \sqrt{-1-x^3} \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(c - (1+\sqrt{3}) d)^2}{(c - (1-\sqrt{3}) d)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x} \right], -7-4\sqrt{3} \right] \right) / \\
& \left((c - (1-\sqrt{3}) d) \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)
\end{aligned}$$

Result (type 4, 216 leaves):

$$\begin{aligned}
& \frac{1}{d \sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} \left((-1)^{1/3} - x \right) \right. \\
& \left. + \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \frac{1}{c+(-1)^{1/3} d} \right. \\
& \left. \pm \left(c - (1+\sqrt{3}) d \right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right) \right)
\end{aligned}$$

Problem 125: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}+x}{(c+d x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 360 leaves, 6 steps):

$$\begin{aligned}
& - \left(\left(c - (1-\sqrt{3}) d \right) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{ArcTanh}\left[\frac{2 \sqrt{2+\sqrt{3}} \sqrt{c^2+c d+d^2}}{\sqrt{c-d} \sqrt{d} \sqrt{7+4 \sqrt{3}+\frac{(1+\sqrt{3}+x)^2}{(1-\sqrt{3}+x)^2}}}\right] \right) / \\
& \left(\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) + \\
& \left(4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[\frac{(c-(1-\sqrt{3}) d)^2}{(c-(1+\sqrt{3}) d)^2}, \right. \right. \\
& \left. \left. -\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3} \right] \right) / \left((c-d-\sqrt{3} d) \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)
\end{aligned}$$

Result (type 4, 213 leaves):

$$\begin{aligned}
& \frac{1}{d \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} \left((-1)^{1/3} - x \right) \right. \\
& \left. + \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \frac{1}{c+(-1)^{1/3}d} \right. \\
& \left. \pm \left(c+(-1+\sqrt{3})d\right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 126: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}-x}{(c+d x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 348 leaves, 6 steps):

$$\begin{aligned}
& \frac{\left(c+d-\sqrt{3} d\right) (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2-c d+d^2}}{\sqrt{d} \sqrt{c+d}} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2-c d+d^2} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{1-x^3}} - \\
& \left(4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{(c+d-\sqrt{3} d)^2}{(c+d+\sqrt{3} d)^2}, \right. \right. \\
& \left. \left. -\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]\right) / \left(\left(c+d+\sqrt{3} d\right) \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{1-x^3}\right)
\end{aligned}$$

Result (type 4, 235 leaves):

$$\begin{aligned}
& \frac{1}{3 d \sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} \right. \\
& 3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] + \\
& \frac{1}{c - (-1)^{1/3} d} (-1)^{1/3} (1+(-1)^{1/3}) (\sqrt{3} c + (-3 + \sqrt{3}) d) \sqrt{1+x+x^2} \\
& \left. \operatorname{EllipticPi}\left[\frac{\frac{i \sqrt{3} d}{-c + (-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}}{} \right] \right)
\end{aligned}$$

Problem 127: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}-x}{(c+d x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 344 leaves, 6 steps):

$$\begin{aligned}
& \frac{\left(c+d-\sqrt{3} d\right) (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2-c d+d^2}}{\sqrt{d} \sqrt{c+d}} \sqrt{\frac{-\frac{1-x}{(1-\sqrt{3}-x)^2}}{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}}}\right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2-c d+d^2} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}} - \\
& \left(4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{(c+d-\sqrt{3} d)^2}{(c+d+\sqrt{3} d)^2},\right.\right. \\
& \left.\left.-\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]\right) / \left(\left(c+d+\sqrt{3} d\right) \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}\right)
\end{aligned}$$

Result (type 4, 233 leaves):

$$\begin{aligned}
& \frac{1}{3 d \sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} \right. \\
& 3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] + \\
& \frac{1}{c - (-1)^{1/3} d} (-1)^{1/3} (1+(-1)^{1/3}) (\sqrt{3} c + (-3+\sqrt{3}) d) \sqrt{1+x+x^2} \\
& \left. \operatorname{EllipticPi}\left[\frac{\frac{i \sqrt{3} d}{-c + (-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}}{} \right] \right)
\end{aligned}$$

Problem 128: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}+x}{(c+d x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 364 leaves, 6 steps):

$$\begin{aligned}
& - \left(\left(c - (1 - \sqrt{3}) d \right) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{ArcTanh}\left[\frac{2 \sqrt{2+\sqrt{3}} \sqrt{c^2+c d+d^2}}{\sqrt{c-d} \sqrt{d}} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \right] \right. \\
& \left. \left(\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right) \right) + \\
& \left(4 \times 3^{1/4} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[\frac{(c - (1 - \sqrt{3}) d)^2}{(c - (1 + \sqrt{3}) d)^2}, \right. \right. \\
& \left. \left. -\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4\sqrt{3} \right] \right) / \left((c-d-\sqrt{3} d) \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)
\end{aligned}$$

Result (type 4, 215 leaves):

$$\begin{aligned}
& \frac{1}{d \sqrt{-1 - x^3}} 2 \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}} \left((-1)^{1/3} - x \right) \right. \\
& \left. \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3}x}{1 + (-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3}x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}] + \frac{1}{c + (-1)^{1/3}d} \right. \\
& \left. \pm \left(c + (-1 + \sqrt{3})d\right) \sqrt{1 - x + x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c + (-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3}x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 129: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} + x}{x \sqrt{1 + x^3}} dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$\begin{aligned}
& -\frac{2}{3} \left(1 + \sqrt{3}\right) \operatorname{ArcTanh}\left[\sqrt{1 + x^3}\right] + \\
& \left(2 \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4\sqrt{3}\right]\right) / \\
& \left(3^{1/4} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}\right)
\end{aligned}$$

Result (type 4, 149 leaves):

$$\begin{aligned}
& -\frac{2}{3} \operatorname{ArcTanh}\left[\sqrt{1 + x^3}\right] - \frac{2 \operatorname{ArcTanh}\left[\sqrt{1 + x^3}\right]}{\sqrt{3}} - \\
& \left(2 \left((-1)^{1/3} - x\right) \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} \left((-1)^{2/3} + x\right)}{1 + (-1)^{1/3}}}\right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3}x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}\right]\right) / \left(\sqrt{\frac{1 + (-1)^{2/3}x}{1 + (-1)^{1/3}}} \sqrt{1 + x^3}\right)
\end{aligned}$$

Problem 130: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} - x}{x \sqrt{1 - x^3}} dx$$

Optimal (type 4, 139 leaves, 5 steps) :

$$\begin{aligned} & -\frac{2}{3} \left(1 + \sqrt{3}\right) \operatorname{ArcTanh} \left[\sqrt{1 - x^3} \right] + \\ & \left(2 \sqrt{2 + \sqrt{3}} (1 - x) \sqrt{\frac{1 + x + x^2}{(1 + \sqrt{3} - x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1 - \sqrt{3} - x}{1 + \sqrt{3} - x} \right], -7 - 4\sqrt{3} \right] \right) / \\ & \left(3^{1/4} \sqrt{\frac{1 - x}{(1 + \sqrt{3} - x)^2}} \sqrt{1 - x^3} \right) \end{aligned}$$

Result (type 4, 157 leaves) :

$$\begin{aligned} & -\frac{2}{3} \operatorname{ArcTanh} \left[\sqrt{1 - x^3} \right] - \frac{2 \operatorname{ArcTanh} \left[\sqrt{1 - x^3} \right]}{\sqrt{3}} - \\ & \left(2 \sqrt{\frac{1 - x}{1 + (-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \right. \\ & \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1 - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \right], (-1)^{1/3} \right] \right) / \left(\sqrt{\frac{1 - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{1 - x^3} \right) \end{aligned}$$

Problem 131: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} - x}{x \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 142 leaves, 5 steps) :

$$\begin{aligned} & \frac{2}{3} \left(1 + \sqrt{3}\right) \operatorname{ArcTan} \left[\sqrt{-1 + x^3} \right] + \\ & \left(2 \sqrt{2 - \sqrt{3}} (1 - x) \sqrt{\frac{1 + x + x^2}{(1 - \sqrt{3} - x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1 + \sqrt{3} - x}{1 - \sqrt{3} - x} \right], -7 + 4\sqrt{3} \right] \right) / \\ & \left(3^{1/4} \sqrt{-\frac{1 - x}{(1 - \sqrt{3} - x)^2}} \sqrt{-1 + x^3} \right) \end{aligned}$$

Result (type 4, 150 leaves) :

$$\frac{2}{3} \left(\text{ArcTan}[\sqrt{-1+x^3}] + \sqrt{3} \text{ArcTan}[\sqrt{-1+x^3}] - \right. \\ \left. 3 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \right. \\ \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right) / \left(\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{-1+x^3} \right)$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}} dx$$

Optimal (type 4, 136 leaves, 5 steps):

$$\frac{2}{3} \left(1 + \sqrt{3} \right) \text{ArcTan}[\sqrt{-1-x^3}] + \\ \left(2 \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}] \right) / \\ \left(3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3} \right)$$

Result (type 4, 155 leaves):

$$\frac{2}{3} \left(\text{ArcTan}[\sqrt{-1-x^3}] + \sqrt{3} \text{ArcTan}[\sqrt{-1-x^3}] - \right. \\ \left. 3 \left((-1)^{1/3} - x \right) \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \right. \\ \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right) / \left(\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{-1-x^3} \right)$$

Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}} dx$$

Optimal (type 4, 127 leaves, 5 steps) :

$$\begin{aligned} & -\frac{2}{3} \left(1 - \sqrt{3}\right) \operatorname{ArcTanh} \left[\sqrt{1 + x^3} \right] + \\ & \left(2 \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x} \right], -7 - 4\sqrt{3} \right] \right) / \\ & \left(3^{1/4} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3} \right) \end{aligned}$$

Result (type 4, 149 leaves) :

$$\begin{aligned} & -\frac{2}{3} \operatorname{ArcTanh} \left[\sqrt{1 + x^3} \right] + \frac{2 \operatorname{ArcTanh} \left[\sqrt{1 + x^3} \right]}{\sqrt{3}} - \\ & \left(2 \left((-1)^{1/3} - x \right) \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} ((-1)^{2/3} + x)}{1 + (-1)^{1/3}}} \right. \\ & \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \right], (-1)^{1/3} \right] \right) / \left(\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{1 + x^3} \right) \end{aligned}$$

Problem 134: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} - x}{x \sqrt{1 - x^3}} dx$$

Optimal (type 4, 141 leaves, 5 steps) :

$$\begin{aligned} & -\frac{2}{3} \left(1 - \sqrt{3}\right) \operatorname{ArcTanh} \left[\sqrt{1 - x^3} \right] + \\ & \left(2 \sqrt{2 + \sqrt{3}} (1 - x) \sqrt{\frac{1 + x + x^2}{(1 + \sqrt{3} - x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1 - \sqrt{3} - x}{1 + \sqrt{3} - x} \right], -7 - 4\sqrt{3} \right] \right) / \\ & \left(3^{1/4} \sqrt{\frac{1 - x}{(1 + \sqrt{3} - x)^2}} \sqrt{1 - x^3} \right) \end{aligned}$$

Result (type 4, 158 leaves) :

$$\frac{2}{3} \left(-\text{ArcTanh}\left[\sqrt{1-x^3}\right] + \sqrt{3} \text{ArcTanh}\left[\sqrt{1-x^3}\right] - \right. \\ \left. 3 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3}+x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) / \left(\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}} \sqrt{1-x^3} \right)$$

Problem 135: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}-x}{x \sqrt{-1+x^3}} dx$$

Optimal (type 4, 144 leaves, 5 steps):

$$\frac{2}{3} \left(1 - \sqrt{3} \right) \text{ArcTan}\left[\sqrt{-1+x^3}\right] + \\ \left(2 \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right] \right) / \\ \left(3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right)$$

Result (type 4, 151 leaves):

$$\frac{2}{3} \left(\text{ArcTan}\left[\sqrt{-1+x^3}\right] - \sqrt{3} \text{ArcTan}\left[\sqrt{-1+x^3}\right] - \right. \\ \left. 3 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3}+x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3}x}{1+(-1)^{1/3}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) / \left(\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}} \sqrt{-1+x^3} \right)$$

Problem 136: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}+x}{x \sqrt{-1-x^3}} dx$$

Optimal (type 4, 138 leaves, 5 steps) :

$$\begin{aligned} & \frac{2}{3} \left(1 - \sqrt{3} \right) \operatorname{ArcTan} \left[\sqrt{-1 - x^3} \right] + \\ & \left(2 \sqrt{2 - \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x} \right], -7 + 4\sqrt{3} \right] \right) / \\ & \left(3^{1/4} \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}} \sqrt{-1 - x^3} \right) \end{aligned}$$

Result (type 4, 156 leaves) :

$$\begin{aligned} & \frac{2}{3} \left(\operatorname{ArcTan} \left[\sqrt{-1 - x^3} \right] - \sqrt{3} \operatorname{ArcTan} \left[\sqrt{-1 - x^3} \right] - \right. \\ & \left. 3 \left((-1)^{1/3} - x \right) \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \right. \\ & \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \right], (-1)^{1/3} \right] \right) / \left(\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{-1 - x^3} \right) \end{aligned}$$

Problem 137: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 334 leaves, 8 steps) :

$$\begin{aligned}
& - \frac{3 (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}}\right]}{\sqrt{26} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}} \\
& \left(2 \sqrt{2 \left(97+56 \sqrt{3}\right)} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}\right) - \\
& \left(12 \times 3^{1/4} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[97-56 \sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}\right)
\end{aligned}$$

Result (type 4, 194 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} \right. \\
& \left. \left((-1)^{1/3}-x\right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \frac{1}{3+(-1)^{1/3}} 3 i \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{3+(-1)^{1/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 138: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 379 leaves, 8 steps):

$$\begin{aligned}
& \frac{3 (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{2 \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}}\right]}{2 \sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}} - \\
& \left(2 \sqrt{2 \left(37+20 \sqrt{3}\right)} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(13 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}\right) - \\
& \left(12 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{1}{169} \left(553+304 \sqrt{3}\right),\right.\right. \\
& \left.\left.-\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]\right) / \left(13 \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}\right)
\end{aligned}$$

Result (type 4, 195 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \\
& \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} \left((-1)^{1/3} + x\right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \frac{1}{-3+(-1)^{1/3}} 3 \pm \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{5 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]\right)
\end{aligned}$$

Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 375 leaves, 8 steps):

$$\begin{aligned}
& \frac{3 (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{7} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2 \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right]}{2 \sqrt{7} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{-1+x^3}} - \\
& \frac{2 \sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]}{3^{1/4} (4+\sqrt{3}) \sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}} \sqrt{-1+x^3}} - \\
& \left(12 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{EllipticPi}\left[\frac{1}{169} \left(553+304 \sqrt{3}\right), \right.\right. \\
& \left.\left.-\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]\right) / \left(13 \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{-1+x^3}\right)
\end{aligned}$$

Result (type 4, 193 leaves) :

$$\begin{aligned}
& \frac{1}{\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \\
& \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} \left((-1)^{1/3}+x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \frac{1}{-3+(-1)^{1/3}} 3 \pm \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{5 \pm+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]\right)
\end{aligned}$$

Problem 140: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 343 leaves, 8 steps) :

$$\begin{aligned}
& - \frac{3 (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}}{\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}}\right]}{\sqrt{26} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}} \\
& \left(2 \sqrt{14+8 \sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]\right) / \\
& \left(3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}\right) - \\
& \left(12 \times 3^{1/4} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[97-56 \sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}\right)
\end{aligned}$$

Result (type 4, 196 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{1}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} \right. \\
& \left. \left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \frac{1}{3+(-1)^{1/3}} 3 i \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{3+(-1)^{1/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 141: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{(c+d x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 452 leaves, 8 steps):

$$\begin{aligned}
& \frac{(d e - c f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2+c d+d^2}}{\sqrt{c-d} \sqrt{d}} \sqrt{\frac{1+x}{\frac{(1-x+x^2)}{(1+\sqrt{3}+x)^2}}}\right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}} + \\
& \left(2 \sqrt{2+\sqrt{3}} (e-f-\sqrt{3} f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(3^{1/4} (c-d-\sqrt{3} d) \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}\right) + \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (d e - c f) (1+x) \right. \\
& \left. \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[\frac{(c-(1+\sqrt{3}) d)^2}{(c-(1-\sqrt{3}) d)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left((c^2-2 c d-2 d^2) \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}\right)
\end{aligned}$$

Result (type 4, 211 leaves):

$$\begin{aligned}
& \frac{1}{d \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} f\left((-1)^{1/3}-x\right) \right. \\
& \left. \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \frac{1}{c+(-1)^{1/3} d} \right. \\
& \left. \pm (-d e + c f) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{\pm \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 142: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 476 leaves, 8 steps) :

$$\begin{aligned}
& \frac{\left(d e - c f\right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2-c d+d^2} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{\sqrt{d} \sqrt{c+d} \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}}\right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2-c d+d^2} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}} \\
& - \frac{\left(2 \sqrt{2+\sqrt{3}} \left(e+f+\sqrt{3} f\right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]\right)}{3^{1/4} \left(c+d+\sqrt{3} d\right) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}} \\
& + \frac{\left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} \left(d e - c f\right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{\left(c+d+\sqrt{3} d\right)^2}{\left(c+d-\sqrt{3} d\right)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]\right)}{\left(c^2+2 c d-2 d^2\right) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}
\end{aligned}$$

Result (type 4, 233 leaves) :

$$\begin{aligned}
 & \frac{1}{3 d \sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\begin{array}{l} \frac{1}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} \\ \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \right. \\
 & 3 f \left((-1)^{1/3} + x\right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \\
 & \left. \frac{1}{-c + (-1)^{1/3} d} (-1)^{1/3} \sqrt{3} \left(1 + (-1)^{1/3}\right) (-d e + c f) \sqrt{1+x+x^2} \right. \\
 & \left. \text{EllipticPi}\left[\frac{\frac{i}{2} \sqrt{3} d}{-c + (-1)^{1/3} d}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
 \end{aligned}$$

Problem 143: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 477 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\left(d e - c f \right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh} \left[\frac{\sqrt{c^2 - c d + d^2}}{\sqrt{d} \sqrt{c+d}} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2 - c d + d^2} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{-1+x^3}} - \\
& \left(2 \sqrt{2-\sqrt{3}} (e+f+\sqrt{3} f) (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \right. \\
& \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x} \right], -7+4\sqrt{3} \right] \right) / \\
& \left(3^{1/4} (c+d+\sqrt{3} d) \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right) + \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (d e - c f) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \right. \\
& \left. \operatorname{EllipticPi} \left[\frac{(c+d+\sqrt{3} d)^2}{(c+d-\sqrt{3} d)^2}, -\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right] \right) / \\
& \left((c^2 + 2 c d - 2 d^2) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3} \right)
\end{aligned}$$

Result (type 4, 231 leaves):

$$\begin{aligned}
& \frac{1}{3 d \sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} \right. \\
& 3 f \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3}] + \\
& \frac{1}{-c+(-1)^{1/3} d} (-1)^{1/3} \sqrt{3} \left(1+(-1)^{1/3} \right) (-d e + c f) \sqrt{1+x+x^2} \\
& \left. \text{EllipticPi}\left[\frac{\frac{i}{2} \sqrt{3} d}{-c+(-1)^{1/3} d}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] \right)
\end{aligned}$$

Problem 144: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{(c+d x) \sqrt{-1-x^3}} d x$$

Optimal (type 4, 465 leaves, 8 steps):

$$\begin{aligned}
& \frac{(d e - c f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2+c d+d^2}}{\sqrt{c-d} \sqrt{d}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2}} + \\
& \frac{2 \sqrt{2-\sqrt{3}} (e-f-\sqrt{3} f) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}}}{\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]\Bigg] / \\
& \left(3^{1/4} (c-d-\sqrt{3} d) \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}\right) + \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (d e - c f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}\right. \\
& \left.\operatorname{EllipticPi}\left[\frac{\left(c-\left(1+\sqrt{3}\right) d\right)^2}{\left(c-\left(1-\sqrt{3}\right) d\right)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\Bigg) / \\
& \left((c^2-2 c d-2 d^2) \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}\right)
\end{aligned}$$

Result (type 4, 213 leaves):

$$\begin{aligned}
& \frac{1}{d \sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{1}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} f\left((-1)^{1/3}-x\right) \right. \\
& \left. \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \frac{1}{c+(-1)^{1/3} d} \right. \\
& \left. \pm (-d e + c f) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{\pm \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 145: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{x \sqrt{1+x^3}} dx$$

Optimal (type 4, 120 leaves, 6 steps):

$$\begin{aligned} & -\frac{2}{3} e \operatorname{ArcTanh}\left[\sqrt{1+x^3}\right] + \\ & \left(2 \sqrt{2+\sqrt{3}} f (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]\right) / \\ & \left(3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}\right) \end{aligned}$$

Result (type 4, 134 leaves):

$$\begin{aligned} & -\frac{2}{3} e \operatorname{ArcTanh}\left[\sqrt{1+x^3}\right] - \left(2 f \left((-1)^{1/3}-x\right) \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} \left((-1)^{2/3}+x\right)}{1+(-1)^{1/3}}}\right. \\ & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]\right) / \left(\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1+x^3}\right) \end{aligned}$$

Problem 146: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{x \sqrt{1-x^3}} dx$$

Optimal (type 4, 134 leaves, 6 steps):

$$\begin{aligned} & -\frac{2}{3} e \operatorname{ArcTanh}\left[\sqrt{1-x^3}\right] - \\ & \left(2 \sqrt{2+\sqrt{3}} f (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]\right) / \\ & \left(3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}\right) \end{aligned}$$

Result (type 4, 140 leaves):

$$\begin{aligned}
 & -\frac{2}{3} e \operatorname{ArcTanh}\left[\sqrt{1-x^3}\right] + \left(2 f \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3}+x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right. \\
 & \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]\right) / \left(\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1-x^3}\right)
 \end{aligned}$$

Problem 147: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{x \sqrt{-1+x^3}} dx$$

Optimal (type 4, 137 leaves, 6 steps):

$$\begin{aligned}
 & \frac{2}{3} e \operatorname{ArcTan}\left[\sqrt{-1+x^3}\right] - \\
 & \left(2 \sqrt{2-\sqrt{3}} f (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]\right) / \\
 & \left(3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}\right)
 \end{aligned}$$

Result (type 4, 136 leaves):

$$\begin{aligned}
 & \frac{2}{3} e \operatorname{ArcTan}\left[\sqrt{-1+x^3}\right] + \left(2 f \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3}+x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right. \\
 & \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]\right) / \left(\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{-1+x^3}\right)
 \end{aligned}$$

Problem 148: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{x \sqrt{-1-x^3}} dx$$

Optimal (type 4, 131 leaves, 6 steps):

$$\begin{aligned} & \frac{2}{3} e \operatorname{ArcTan}\left[\sqrt{-1-x^3}\right] + \\ & \left(2 \sqrt{2-\sqrt{3}} f(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}\right]\right) / \\ & \left(3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}\right) \end{aligned}$$

Result (type 4, 138 leaves):

$$\begin{aligned} & \frac{2}{3} e \operatorname{ArcTan}\left[\sqrt{-1-x^3}\right] - \left(2 f\left(\left(-1\right)^{1/3}-x\right) \sqrt{\frac{1+x}{1+\left(-1\right)^{1/3}}} \sqrt{-\frac{\left(-1\right)^{2/3} \left(\left(-1\right)^{2/3}+x\right)}{1+\left(-1\right)^{1/3}}}\right. \\ & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+\left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}}\right], \left(-1\right)^{1/3}\right]\right) / \left(\sqrt{\frac{1+\left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}} \sqrt{-1-x^3}\right) \end{aligned}$$

Problem 149: Unable to integrate problem.

$$\int \frac{c-dx}{(c+dx)(2c^3+d^3x^3)^{1/3}} dx$$

Optimal (type 3, 95 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1+\frac{2(c+d x)}{(2 c^3+d^3 x^3)^{1/3}}}{\sqrt{3}}\right]}{d}-\frac{\operatorname{Log}[c+d x]}{d}+\frac{3 \operatorname{Log}[d(2 c+d x)-d(2 c^3+d^3 x^3)^{1/3}]}{2 d}$$

Result (type 8, 33 leaves):

$$\int \frac{c-dx}{(c+dx)(2c^3+d^3x^3)^{1/3}} dx$$

Problem 150: Unable to integrate problem.

$$\int \frac{e+f x}{(c+d x)\left(-c^3+d^3 x^3\right)^{1/3}} dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$\begin{aligned} & \frac{f \operatorname{ArcTan}\left[\frac{1+\frac{2 d x}{\left(-c^3+d^3 x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2}+\frac{\sqrt{3} (d e-c f) \operatorname{ArcTan}\left[\frac{1+\frac{2^{1/3} (c-d x)}{\left(-c^3+d^3 x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3} c d^2}+\frac{(d e-c f) \operatorname{Log}\left[(c-d x)(c+d x)^2\right]}{4 \times 2^{1/3} c d^2}- \\ & \frac{f \operatorname{Log}\left[-d x+\left(-c^3+d^3 x^3\right)^{1/3}\right]}{2 d^2}-\frac{3 (d e-c f) \operatorname{Log}\left[d(c-d x)+2^{2/3} d\left(-c^3+d^3 x^3\right)^{1/3}\right]}{4 \times 2^{1/3} c d^2} \end{aligned}$$

Result (type 8, 32 leaves) :

$$\int \frac{e + f x}{(c + d x) (-c^3 + d^3 x^3)^{1/3}} dx$$

Problem 158: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^n (c + d x^3)^2}{x} dx$$

Optimal (type 5, 209 leaves, 3 steps) :

$$\begin{aligned} & \frac{a^2 d (2 b^3 c - a^3 d) (a + b x)^{1+n}}{b^6 (1+n)} - \frac{a d (4 b^3 c - 5 a^3 d) (a + b x)^{2+n}}{b^6 (2+n)} + \\ & \frac{2 d (b^3 c - 5 a^3 d) (a + b x)^{3+n}}{b^6 (3+n)} + \frac{10 a^2 d^2 (a + b x)^{4+n}}{b^6 (4+n)} - \frac{5 a d^2 (a + b x)^{5+n}}{b^6 (5+n)} + \\ & \frac{d^2 (a + b x)^{6+n}}{b^6 (6+n)} - \frac{c^2 (a + b x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, 1 + \frac{b x}{a}]}{a (1+n)} \end{aligned}$$

Result (type 5, 420 leaves) :

$$\begin{aligned} & (a + b x)^n \left(\left(2 c d \left(1 + \frac{b x}{a} \right)^{-n} \left(-2 a^2 b n x \left(1 + \frac{b x}{a} \right)^n + a b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + b^3 (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n + \right. \right. \right. \right. \\ & \left. \left. \left. \left. 2 a^3 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) \right) \right) / \left(b^3 (1+n) (2+n) (3+n) \right) + \\ & \left(d^2 \left(1 + \frac{b x}{a} \right)^{-n} \left(120 a^5 b n x \left(1 + \frac{b x}{a} \right)^n - 60 a^4 b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + \right. \right. \\ & \left. \left. 20 a^3 b^3 n (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n - 5 a^2 b^4 n (6+11n+6n^2+n^3) x^4 \left(1 + \frac{b x}{a} \right)^n + \right. \right. \\ & \left. \left. a b^5 n (24+50n+35n^2+10n^3+n^4) x^5 \left(1 + \frac{b x}{a} \right)^n + \right. \right. \\ & \left. \left. b^6 (120+274n+225n^2+85n^3+15n^4+n^5) x^6 \left(1 + \frac{b x}{a} \right)^n - 120 a^6 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) \right) / \\ & \left(b^6 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) \right) + \\ & \left. \left. c^2 \left(1 + \frac{a}{b x} \right)^{-n} \text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{a}{b x}] \right) \right) \end{aligned}$$

Problem 159: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b x)^n (c + d x^3)^3 dx$$

Optimal (type 3, 459 leaves, 2 steps) :

$$\begin{aligned}
& \frac{a^2 (b^3 c - a^3 d)^3 (a + b x)^{1+n}}{b^{12} (1+n)} - \frac{a (2 b^3 c - 11 a^3 d) (b^3 c - a^3 d)^2 (a + b x)^{2+n}}{b^{12} (2+n)} + \\
& \frac{(b^3 c - a^3 d) (b^6 c^2 - 29 a^3 b^3 c d + 55 a^6 d^2) (a + b x)^{3+n}}{b^{12} (3+n)} + \\
& \frac{3 a^2 d (10 b^6 c^2 - 56 a^3 b^3 c d + 55 a^6 d^2) (a + b x)^{4+n}}{b^{12} (4+n)} - \\
& \frac{15 a d (b^6 c^2 - 14 a^3 b^3 c d + 22 a^6 d^2) (a + b x)^{5+n}}{b^{12} (5+n)} + \frac{3 d (b^6 c^2 - 56 a^3 b^3 c d + 154 a^6 d^2) (a + b x)^{6+n}}{b^{12} (6+n)} + \\
& \frac{42 a^2 d^2 (2 b^3 c - 11 a^3 d) (a + b x)^{7+n}}{b^{12} (7+n)} - \frac{6 a d^2 (4 b^3 c - 55 a^3 d) (a + b x)^{8+n}}{b^{12} (8+n)} + \\
& \frac{3 d^2 (b^3 c - 55 a^3 d) (a + b x)^{9+n}}{b^{12} (9+n)} + \frac{55 a^2 d^3 (a + b x)^{10+n}}{b^{12} (10+n)} - \frac{11 a d^3 (a + b x)^{11+n}}{b^{12} (11+n)} + \frac{d^3 (a + b x)^{12+n}}{b^{12} (12+n)}
\end{aligned}$$

Result (type 3, 1134 leaves):

$$\begin{aligned}
& \left((a + b x)^{1+n} (-39916800 a^{11} d^3 + 39916800 a^{10} b d^3 (1+n) x - 19958400 a^9 b^2 d^3 (2+3n+n^2) x^2 + \right. \\
& \quad 120960 a^8 b^3 d^2 (c (1320 + 362 n + 33 n^2 + n^3) + 55 d (6 + 11 n + 6 n^2 + n^3) x^3) - \\
& \quad 30240 a^7 b^4 d^2 (1+n) x (4 c (1320 + 362 n + 33 n^2 + n^3) + 55 d (24 + 26 n + 9 n^2 + n^3) x^3) + \\
& \quad 30240 a^6 b^5 d^2 (2+3n+n^2) x^2 (2 c (1320 + 362 n + 33 n^2 + n^3) + 11 d (60 + 47 n + 12 n^2 + n^3) x^3) - \\
& \quad 360 a^5 b^6 d (c^2 (665280 + 434568 n + 117454 n^2 + 16815 n^3 + 1345 n^4 + 57 n^5 + n^6) + \\
& \quad 56 c d (7920 + 16692 n + 12100 n^2 + 3861 n^3 + 571 n^4 + 39 n^5 + n^6) x^3 + \\
& \quad 154 d^2 (720 + 1764 n + 1624 n^2 + 735 n^3 + 175 n^4 + 21 n^5 + n^6) x^6) + \\
& \quad 360 a^4 b^7 d (1+n) x (c^2 (665280 + 434568 n + 117454 n^2 + 16815 n^3 + 1345 n^4 + 57 n^5 + n^6) + \\
& \quad 14 c d (31680 + 43008 n + 22084 n^2 + 5460 n^3 + 685 n^4 + 42 n^5 + n^6) x^3 + \\
& \quad 22 d^2 (5040 + 8028 n + 5104 n^2 + 1665 n^3 + 295 n^4 + 27 n^5 + n^6) x^6) - 18 a^3 b^8 d (2+3n+n^2) \\
& \quad x^2 (10 c^2 (665280 + 434568 n + 117454 n^2 + 16815 n^3 + 1345 n^4 + 57 n^5 + n^6) + \\
& \quad 56 c d (79200 + 83760 n + 34834 n^2 + 7275 n^3 + 805 n^4 + 45 n^5 + n^6) x^3 + \\
& \quad 55 d^2 (20160 + 24552 n + 12154 n^2 + 3135 n^3 + 445 n^4 + 33 n^5 + n^6) x^6) + \\
& b^{11} (246400 + 593520 n + 541508 n^2 + 251352 n^3 + 66489 n^4 + 10440 n^5 + 962 n^6 + 48 n^7 + n^8) \\
& \quad x^2 (c^3 (648 + 234 n + 27 n^2 + n^3) + 3 c^2 d (324 + 171 n + 24 n^2 + n^3) x^3 + \\
& \quad 3 c d^2 (216 + 126 n + 21 n^2 + n^3) x^6 + d^3 (162 + 99 n + 18 n^2 + n^3) x^9) - \\
& a b^{10} (280 + 418 n + 159 n^2 + 22 n^3 + n^4) x \\
& \quad (2 c^3 (285120 + 221544 n + 70254 n^2 + 11645 n^3 + 1065 n^4 + 51 n^5 + n^6) + \\
& \quad 15 c^2 d (57024 + 70920 n + 32574 n^2 + 7115 n^3 + 801 n^4 + 45 n^5 + n^6) x^3 + \\
& \quad 24 c d^2 (23760 + 32652 n + 17160 n^2 + 4421 n^3 + 591 n^4 + 39 n^5 + n^6) x^6 + \\
& \quad 11 d^3 (12960 + 18612 n + 10404 n^2 + 2915 n^3 + 435 n^4 + 33 n^5 + n^6) x^9) + \\
& 2 a^2 b^9 (c^3 (79833600 + 101378880 n + 56231712 n^2 + 17893196 n^3 + 3602088 n^4 + \\
& \quad 476049 n^5 + 41328 n^6 + 2274 n^7 + 72 n^8 + n^9) + 30 c^2 d (3991680 + 9925488 n + \\
& \quad 9476652 n^2 + 4665572 n^3 + 1332327 n^4 + 233481 n^5 + 25518 n^6 + 1698 n^7 + 63 n^8 + n^9) x^3 + \\
& 84 c d^2 (950400 + 2589120 n + 2806008 n^2 + 1617020 n^3 + 552426 n^4 + 116949 n^5 + \\
& \quad 15432 n^6 + 1230 n^7 + 54 n^8 + n^9) x^6 + 55 d^3 (362880 + 1026576 n + 1172700 n^2 + \\
& \quad 723680 n^3 + 269325 n^4 + 63273 n^5 + 9450 n^6 + 870 n^7 + 45 n^8 + n^9) x^9)) / \\
& (b^{12} (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) \\
& \quad (8+n) \\
& \quad (9+n) \\
& \quad (10+n) \\
& \quad (11+n) \\
& \quad (12+n))
\end{aligned}$$

Problem 160: Result more than twice size of optimal antiderivative.

$$\int x (a + b x)^n (c + d x^3)^3 dx$$

Optimal (type 3, 396 leaves, 2 steps):

$$\begin{aligned}
& - \frac{a (b^3 c - a^3 d)^3 (a + b x)^{1+n}}{b^{11} (1+n)} + \frac{(b^3 c - 10 a^3 d) (b^3 c - a^3 d)^2 (a + b x)^{2+n}}{b^{11} (2+n)} + \\
& \frac{9 a^2 d (2 b^3 c - 5 a^3 d) (b^3 c - a^3 d) (a + b x)^{3+n}}{b^{11} (3+n)} - \frac{3 a d (4 b^6 c^2 - 35 a^3 b^3 c d + 40 a^6 d^2) (a + b x)^{4+n}}{b^{11} (4+n)} + \\
& \frac{3 d (b^6 c^2 - 35 a^3 b^3 c d + 70 a^6 d^2) (a + b x)^{5+n}}{b^{11} (5+n)} + \frac{63 a^2 d^2 (b^3 c - 4 a^3 d) (a + b x)^{6+n}}{b^{11} (6+n)} - \\
& \frac{21 a d^2 (b^3 c - 10 a^3 d) (a + b x)^{7+n}}{b^{11} (7+n)} + \frac{3 d^2 (b^3 c - 40 a^3 d) (a + b x)^{8+n}}{b^{11} (8+n)} + \\
& \frac{45 a^2 d^3 (a + b x)^{9+n}}{b^{11} (9+n)} - \frac{10 a d^3 (a + b x)^{10+n}}{b^{11} (10+n)} + \frac{d^3 (a + b x)^{11+n}}{b^{11} (11+n)}
\end{aligned}$$

Result (type 3, 903 leaves):

$$\begin{aligned}
& \left((a + b x)^{1+n} (3628800 a^{10} d^3 - 3628800 a^9 b d^3 (1+n) x + 1814400 a^8 b^2 d^3 (2+3 n+n^2) x^2 - \right. \\
& 15120 a^7 b^3 d^2 (c (990+299 n+30 n^2+n^3) + 40 d (6+11 n+6 n^2+n^3) x^3) + \\
& 15120 a^6 b^4 d^2 (1+n) x (c (990+299 n+30 n^2+n^3) + 10 d (24+26 n+9 n^2+n^3) x^3) - \\
& 7560 a^5 b^5 d^2 (2+3 n+n^2) x^2 (c (990+299 n+30 n^2+n^3) + 4 d (60+47 n+12 n^2+n^3) x^3) + \\
& 72 a^4 b^6 d (c^2 (332640+245004 n+74524 n^2+11985 n^3+1075 n^4+51 n^5+n^6) + \\
& 35 c d (5940+12684 n+9409 n^2+3120 n^3+490 n^4+36 n^5+n^6) x^3 + \\
& 70 d^2 (720+1764 n+1624 n^2+735 n^3+175 n^4+21 n^5+n^6) x^6) - \\
& 18 a^3 b^7 d (1+n) x (4 c^2 (332640+245004 n+74524 n^2+11985 n^3+1075 n^4+51 n^5+n^6) + \\
& 35 c d (23760+32916 n+17404 n^2+4485 n^3+595 n^4+39 n^5+n^6) x^3 + \\
& 40 d^2 (5040+8028 n+5104 n^2+1665 n^3+295 n^4+27 n^5+n^6) x^6) + \\
& 18 a^2 b^8 d (2+3 n+n^2) x^2 (2 c^2 (332640+245004 n+74524 n^2+11985 n^3+1075 n^4+51 n^5+n^6) + \\
& 7 c d (59400+64470 n+27733 n^2+6048 n^3+706 n^4+42 n^5+n^6) x^3 + \\
& 5 d^2 (20160+24552 n+12154 n^2+3135 n^3+445 n^4+33 n^5+n^6) x^6) + \\
& b^{10} (45360+95436 n+72180 n^2+27109 n^3+5620 n^4+654 n^5+40 n^6+n^7) x \\
& (c^3 (440+183 n+24 n^2+n^3) + 3 c^2 d (176+126 n+21 n^2+n^3) x^3 + \\
& 3 c d^2 (110+87 n+18 n^2+n^3) x^6 + d^3 (80+66 n+15 n^2+n^3) x^9) - \\
& a b^9 (162+99 n+18 n^2+n^3) (c^3 (123200+111960 n+41214 n^2+7875 n^3+825 n^4+45 n^5+n^6) + \\
& 12 c^2 d (12320+24132 n+15600 n^2+4341 n^3+591 n^4+39 n^5+n^6) x^3 + \\
& 21 c d^2 (4400+9420 n+7068 n^2+2427 n^3+411 n^4+33 n^5+n^6) x^6 + \\
& 10 d^3 (2240+4968 n+3954 n^2+1485 n^3+285 n^4+27 n^5+n^6) x^9)) / \\
& (b^{11} (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) \\
& (8+n) \\
& (9+n) \\
& (10+n) \\
& (11+n))
\end{aligned}$$

Problem 161: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^n (c + d x^3)^3 dx$$

Optimal (type 3, 337 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b^3 c - a^3 d)^3 (a + b x)^{1+n}}{b^{10} (1+n)} + \frac{9 a^2 d (b^3 c - a^3 d)^2 (a + b x)^{2+n}}{b^{10} (2+n)} - \\
& \frac{9 a d (b^3 c - 4 a^3 d) (b^3 c - a^3 d) (a + b x)^{3+n}}{b^{10} (3+n)} + \frac{3 d (b^6 c^2 - 20 a^3 b^3 c d + 28 a^6 d^2) (a + b x)^{4+n}}{b^{10} (4+n)} + \\
& \frac{9 a^2 d^2 (5 b^3 c - 14 a^3 d) (a + b x)^{5+n}}{b^{10} (5+n)} - \frac{18 a d^2 (b^3 c - 7 a^3 d) (a + b x)^{6+n}}{b^{10} (6+n)} + \\
& \frac{3 d^2 (b^3 c - 28 a^3 d) (a + b x)^{7+n}}{b^{10} (7+n)} + \frac{36 a^2 d^3 (a + b x)^{8+n}}{b^{10} (8+n)} - \frac{9 a d^3 (a + b x)^{9+n}}{b^{10} (9+n)} + \frac{d^3 (a + b x)^{10+n}}{b^{10} (10+n)}
\end{aligned}$$

Result (type 3, 706 leaves) :

$$\begin{aligned}
& \left((a + b x)^{1+n} (-362880 a^9 d^3 + 362880 a^8 b d^3 (1+n) x - 181440 a^7 b^2 d^3 (2+3 n+n^2) x^2 + \right. \\
& 2160 a^6 b^3 d^2 (c (720+242 n+27 n^2+n^3) + 28 d (6+11 n+6 n^2+n^3) x^3) - \\
& 2160 a^5 b^4 d^2 (1+n) x (c (720+242 n+27 n^2+n^3) + 7 d (24+26 n+9 n^2+n^3) x^3) + \\
& 216 a^4 b^5 d^2 (2+3 n+n^2) x^2 (5 c (720+242 n+27 n^2+n^3) + 14 d (60+47 n+12 n^2+n^3) x^3) - \\
& 9 a b^8 d (80+146 n+81 n^2+16 n^3+n^4) x^2 (c^2 (3780+1968 n+379 n^2+32 n^3+n^4) + \\
& 2 c d (1080+858 n+235 n^2+26 n^3+n^4) x^3 + d^2 (504+450 n+145 n^2+20 n^3+n^4) x^6) - \\
& 18 a^3 b^6 d (c^2 (151200+127860 n+44524 n^2+8175 n^3+835 n^4+45 n^5+n^6) + \\
& 20 c d (4320+9372 n+7144 n^2+2475 n^3+415 n^4+33 n^5+n^6) x^3 + \\
& 28 d^2 (720+1764 n+1624 n^2+735 n^3+175 n^4+21 n^5+n^6) x^6) + \\
& 18 a^2 b^7 d (1+n) x (c^2 (151200+127860 n+44524 n^2+8175 n^3+835 n^4+45 n^5+n^6) + \\
& 5 c d (17280+24528 n+13420 n^2+3624 n^3+511 n^4+36 n^5+n^6) x^3 + \\
& 4 d^2 (5040+8028 n+5104 n^2+1665 n^3+295 n^4+27 n^5+n^6) x^6) + \\
& b^9 (12960+18612 n+10404 n^2+2915 n^3+435 n^4+33 n^5+n^6) \\
& \left. (c^3 (280+138 n+21 n^2+n^3) + 3 c^2 d (70+87 n+18 n^2+n^3) x^3 + \right. \\
& \left. 3 c d^2 (40+54 n+15 n^2+n^3) x^6 + d^3 (28+39 n+12 n^2+n^3) x^9) \right) / \\
& (b^{10} (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) \\
& (9+n) \\
& (10+n))
\end{aligned}$$

Problem 162: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^n (c + d x^3)^3}{x} dx$$

Optimal (type 5, 358 leaves, 3 steps) :

$$\begin{aligned}
& \frac{a^2 d (3 b^6 c^2 - 3 a^3 b^3 c d + a^6 d^2) (a + b x)^{1+n}}{b^9 (1+n)} - \frac{a d (6 b^6 c^2 - 15 a^3 b^3 c d + 8 a^6 d^2) (a + b x)^{2+n}}{b^9 (2+n)} + \\
& \frac{d (3 b^6 c^2 - 30 a^3 b^3 c d + 28 a^6 d^2) (a + b x)^{3+n}}{b^9 (3+n)} + \frac{2 a^2 d^2 (15 b^3 c - 28 a^3 d) (a + b x)^{4+n}}{b^9 (4+n)} - \\
& \frac{5 a d^2 (3 b^3 c - 14 a^3 d) (a + b x)^{5+n}}{b^9 (5+n)} + \frac{d^2 (3 b^3 c - 56 a^3 d) (a + b x)^{6+n}}{b^9 (6+n)} + \frac{28 a^2 d^3 (a + b x)^{7+n}}{b^9 (7+n)} - \\
& \frac{8 a d^3 (a + b x)^{8+n}}{b^9 (8+n)} + \frac{d^3 (a + b x)^{9+n}}{b^9 (9+n)} - \frac{c^3 (a + b x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, 1 + \frac{b x}{a}]}{a (1+n)}
\end{aligned}$$

Result (type 5, 856 leaves):

$$\begin{aligned}
& (a + b x)^n \left(\left(3 c^2 d \left(1 + \frac{b x}{a} \right)^{-n} \left(-2 a^2 b n x \left(1 + \frac{b x}{a} \right)^n + a b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + \right. \right. \right. \right. \\
& \left. \left. \left. \left. b^3 (2 + 3 n + n^2) x^3 \left(1 + \frac{b x}{a} \right)^n + 2 a^3 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) \right) \right) / \left(b^3 (1+n) (2+n) (3+n) \right) + \\
& \frac{1}{b^6 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n)} 3 c d^2 \left(1 + \frac{b x}{a} \right)^{-n} \\
& \left(120 a^5 b n x \left(1 + \frac{b x}{a} \right)^n - 60 a^4 b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + 20 a^3 b^3 n (2 + 3 n + n^2) x^3 \left(1 + \frac{b x}{a} \right)^n - \right. \\
& \left. 5 a^2 b^4 n (6 + 11 n + 6 n^2 + n^3) x^4 \left(1 + \frac{b x}{a} \right)^n + a b^5 n (24 + 50 n + 35 n^2 + 10 n^3 + n^4) x^5 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& \left. b^6 (120 + 274 n + 225 n^2 + 85 n^3 + 15 n^4 + n^5) x^6 \left(1 + \frac{b x}{a} \right)^n - 120 a^6 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) + \\
& \left(1 / \left(b^9 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) (9+n) \right) \right) \\
& d^3 \left(1 + \frac{b x}{a} \right)^{-n} \left(-40320 a^8 b n x \left(1 + \frac{b x}{a} \right)^n + 20160 a^7 b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n - \right. \\
& \left. 6720 a^6 b^3 n (2 + 3 n + n^2) x^3 \left(1 + \frac{b x}{a} \right)^n + 1680 a^5 b^4 n (6 + 11 n + 6 n^2 + n^3) x^4 \left(1 + \frac{b x}{a} \right)^n - \right. \\
& \left. 336 a^4 b^5 n (24 + 50 n + 35 n^2 + 10 n^3 + n^4) x^5 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& \left. 56 a^3 b^6 n (120 + 274 n + 225 n^2 + 85 n^3 + 15 n^4 + n^5) x^6 \left(1 + \frac{b x}{a} \right)^n - \right. \\
& \left. 8 a^2 b^7 n (720 + 1764 n + 1624 n^2 + 735 n^3 + 175 n^4 + 21 n^5 + n^6) x^7 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& \left. a b^8 n (5040 + 13068 n + 13132 n^2 + 6769 n^3 + 1960 n^4 + 322 n^5 + 28 n^6 + n^7) x^8 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& \left. b^9 (40320 + 109584 n + 118124 n^2 + 67284 n^3 + 22449 n^4 + 4536 n^5 + 546 n^6 + 36 n^7 + n^8) \right. \\
& \left. x^9 \left(1 + \frac{b x}{a} \right)^n + 40320 a^9 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) + \\
& \frac{c^3 \left(1 + \frac{a}{b x} \right)^{-n} \text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{a}{b x}]}{n}
\end{aligned}$$

Problem 163: Result is not expressed in closed-form.

$$\int \frac{x^5 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 324 leaves, 7 steps) :

$$\begin{aligned} & \frac{e^2 (e + f x)^{1+n}}{b f^3 (1+n)} - \frac{2 e (e + f x)^{2+n}}{b f^3 (2+n)} + \frac{(e + f x)^{3+n}}{b f^3 (3+n)} + \\ & \frac{a (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}]}{3 b^{5/3} (b^{1/3} e - a^{1/3} f) (1+n)} + \\ & \frac{a (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e + (-1)^{1/3} a^{1/3} f}]}{3 b^{5/3} (b^{1/3} e + (-1)^{1/3} a^{1/3} f) (1+n)} + \\ & \frac{a (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - (-1)^{2/3} a^{1/3} f}]}{3 b^{5/3} (b^{1/3} e - (-1)^{2/3} a^{1/3} f) (1+n)} \end{aligned}$$

Result (type 7, 423 leaves) :

$$\begin{aligned} & \frac{1}{3 b f^3} (e + f x)^n \left(\left(3 \left(-2 e^2 f n x + e f^2 n (1+n) x^2 + f^3 (2+3 n+n^2) x^3 + e^3 \left(2 - 2 \left(1 + \frac{f x}{e} \right)^{-n} \right) \right) \right) / \right. \\ & (6 + 11 n + 6 n^2 + n^3) - \frac{1}{b n} a f^3 \left(e^2 \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \\ & \left. \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] - \right. \\ & 2 e \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \\ & \left. \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] + \\ & \left. \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \\ & \left. \left. \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1^2}{e^2 - 2 e \#1 + \#1^2} \&] \right) \right) \end{aligned}$$

Problem 164: Result is not expressed in closed-form.

$$\int \frac{x^4 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 332 leaves, 7 steps) :

$$\begin{aligned}
& - \frac{e (e + f x)^{1+n}}{b f^2 (1+n)} + \frac{(e + f x)^{2+n}}{b f^2 (2+n)} - \frac{a^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}]}{3 b^{4/3} (b^{1/3} e - a^{1/3} f) (1+n)} + \\
& \left((-1)^{1/3} a^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}] \right) / \\
& \left(3 b^{4/3} ((-1)^{2/3} b^{1/3} e - a^{1/3} f) (1+n) \right) + \\
& \left((-1)^{2/3} a^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}] \right) / \\
& \left(3 b^{4/3} ((-1)^{1/3} b^{1/3} e + a^{1/3} f) (1+n) \right)
\end{aligned}$$

Result (type 7, 298 leaves):

$$\begin{aligned}
& \frac{1}{3 b f^2} (e + f x)^n \left(- \frac{3 (-e f n x - f^2 (1+n) x^2 + e^2 (1 - (1 + \frac{f x}{e})^{-n}))}{2 + 3 n + n^2} + \right. \\
& \frac{1}{b n} a e f^3 \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \\
& \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&] - \\
& \frac{1}{b n} a f^3 \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \\
& \left. \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] \right)
\end{aligned}$$

Problem 165: Result is not expressed in closed-form.

$$\int \frac{x^3 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 293 leaves, 7 steps):

$$\begin{aligned}
& \frac{(e + f x)^{1+n}}{b f (1+n)} + \frac{a^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}]}{3 b (b^{1/3} e - a^{1/3} f) (1+n)} + \\
& \frac{a^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}]}{3 b ((-1)^{2/3} b^{1/3} e - a^{1/3} f) (1+n)} - \\
& \frac{a^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}]}{3 b ((-1)^{1/3} b^{1/3} e + a^{1/3} f) (1+n)}
\end{aligned}$$

Result (type 7, 142 leaves):

$$\frac{1}{3 b^2 f} (e + f x)^n \left(\frac{3 b (e + f x)}{1 + n} - \frac{1}{n} a f^3 \text{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \right.$$

$$\left. \left. \frac{\text{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1}\right] \left(\frac{e + f x}{e + f x - \#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \& \right] \right)$$

Problem 166: Result is not expressed in closed-form.

$$\int \frac{x^2 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 253 leaves, 5 steps) :

$$-\frac{(e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/3} (e + f x)}{b^{1/3} e - a^{1/3} f}\right]}{3 b^{2/3} (b^{1/3} e - a^{1/3} f) (1 + n)} -$$

$$\frac{(e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/3} (e + f x)}{b^{1/3} e + (-1)^{1/3} a^{1/3} f}\right]}{3 b^{2/3} (b^{1/3} e + (-1)^{1/3} a^{1/3} f) (1 + n)} -$$

$$\frac{(e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{b^{1/3} (e + f x)}{b^{1/3} e - (-1)^{2/3} a^{1/3} f}\right]}{3 b^{2/3} (b^{1/3} e - (-1)^{2/3} a^{1/3} f) (1 + n)}$$

Result (type 7, 337 leaves) :

$$\frac{1}{3 b n} (e + f x)^n \left(e^2 \text{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \right.$$

$$\left. \left. \frac{\text{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1}\right] \left(\frac{e + f x}{e + f x - \#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \& \right] - \right.$$

$$2 e \text{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right.$$

$$\left. \frac{\text{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1}\right] \left(\frac{e + f x}{e + f x - \#1}\right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \& \right] +$$

$$\left. \text{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \right.$$

$$\left. \left. \frac{\text{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{\#1}{e + f x - \#1}\right] \left(\frac{e + f x}{e + f x - \#1}\right)^{-n} \#1^2}{e^2 - 2 e \#1 + \#1^2} \& \right] \right)$$

Problem 167: Result is not expressed in closed-form.

$$\int \frac{x (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 288 leaves, 5 steps) :

$$\begin{aligned}
& \frac{\left(e + fx\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3} (e+fx)}{b^{1/3} e - a^{1/3} f}\right]}{3 a^{1/3} b^{1/3} (b^{1/3} e - a^{1/3} f) (1+n)} - \\
& \left((-1)^{1/3} (e + fx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+fx)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}\right] \right) / \\
& \quad \left(3 a^{1/3} b^{1/3} \left((-1)^{2/3} b^{1/3} e - a^{1/3} f \right) (1+n) \right) - \\
& \left((-1)^{2/3} (e + fx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+fx)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}\right] \right) / \\
& \quad \left(3 a^{1/3} b^{1/3} \left((-1)^{1/3} b^{1/3} e + a^{1/3} f \right) (1+n) \right)
\end{aligned}$$

Result (type 7, 229 leaves):

$$\begin{aligned}
& -\frac{1}{3 b n} f (e + fx)^n \left(e \text{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \right. \\
& \quad \left. \left. \frac{\text{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}\right] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&\right] - \right. \\
& \quad \left. \text{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \right. \\
& \quad \left. \left. \frac{\text{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}\right] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&\right] \right)
\end{aligned}$$

Problem 168: Result is not expressed in closed-form.

$$\int \frac{(e + fx)^n}{a + b x^3} dx$$

Optimal (type 5, 263 leaves, 5 steps):

$$\begin{aligned}
& -\frac{\left(e + fx\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3} (e+fx)}{b^{1/3} e - a^{1/3} f}\right]}{3 a^{2/3} (b^{1/3} e - a^{1/3} f) (1+n)} - \\
& \frac{\left(e + fx\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+fx)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}\right]}{3 a^{2/3} \left((-1)^{2/3} b^{1/3} e - a^{1/3} f \right) (1+n)} + \\
& \frac{\left(e + fx\right)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+fx)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}\right]}{3 a^{2/3} \left((-1)^{1/3} b^{1/3} e + a^{1/3} f \right) (1+n)}
\end{aligned}$$

Result (type 7, 122 leaves):

$$\begin{aligned}
& \frac{1}{3 b n} f^2 (e + fx)^n \text{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \\
& \quad \left. \frac{\text{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}\right] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&\right]
\end{aligned}$$

Problem 169: Result is not expressed in closed-form.

$$\int \frac{(e + f x)^n}{x (a + b x^3)} dx$$

Optimal (type 5, 300 leaves, 8 steps) :

$$\begin{aligned} & \frac{b^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}]}{3 a (b^{1/3} e - a^{1/3} f) (1+n)} + \\ & \frac{b^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e + (-1)^{1/3} a^{1/3} f}]}{3 a (b^{1/3} e + (-1)^{1/3} a^{1/3} f) (1+n)} + \\ & \frac{b^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - (-1)^{2/3} a^{1/3} f}]}{3 a (b^{1/3} e - (-1)^{2/3} a^{1/3} f) (1+n)} - \\ & \frac{(e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, 1 + \frac{f x}{e}]}{a e (1+n)} \end{aligned}$$

Result (type 7, 377 leaves) :

$$\begin{aligned} & \frac{1}{3 a n} (e + f x)^n \left(3 \left(1 + \frac{e}{f x} \right)^{-n} \text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{e}{f x}] - \right. \\ & \left. e^2 \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \\ & \left. \text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \&] + \\ & \left. e^2 - 2 e \#1 + \#1^2 \right. \\ & 2 e \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \\ & \left. \text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1 \&] - \\ & \left. e^2 - 2 e \#1 + \#1^2 \right. \\ & \left. \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \right. \\ & \left. \text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1^2 \&] \right) \end{aligned}$$

Problem 170: Result is not expressed in closed-form.

$$\int \frac{(e + f x)^n}{x^2 (a + b x^3)} dx$$

Optimal (type 5, 326 leaves, 8 steps) :

$$\begin{aligned}
& - \frac{b^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}]}{3 a^{4/3} (b^{1/3} e - a^{1/3} f) (1+n)} + \\
& \left((-1)^{1/3} b^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}] \right) / \\
& \left(3 a^{4/3} \left((-1)^{2/3} b^{1/3} e - a^{1/3} f \right) (1+n) \right) + \\
& \left((-1)^{2/3} b^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}] \right) / \\
& \left(3 a^{4/3} \left((-1)^{1/3} b^{1/3} e + a^{1/3} f \right) (1+n) \right) + \frac{f (e + f x)^{1+n} \text{Hypergeometric2F1}[2, 1+n, 2+n, 1 + \frac{f x}{e}]}{a e^2 (1+n)}
\end{aligned}$$

Result (type 7, 280 leaves) :

$$\begin{aligned}
& \frac{1}{3 a} (e + f x)^n \left(\frac{3 \left(1 + \frac{e}{f x} \right)^{-n} \text{Hypergeometric2F1}[1-n, -n, 2-n, -\frac{e}{f x}]}{(-1+n) x} + \right. \\
& \frac{1}{n} e f \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \\
& \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&] - \\
& \frac{1}{n} f \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \\
& \left. \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] \right)
\end{aligned}$$

Problem 171: Result is not expressed in closed-form.

$$\int \frac{x^2 (c + d x)^{1+n}}{a + b x^3} dx$$

Optimal (type 5, 253 leaves, 5 steps) :

$$\begin{aligned}
& - \frac{(c + d x)^{2+n} \text{Hypergeometric2F1}[1, 2+n, 3+n, \frac{b^{1/3} (c+d x)}{b^{1/3} c - a^{1/3} d}]}{3 b^{2/3} (b^{1/3} c - a^{1/3} d) (2+n)} - \\
& \frac{(c + d x)^{2+n} \text{Hypergeometric2F1}[1, 2+n, 3+n, \frac{b^{1/3} (c+d x)}{b^{1/3} c + (-1)^{1/3} a^{1/3} d}]}{3 b^{2/3} (b^{1/3} c + (-1)^{1/3} a^{1/3} d) (2+n)} - \\
& \frac{(c + d x)^{2+n} \text{Hypergeometric2F1}[1, 2+n, 3+n, \frac{b^{1/3} (c+d x)}{b^{1/3} c - (-1)^{2/3} a^{1/3} d}]}{3 b^{2/3} (b^{1/3} c - (-1)^{2/3} a^{1/3} d) (2+n)}
\end{aligned}$$

Result (type 7, 375 leaves) :

$$\begin{aligned}
& \frac{1}{3 b^2 n (1+n)} (c+d x)^n \left((b c^3 - a d^3) (1+n) \operatorname{RootSum}[b c^3 - a d^3 - 3 b c^2 \#1 + 3 b c \#1^2 - b \#1^3 \&, \right. \\
& \quad \left. \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n}}{c^2 - 2 c \#1 + \#1^2} \&] + \right. \\
& \quad b \left(3 n (c+d x) - 2 c^2 (1+n) \operatorname{RootSum}[b c^3 - a d^3 - 3 b c^2 \#1 + 3 b c \#1^2 - b \#1^3 \&, \right. \\
& \quad \left. \left. \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1}{c^2 - 2 c \#1 + \#1^2} \&] + \right. \\
& \quad c (1+n) \operatorname{RootSum}[b c^3 - a d^3 - 3 b c^2 \#1 + 3 b c \#1^2 - b \#1^3 \&, \\
& \quad \left. \left. \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^2}{c^2 - 2 c \#1 + \#1^2} \&] \right) \right)
\end{aligned}$$

Problem 172: Unable to integrate problem.

$$\int \frac{x^m (e+f x)^n}{a+b x^3} dx$$

Optimal (type 6, 211 leaves, 8 steps):

$$\begin{aligned}
& \frac{x^{1+m} (e+f x)^n \left(1+\frac{f x}{e}\right)^{-n} \operatorname{AppellF1}[1+m, -n, 1, 2+m, -\frac{f x}{e}, -\frac{b^{1/3} x}{a^{1/3}}]}{3 a (1+m)} + \frac{1}{3 a (1+m)} \\
& x^{1+m} (e+f x)^n \left(1+\frac{f x}{e}\right)^{-n} \operatorname{AppellF1}[1+m, -n, 1, 2+m, -\frac{f x}{e}, \frac{(-1)^{1/3} b^{1/3} x}{a^{1/3}}] + \\
& \frac{1}{3 a (1+m)} x^{1+m} (e+f x)^n \left(1+\frac{f x}{e}\right)^{-n} \operatorname{AppellF1}[1+m, -n, 1, 2+m, -\frac{f x}{e}, -\frac{(-1)^{2/3} b^{1/3} x}{a^{1/3}}]
\end{aligned}$$

Result (type 8, 22 leaves):

$$\int \frac{x^m (e+f x)^n}{a+b x^3} dx$$

Problem 173: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c+d x^3}}{a+b x} dx$$

Optimal (type 4, 1482 leaves, 13 steps):

$$\frac{2 \sqrt{c+d x^3}}{3 b} - \frac{2 a d^{1/3} \sqrt{c+d x^3}}{b^2 \left(\left(1+\sqrt{3}\right) c^{1/3} + d^{1/3} x\right)}$$

$$\begin{aligned}
& \left(c^{1/6} \sqrt{b c^{1/3} - a d^{1/3}} \sqrt{b^2 c^{2/3} + a b c^{1/3} d^{1/3} + a^2 d^{2/3}} (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} \left(1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}}\right)}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \right. \\
& \left. \operatorname{ArcTanh}\left[\sqrt{2 - \sqrt{3}} \sqrt{b^2 c^{2/3} + a b c^{1/3} d^{1/3} + a^2 d^{2/3}} \sqrt{1 - \frac{\left(\left(1 - \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}}\right] \right. \\
& \left. \left. \left. 3^{1/4} \sqrt{b} c^{1/6} \sqrt{b c^{1/3} - a d^{1/3}} \sqrt{7 - 4 \sqrt{3} + \frac{\left(\left(1 - \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}}\right]\right) \\
& \left(b^{5/2} \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \sqrt{c + d x^3} \right) + \left(3^{1/4} \sqrt{2 - \sqrt{3}} a c^{1/3} d^{1/3} (c^{1/3} + d^{1/3} x) \right. \\
& \left. \left. \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) c^{1/3} + d^{1/3} x}{\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \\
& \left(b^2 \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \sqrt{c + d x^3} \right) + \\
& \left(2 \sqrt{2 + \sqrt{3}} a \left(\left(1 - \sqrt{3}\right) b c^{1/3} + a d^{1/3}\right) d^{1/3} (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) c^{1/3} + d^{1/3} x}{\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \\
& \left(3^{1/4} b^3 \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \sqrt{c + d x^3} \right) - \\
& \left(2 \sqrt{2 + \sqrt{3}} (b^3 c - a^3 d) (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) c^{1/3} + d^{1/3} x}{\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right)
\end{aligned}$$

$$\begin{aligned}
& \left(3^{1/4} b^3 \left((1 + \sqrt{3}) b c^{1/3} - a d^{1/3} \right) \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \sqrt{c + d x^3} \right) - \\
& \left(4 \times 3^{1/4} \sqrt{2 + \sqrt{3}} c^{1/3} (b^3 c - a^3 d) (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} \left(1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}} \right)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \right. \\
& \left. \text{EllipticPi} \left[\frac{\left((1 + \sqrt{3}) b c^{1/3} - a d^{1/3} \right)^2}{\left((1 - \sqrt{3}) b c^{1/3} - a d^{1/3} \right)^2}, -\text{ArcSin} \left[\frac{(1 - \sqrt{3}) c^{1/3} + d^{1/3} x}{(1 + \sqrt{3}) c^{1/3} + d^{1/3} x}, -7 - 4 \sqrt{3} \right] \right] \right) / \\
& \left(b^2 (2 b^2 c^{2/3} + 2 a b c^{1/3} d^{1/3} - a^2 d^{2/3}) \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \sqrt{c + d x^3} \right)
\end{aligned}$$

Result (type 4, 820 leaves) :

$$\begin{aligned}
& \frac{1}{3 b \sqrt{c + d x^3}} \\
& 2 \left(c + d x^3 - \left(3^{3/4} a^2 d^{2/3} \left((-1)^{1/3} c^{1/3} - d^{1/3} x \right) \sqrt{\frac{c^{1/3} + d^{1/3} x}{\left(1 + (-1)^{1/3} \right) c^{1/3}}} \sqrt{(-1)^{1/6} - \frac{i d^{1/3} x}{c^{1/3}}} \right. \right. \\
& \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{\left(1 + (-1)^{1/3} \right) c^{1/3}}} \right], (-1)^{1/3} \right] \right) / \left(b^2 \sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{\left(1 + (-1)^{1/3} \right) c^{1/3}}} \right) + \right. \\
& \left(3^{3/4} a c^{1/3} d^{1/3} \left((-1)^{1/3} c^{1/3} - d^{1/3} x \right) \sqrt{\frac{i + \sqrt{3} - 2 i d^{1/3} x}{c^{1/3}}} \sqrt{\frac{i \left(1 + \frac{d^{1/3} x}{c^{1/3}} \right)}{3 i + \sqrt{3}}} \right. \\
& \left. \left(-1 + (-1)^{2/3} \right) \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{(-1)^{1/6} - \frac{i d^{1/3} x}{c^{1/3}}}{3^{1/4}}} \right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}} \right] + \right. \\
& \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{(-1)^{1/6} - \frac{i d^{1/3} x}{c^{1/3}}}{3^{1/4}}} \right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}} \right] \right) / \left(b \sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{\left(1 + (-1)^{1/3} \right) c^{1/3}}} \right) - \right)
\end{aligned}$$

$$\begin{aligned}
& \left(3 \pm b c^{4/3} \sqrt{\frac{c^{1/3} + d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}} \sqrt{1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}}} \operatorname{EllipticPi}\left[\frac{\pm \sqrt{3} b c^{1/3}}{(-1)^{1/3} b c^{1/3} + a d^{1/3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}}, (-1)^{1/3}\right]\right] \middle/ \left((-1)^{1/3} b c^{1/3} + a d^{1/3}\right) + \right. \\
& \left. \left((-1)^{1/3} \sqrt{3} \left(1 + (-1)^{1/3}\right) a^3 c^{1/3} d \sqrt{\frac{c^{1/3} + d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}} \sqrt{1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}}} \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{\pm \sqrt{3} b c^{1/3}}{(-1)^{1/3} b c^{1/3} + a d^{1/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}}, (-1)^{1/3}\right]\right] \right) \middle/ (b^2 \right. \\
& \left. \left. \left((-1)^{1/3} b c^{1/3} + a d^{1/3}\right)\right) \right)
\end{aligned}$$

Problem 174: Unable to integrate problem.

$$\int \frac{(d^3 + e^3 x^3)^p}{d + e x} dx$$

Optimal (type 6, 135 leaves, ? steps):

$$\frac{1}{e p} \left(d^3 + e^3 x^3 \right)^p \left(1 + \frac{2 (d + e x)}{\left(-3 + i \sqrt{3} \right) d} \right)^{-p} \left(1 - \frac{2 (d + e x)}{\left(3 + i \sqrt{3} \right) d} \right)^{-p}$$

$$\text{AppellF1}\left[p, -p, -p, 1 + p, -\frac{2 (d + e x)}{\left(-3 + i \sqrt{3} \right) d}, \frac{2 (d + e x)}{\left(3 + i \sqrt{3} \right) d}\right]$$

Result (type 8, 23 leaves):

$$\int \frac{(d^3 + e^3 x^3)^p}{d + e x} dx$$

Problem 175: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2 - 2x - x^2}{(2 + x^2) \sqrt{1 + x^3}} dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$2 \operatorname{ArcTan}\left[\frac{1+x}{\sqrt{1+x^3}}\right]$$

Result (type 4, 296 leaves) :

$$\begin{aligned} & \frac{1}{3 \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2} \left(\frac{1}{1+(-1)^{2/3} x} \right. \\ & \left. \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}-x\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] - \frac{1}{(-1)^{5/6}+\sqrt{2}} \right. \\ & \left. 3 i \left(-\frac{i}{2}+\sqrt{2}\right) \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-\frac{i}{2}-2 \sqrt{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\ & \left. \left(3 \left(5+i \sqrt{2}+i \sqrt{3}+\sqrt{6}\right) \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-\frac{i}{2}+2 \sqrt{2}+\sqrt{3}}, \right. \right. \right. \\ & \left. \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]\right) \Big/ \left(5 \frac{i}{2}+2 \sqrt{2}+\sqrt{3}+2 i \sqrt{6}\right) \right) \end{aligned}$$

Problem 176: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2+2 x-x^2}{(2+x^2) \sqrt{1-x^3}} dx$$

Optimal (type 3, 20 leaves, 2 steps) :

$$-2 \operatorname{ArcTan}\left[\frac{1-x}{\sqrt{1-x^3}}\right]$$

Result (type 4, 280 leaves) :

$$\begin{aligned}
& \frac{1}{3 \sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2} \\
& \left(\frac{1}{-1+(-1)^{2/3} x} \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}+x\right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \right. \\
& \left. \frac{1}{i+2 \sqrt{2}-\sqrt{3}} 6 \left(1+i \sqrt{2}\right) \text{EllipticPi}\left[\frac{2 \sqrt{3}}{-i-2 \sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \frac{1}{(-1)^{5/6}-\sqrt{2}} 3 \left(1-i \sqrt{2}\right) \text{EllipticPi}\left[\frac{2 \sqrt{3}}{-i+2 \sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 177: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2+2x-x^2}{(2+x^2) \sqrt{-1+x^3}} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$-2 \text{ArcTanh}\left[\frac{1-x}{\sqrt{-1+x^3}}\right]$$

Result (type 4, 278 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2} \\
& \left(\frac{1}{-1+(-1)^{2/3} x} \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}+x\right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \right. \\
& \left. \frac{1}{i+2 \sqrt{2}-\sqrt{3}} 6 \left(1+i \sqrt{2}\right) \text{EllipticPi}\left[\frac{2 \sqrt{3}}{-i-2 \sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \frac{1}{(-1)^{5/6}-\sqrt{2}} 3 \left(1-i \sqrt{2}\right) \text{EllipticPi}\left[\frac{2 \sqrt{3}}{-i+2 \sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 178: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2 - 2x - x^2}{(2 + x^2) \sqrt{-1 - x^3}} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{1+x}{\sqrt{-1-x^3}}\right]}{\sqrt{-1-x^3}}$$

Result (type 4, 298 leaves):

$$\begin{aligned} & \frac{1}{3 \sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2} \left(\frac{1}{1+(-1)^{2/3} x} \right. \\ & \quad \left. \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}-x\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] - \frac{1}{(-1)^{5/6}+\sqrt{2}} \right. \\ & \quad \left. 3 \frac{i}{\left(-\frac{i}{2}+\sqrt{2}\right)} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-\frac{i}{2}-2 \sqrt{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\ & \quad \left. 3 \left(5+\frac{i}{2} \sqrt{2}+\frac{i}{2} \sqrt{3}+\sqrt{6}\right) \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-\frac{i}{2}+2 \sqrt{2}+\sqrt{3}}, \right. \right. \\ & \quad \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right) \Big/ \left(5 \frac{i}{2}+2 \sqrt{2}+\sqrt{3}+2 \frac{i}{2} \sqrt{6}\right) \end{aligned}$$

Problem 179: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2 - 2x - x^2}{(2 + d + d x + x^2) \sqrt{1+x^3}} dx$$

Optimal (type 3, 30 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{1+d} (1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{1+d}}$$

Result (type 4, 424 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{1+x^3}} \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2} \\
& \left(\frac{1}{1+(-1)^{2/3} x} 2 \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}-x\right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] - \right. \\
& \frac{1}{\left(2+(-1)^{2/3}+d+(-1)^{1/3} d\right) \sqrt{-8-4 d+d^2}} \\
& 3 \pm \left(\left(8+8(-1)^{1/3}-\left(1+(-1)^{1/3}\right) d^2+4 \sqrt{-8-4 d+d^2}\right) - 2(-1)^{1/3} \sqrt{-8-4 d+d^2} + \right. \\
& \left. \left(1+(-1)^{1/3}\right) d \left(4+\sqrt{-8-4 d+d^2}\right) \right) \text{EllipticPi}\left[\frac{2 \pm \sqrt{3}}{2 (-1)^{1/3}+d-\sqrt{-8-4 d+d^2}}, \right. \\
& \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \left(\left(1+(-1)^{1/3}\right) d^2+\left(1+(-1)^{1/3}\right) d \right. \\
& \left. \left(-4+\sqrt{-8-4 d+d^2}\right) - 2 \left(4+4(-1)^{1/3}-2 \sqrt{-8-4 d+d^2}+(-1)^{1/3} \sqrt{-8-4 d+d^2}\right) \right) \\
& \text{EllipticPi}\left[\frac{2 \pm \sqrt{3}}{2 (-1)^{1/3}+d+\sqrt{-8-4 d+d^2}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right)
\end{aligned}$$

Problem 180: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2+2x-x^2}{(2-d+d x+x^2) \sqrt{1-x^3}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \text{ArcTan}\left[\frac{\sqrt{1-d} (1-x)}{\sqrt{1-x^3}}\right]}{\sqrt{1-d}}
\end{aligned}$$

Result (type 4, 427 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{1-x^3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2} \\
& \left(\frac{1}{-1+(-1)^{2/3} x} 2 \sqrt{3} (1+(-1)^{1/3}) ((-1)^{1/3}+x) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \right. \\
& \frac{1}{\left(-2-(-1)^{2/3}+d+(-1)^{1/3} d\right) \sqrt{-8+4 d+d^2}} \\
& 3 \pm \left(\left(8+8(-1)^{1/3}-\left(1+(-1)^{1/3}\right) d^2-4 \sqrt{-8+4 d+d^2}+2(-1)^{1/3} \sqrt{-8+4 d+d^2}\right. \right. + \\
& \left. \left. \left(1+(-1)^{1/3}\right) d \left(-4+\sqrt{-8+4 d+d^2}\right)\right) \operatorname{EllipticPi}\left[\frac{2 \pm \sqrt{3}}{2(-1)^{1/3}-d+\sqrt{-8+4 d+d^2}}, \right. \\
& \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \left(-8-8(-1)^{1/3}+\left(1+(-1)^{1/3}\right) d^2-\right. \\
& \left. 4 \sqrt{-8+4 d+d^2}+2(-1)^{1/3} \sqrt{-8+4 d+d^2}+\left(1+(-1)^{1/3}\right) d \left(4+\sqrt{-8+4 d+d^2}\right)\right) \\
& \left. \operatorname{EllipticPi}\left[-\frac{2 \pm \sqrt{3}}{-2(-1)^{1/3}+d+\sqrt{-8+4 d+d^2}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]\right)
\end{aligned}$$

Problem 181: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2+2 x-x^2}{(2-d+d x+x^2) \sqrt{-1+x^3}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{1-d} (1-x)}{\sqrt{-1+x^3}}\right]}{\sqrt{1-d}}
\end{aligned}$$

Result (type 4, 425 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{-1+x^3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2} \\
& \left(\frac{1}{-1+(-1)^{2/3} x} 2 \sqrt{3} (1+(-1)^{1/3}) ((-1)^{1/3}+x) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \right. \\
& \frac{1}{\left(-2-(-1)^{2/3}+d+(-1)^{1/3} d\right) \sqrt{-8+4 d+d^2}} \\
& 3 \pm \left(\left(8+8 (-1)^{1/3}-\left(1+(-1)^{1/3}\right) d^2-4 \sqrt{-8+4 d+d^2}+2 (-1)^{1/3} \sqrt{-8+4 d+d^2}\right. \right. + \\
& \left. \left. \left(1+(-1)^{1/3}\right) d \left(-4+\sqrt{-8+4 d+d^2}\right)\right) \text{EllipticPi}\left[\frac{2 \pm \sqrt{3}}{2 (-1)^{1/3}-d+\sqrt{-8+4 d+d^2}}, \right. \\
& \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \left(-8-8 (-1)^{1/3}+\left(1+(-1)^{1/3}\right) d^2-\right. \\
& \left. 4 \sqrt{-8+4 d+d^2}+2 (-1)^{1/3} \sqrt{-8+4 d+d^2}+\left(1+(-1)^{1/3}\right) d \left(4+\sqrt{-8+4 d+d^2}\right)\right) \\
& \left. \text{EllipticPi}\left[-\frac{2 \pm \sqrt{3}}{-2 (-1)^{1/3}+d+\sqrt{-8+4 d+d^2}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}]\right)
\end{aligned}$$

Problem 182: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2-2 x-x^2}{(2+d+d x+x^2) \sqrt{-1-x^3}} dx$$

Optimal (type 3, 32 leaves, 2 steps):

$$\frac{2 \text{ArcTanh}\left[\frac{\sqrt{1+d} (1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{1+d}}$$

Result (type 4, 426 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{-1-x^3}} \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2} \\
& \left(\frac{1}{1+(-1)^{2/3} x} 2 \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}-x\right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] - \right. \\
& \frac{1}{\left(2+(-1)^{2/3}+d+(-1)^{1/3} d\right) \sqrt{-8-4 d+d^2}} \\
& 3 \pm \left(\left(8+8(-1)^{1/3}-\left(1+(-1)^{1/3}\right) d^2+4 \sqrt{-8-4 d+d^2}\right) - 2(-1)^{1/3} \sqrt{-8-4 d+d^2} + \right. \\
& \left. \left(1+(-1)^{1/3}\right) d \left(4+\sqrt{-8-4 d+d^2}\right) \right) \text{EllipticPi}\left[\frac{2 \pm \sqrt{3}}{2 (-1)^{1/3}+d-\sqrt{-8-4 d+d^2}}, \right. \\
& \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \left(\left(1+(-1)^{1/3}\right) d^2+\left(1+(-1)^{1/3}\right) d \right. \\
& \left. \left(-4+\sqrt{-8-4 d+d^2}\right) - 2 \left(4+4(-1)^{1/3}-2 \sqrt{-8-4 d+d^2}+(-1)^{1/3} \sqrt{-8-4 d+d^2}\right)\right) \\
& \text{EllipticPi}\left[\frac{2 \pm \sqrt{3}}{2 (-1)^{1/3}+d+\sqrt{-8-4 d+d^2}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] \right)
\end{aligned}$$

Problem 183: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e x)^3 \sqrt{a+c x^4} dx$$

Optimal (type 4, 355 leaves, 11 steps):

$$\begin{aligned}
& \frac{3}{4} d^2 e x^2 \sqrt{a+c x^4} + \frac{6 a d e^2 x \sqrt{a+c x^4}}{5 \sqrt{c} (\sqrt{a}+\sqrt{c} x^2)} + \frac{1}{15} d x (5 d^2+9 e^2 x^2) \sqrt{a+c x^4} + \\
& \frac{e^3 (a+c x^4)^{3/2}}{6 c} + \frac{3 a d^2 e \text{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{4 \sqrt{c}} - \frac{1}{5 c^{3/4} \sqrt{a+c x^4}} \\
& 6 a^{5/4} d e^2 (\sqrt{a}+\sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] + \frac{1}{15 c^{3/4} \sqrt{a+c x^4}} \\
& a^{3/4} d (5 \sqrt{c} d^2+9 \sqrt{a} e^2) (\sqrt{a}+\sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]
\end{aligned}$$

Result (type 4, 310 leaves):

$$\begin{aligned}
& \frac{1}{60 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c \sqrt{a+c x^4}} \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(10 a^2 e^3 + c^2 x^5 (20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 10 e^3 x^3) + \right. \right. \\
& a c x (20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 20 e^3 x^3) + 45 a \sqrt{c} d^2 e \sqrt{a+c x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right] \left. \right) + \\
& 72 a^{3/2} \sqrt{c} d e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \\
& \left. \left. 8 a \sqrt{c} d (5 i \sqrt{c} d^2 + 9 \sqrt{a} e^2) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right)
\end{aligned}$$

Problem 184: Result unnecessarily involves imaginary or complex numbers.

$$\int (d + e x)^2 \sqrt{a + c x^4} dx$$

Optimal (type 4, 326 leaves, 10 steps):

$$\begin{aligned}
& \frac{1}{2} d e x^2 \sqrt{a + c x^4} + \frac{2 a e^2 x \sqrt{a + c x^4}}{5 \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \\
& \frac{1}{15} x (5 d^2 + 3 e^2 x^2) \sqrt{a + c x^4} + \frac{a d e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{2 \sqrt{c}} - \frac{1}{5 c^{3/4} \sqrt{a+c x^4}} \\
& 2 a^{5/4} e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] + \frac{1}{15 c^{3/4} \sqrt{a+c x^4}} \\
& a^{3/4} (5 \sqrt{c} d^2 + 3 \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]
\end{aligned}$$

Result (type 4, 247 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(\sqrt{c} x (10 d^2 + 15 d e x + 6 e^2 x^2) (a + c x^4) + 15 a d e \sqrt{a + c x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right] \right) + \right. \\
& 12 a^{3/2} e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - 4 a (5 i \sqrt{c} d^2 + 3 \sqrt{a} e^2) \\
& \left. \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right) / \left(30 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{c} \sqrt{a + c x^4} \right)
\end{aligned}$$

Problem 185: Result unnecessarily involves imaginary or complex numbers.

$$\int (d + e x) \sqrt{a + c x^4} dx$$

Optimal (type 4, 158 leaves, 8 steps):

$$\frac{1}{3} d x \sqrt{a + c x^4} + \frac{1}{4} e x^2 \sqrt{a + c x^4} + \frac{a e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{4 \sqrt{c}} +$$

$$\frac{a^{3/4} d \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{3 c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 132 leaves):

$$\frac{1}{12} \left(x (4 d + 3 e x) \sqrt{a + c x^4} + \frac{3 a e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{\sqrt{c}} - \right.$$

$$\left. \frac{8 i a d \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a + c x^4}} \right)$$

Problem 186: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + c x^4} dx$$

Optimal (type 4, 105 leaves, 2 steps):

$$\frac{1}{3} x \sqrt{a + c x^4} + \frac{a^{3/4} \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{3 c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 89 leaves):

$$x (a + c x^4) - \frac{2 i a \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}$$

$$\frac{3 \sqrt{a + c x^4}}{3 \sqrt{a + c x^4}}$$

Problem 187: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + c x^4}}{d + e x} dx$$

Optimal (type 4, 730 leaves, 15 steps):

$$\begin{aligned} & \frac{\sqrt{a + c x^4}}{2 e} - \frac{\sqrt{c} d x \sqrt{a + c x^4}}{e^2 (\sqrt{a} + \sqrt{c} x^2)} - \frac{\sqrt{-c d^4 - a e^4} \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a + c x^4}}\right]}{2 e^3} + \\ & \frac{\sqrt{c} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}}\right]}{2 e^3} - \frac{\sqrt{c d^4 + a e^4} \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a + c x^4}}\right]}{2 e^3} + \frac{1}{e^2 \sqrt{a + c x^4}} \\ & a^{1/4} c^{1/4} d \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] - \frac{1}{2 e^4 \sqrt{a + c x^4}} \\ & a^{1/4} c^{1/4} d \left(\frac{\sqrt{c} d^2}{\sqrt{a}} + e^2\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] + \\ & \left(c^{1/4} d (c d^4 + a e^4) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) / \\ & \left(2 a^{1/4} e^4 \left(\sqrt{c} d^2 + \sqrt{a} e^2\right) \sqrt{a + c x^4}\right) - \\ & \left(\left(\sqrt{c} d^2 - \sqrt{a} e^2\right) (c d^4 + a e^4) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticPi}\left[\frac{\left(\sqrt{c} d^2 + \sqrt{a} e^2\right)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) / \left(4 a^{1/4} c^{1/4} d e^4 \left(\sqrt{c} d^2 + \sqrt{a} e^2\right) \sqrt{a + c x^4}\right) \end{aligned}$$

Result (type 4, 451 leaves):

$$\begin{aligned}
& \frac{1}{2 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c^{1/4} d e^4 \sqrt{a+c x^4}} \left(-2 \sqrt{a} c^{3/4} d^2 e^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \\
& 2 c^{3/4} d^2 \left(i \sqrt{c} d^2 + \sqrt{a} e^2 \right) \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \\
& \left(-2 (-1)^{1/4} a^{1/4} (c d^4 + a e^4) \sqrt{1 + \frac{c x^4}{a}} \text{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \text{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + \right. \\
& c^{1/4} d e \left(a e^2 + c e^2 x^4 + \sqrt{c d^4 + a e^4} \sqrt{a+c x^4} \text{Log}\left[-d^2 + e^2 x^2\right] + \sqrt{c} d^2 \sqrt{a+c x^4} \text{Log}\left[c x^2 + \right. \right. \\
& \left. \left. \sqrt{c} \sqrt{a+c x^4}\right] - \sqrt{c d^4 + a e^4} \sqrt{a+c x^4} \text{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a+c x^4}\right]\right)\left.\right)
\end{aligned}$$

Problem 188: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+c x^4}}{(d+e x)^2} dx$$

Optimal (type 4, 1221 leaves, 32 steps):

$$\begin{aligned}
& \frac{2 \sqrt{c} x \sqrt{a+c x^4}}{e^2 (\sqrt{a} + \sqrt{c} x^2)} - \frac{d \sqrt{a+c x^4}}{e (d^2 - e^2 x^2)} + \frac{x \sqrt{a+c x^4}}{d^2 - e^2 x^2} + \\
& \frac{\sqrt{-c d^4 - a e^4} \text{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{2 d e^3} - \frac{(c d^4 - a e^4) \text{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{2 d e^3 \sqrt{-c d^4 - a e^4}} - \\
& \frac{\sqrt{c} d \text{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{e^3} + \frac{c d^3 \text{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a+c x^4}}\right]}{e^3 \sqrt{c d^4 + a e^4}} - \frac{1}{e^2 \sqrt{a+c x^4}} \\
& 2 a^{1/4} c^{1/4} \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] + \frac{1}{4 e^4 \sqrt{a+c x^4}} \\
& 3 a^{1/4} c^{1/4} \left(\frac{\sqrt{c} d^2}{\sqrt{a}} + e^2\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] - \\
& \left(c^{1/4} \left(\sqrt{c} d^2 - \sqrt{a} e^2\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) / \\
& \left(2 a^{1/4} e^4 \sqrt{a+c x^4}\right) +
\end{aligned}$$

$$\begin{aligned}
& \left(c^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \\
& \left(4 a^{1/4} e^4 \sqrt{a + c x^4} \right) - \\
& \left(c^{1/4} (c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \\
& \left(2 a^{1/4} e^4 (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a + c x^4} \right) + \left((\sqrt{c} d^2 - \sqrt{a} e^2)^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \left(4 a^{1/4} c^{1/4} d^2 e^4 \sqrt{a + c x^4} \right) + \\
& \left((\sqrt{c} d^2 - \sqrt{a} e^2) (c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\right. \right. \\
& \left. \left. \frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \left(4 a^{1/4} c^{1/4} d^2 e^4 (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a + c x^4} \right)
\end{aligned}$$

Result (type 4, 531 leaves):

$$\frac{1}{e^4 \sqrt{a + c x^4}} \left(-2 \frac{i}{\sqrt{a}} \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} e^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \frac{1}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} 2 \sqrt{c} \left(i \sqrt{c} d^2 + \sqrt{a} e^2\right) \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + 2 (-1)^{1/4} a^{1/4} c^{3/4} d^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \text{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] - \left(e \left(a e^2 \sqrt{c d^4 + a e^4} + c e^2 \sqrt{c d^4 + a e^4} x^4 + c d^3 (d + e x) \sqrt{a + c x^4} \text{Log}[-d^2 + e^2 x^2]\right) + \sqrt{c} d \sqrt{c d^4 + a e^4} (d + e x) \sqrt{a + c x^4} \text{Log}[c x^2 + \sqrt{c} \sqrt{a + c x^4}] - c d^4 \sqrt{a + c x^4} \text{Log}[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}] - c d^3 e x \sqrt{a + c x^4} \text{Log}[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}]\right) \right) \Bigg)$$

Problem 189: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^3}{\sqrt{a + c x^4}} dx$$

Optimal (type 4, 295 leaves, 9 steps) :

$$\begin{aligned} & \frac{e^3 \sqrt{a + c x^4}}{2 c} + \frac{3 d e^2 x \sqrt{a + c x^4}}{\sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{3 d^2 e \text{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{2 \sqrt{c}} - \frac{1}{c^{3/4} \sqrt{a + c x^4}} \\ & 3 a^{1/4} d e^2 \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] + \\ & \left(d \left(\sqrt{c} d^2 + 3 \sqrt{a} e^2\right) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Bigg) \end{aligned}$$

Result (type 4, 240 leaves) :

$$\begin{aligned} & \left(\sqrt{\frac{\frac{i}{\sqrt{c}}}{\sqrt{a}}} e \left(e^2 (a + c x^4) + 3 \sqrt{c} d^2 \sqrt{a + c x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}}\right] \right) + \right. \\ & 6 \sqrt{a} \sqrt{c} d e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{\frac{i}{\sqrt{c}}}{\sqrt{a}}} x\right], -1\right] - 2 \sqrt{c} d \left(i \sqrt{c} d^2 + 3 \sqrt{a} e^2\right) \\ & \left. \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{\frac{i}{\sqrt{c}}}{\sqrt{a}}} x\right], -1\right] \right) / \left(2 \sqrt{\frac{\frac{i}{\sqrt{c}}}{\sqrt{a}}} c \sqrt{a + c x^4} \right) \end{aligned}$$

Problem 190: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^2}{\sqrt{a + c x^4}} dx$$

Optimal (type 4, 263 leaves, 8 steps):

$$\begin{aligned} & \frac{e^2 x \sqrt{a + c x^4}}{\sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{d e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{\sqrt{c}} - \\ & \frac{a^{1/4} e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{-\frac{a+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4} \sqrt{a + c x^4}} + \frac{1}{2 c^{3/4} \sqrt{a + c x^4}} \\ & a^{1/4} \left(\frac{\sqrt{c} d^2}{\sqrt{a}} + e^2 \right) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \end{aligned}$$

Result (type 4, 204 leaves):

$$\begin{aligned} & \left(\sqrt{\frac{\frac{i}{\sqrt{c}}}{\sqrt{a}}} d e \sqrt{a + c x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}}\right] + \right. \\ & \sqrt{a} e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{\frac{i}{\sqrt{c}}}{\sqrt{a}}} x\right], -1\right] - \left(i \sqrt{c} d^2 + \sqrt{a} e^2 \right) \\ & \left. \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{\frac{i}{\sqrt{c}}}{\sqrt{a}}} x\right], -1\right] \right) / \left(\sqrt{\frac{\frac{i}{\sqrt{c}}}{\sqrt{a}}} \sqrt{c} \sqrt{a + c x^4} \right) \end{aligned}$$

Problem 191: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d + e x}{\sqrt{a + c x^4}} dx$$

Optimal (type 4, 121 leaves, 6 steps) :

$$\frac{e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right] d \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 \sqrt{c} 2 a^{1/4} c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 107 leaves) :

$$\frac{e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right] - \frac{i}{2} d \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{2 \sqrt{c} \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a+c x^4}}$$

Problem 192: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+c x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step) :

$$\frac{\left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 74 leaves) :

$$-\frac{\frac{i}{2} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a+c x^4}}$$

Problem 193: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e x) \sqrt{a+c x^4}} dx$$

Optimal (type 4, 405 leaves, 7 steps) :

$$\begin{aligned}
& \frac{e \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4-a e^4}}{d e \sqrt{a+c x^4}} x\right]}{2 \sqrt{-c d^4-a e^4}}-\frac{e \operatorname{ArcTanh}\left[\frac{a e^2+c d^2 x^2}{\sqrt{c d^4+a e^4} \sqrt{a+c x^4}}\right]}{2 \sqrt{c d^4+a e^4}}+ \\
& \frac{c^{1/4} d \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} \left(\sqrt{c} d^2+\sqrt{a} e^2\right) \sqrt{a+c x^4}}- \\
& \left(\left(\sqrt{c} d^2-\sqrt{a} e^2\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \operatorname{EllipticPi}\left[\frac{\left(\sqrt{c} d^2+\sqrt{a} e^2\right)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2},\right.\right. \\
& \left.\left.2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Bigg/ \left(4 a^{1/4} c^{1/4} d \left(\sqrt{c} d^2+\sqrt{a} e^2\right) \sqrt{a+c x^4}\right)
\end{aligned}$$

Result (type 4, 200 leaves):

$$\begin{aligned}
& \sqrt{1+\frac{c x^4}{a}} \left(-2 (-1)^{1/4} a^{1/4} \sqrt{1+\frac{c d^4}{a e^4}} e \operatorname{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + \right. \\
& \left. c^{1/4} d \log \left[\frac{-d^2+e^2 x^2}{c d^2 x^2+a e^2 \left(1+\sqrt{1+\frac{c d^4}{a e^4}} \sqrt{1+\frac{c x^4}{a}}\right)}\right] \right) \Bigg/ \left(2 c^{1/4} d \sqrt{1+\frac{c d^4}{a e^4}} e \sqrt{a+c x^4}\right)
\end{aligned}$$

Problem 194: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e x)^2 \sqrt{a+c x^4}} dx$$

Optimal (type 4, 610 leaves, 11 steps):

$$\begin{aligned}
& - \frac{e^3 \sqrt{a + c x^4}}{(c d^4 + a e^4) (d + e x)} + \frac{\sqrt{c} e^2 x \sqrt{a + c x^4}}{(c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2)} - \\
& \frac{c d^3 e \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{(-c d^4 - a e^4)^{3/2}} - \frac{c d^3 e \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a+c x^4}}\right]}{(c d^4 + a e^4)^{3/2}} - \\
& \left(a^{1/4} c^{1/4} e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \\
& \left((c d^4 + a e^4) \sqrt{a + c x^4} \right) + \frac{c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a + c x^4}} - \\
& \left(c^{3/4} d^2 (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, \right. \right. \\
& \left. \left. 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \left(2 a^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) (c d^4 + a e^4) \sqrt{a + c x^4} \right)
\end{aligned}$$

Result (type 4, 462 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{(c d^4 + a e^4)^{3/2}} (d + e x) \sqrt{a + c x^4}}} \\
& \left(\sqrt{a} \sqrt{c} e^2 \sqrt{c d^4 + a e^4} (d + e x) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}} x}{-1}}\right], -1\right] + i \sqrt{c} \right. \\
& (\sqrt{c} d^2 + i \sqrt{a} e^2) \sqrt{c d^4 + a e^4} (d + e x) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}} x}{-1}}\right], -1\right] - \\
& \left. \sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{\sqrt{c} d^2}} \left(e^3 \sqrt{c d^4 + a e^4} (a + c x^4) + 2 (-1)^{1/4} a^{1/4} c^{3/4} d^2 \sqrt{c d^4 + a e^4} (d + e x) \sqrt{1 + \frac{c x^4}{a}} \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] - c d^3 e (d + e x) \sqrt{a + c x^4} \right. \right. \\
& \left. \left. \operatorname{Log}\left[-d^2 + e^2 x^2\right] + c d^3 e (d + e x) \sqrt{a + c x^4} \operatorname{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}\right] \right) \right)
\end{aligned}$$

Problem 195: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d + e x)^3 \sqrt{a + c x^4}} dx$$

Optimal (type 4, 659 leaves, 12 steps):

$$\begin{aligned} & -\frac{e^3 \sqrt{a + c x^4}}{2 (c d^4 + a e^4) (d + e x)^2} - \frac{3 c d^3 e^3 \sqrt{a + c x^4}}{(c d^4 + a e^4)^2 (d + e x)} + \frac{3 c^{3/2} d^3 e^2 x \sqrt{a + c x^4}}{(c d^4 + a e^4)^2 (\sqrt{a} + \sqrt{c} x^2)} + \\ & \frac{3 c d^2 e (c d^4 - a e^4) \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a + c x^4}}\right]}{2 (-c d^4 - a e^4)^{5/2}} - \frac{3 c d^2 e (c d^4 - a e^4) \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a + c x^4}}\right]}{2 (c d^4 + a e^4)^{5/2}} - \\ & \left(\frac{3 a^{1/4} c^{5/4} d^3 e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{\left(c d^4 + a e^4\right)^2 \sqrt{a + c x^4}} \right. \\ & \left. \left. \left. + \frac{c^{3/4} d (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (c d^4 + a e^4) \sqrt{a + c x^4}} \right. \right. \\ & \left. \left. \left. + \frac{3 c^{3/4} d (\sqrt{c} d^2 - \sqrt{a} e^2)^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}}}{\operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]} \right. \right) \right) \end{aligned}$$

Result (type 4, 884 leaves):

$$\begin{aligned}
& \frac{1}{2 (c d^4 + a e^4)^{5/2} (d + e x)^2 \sqrt{a + c x^4}} \\
& \left(-e^3 (c d^4 + a e^4)^{3/2} (a + c x^4) - 6 c d^3 e^3 \sqrt{c d^4 + a e^4} (d + e x) (a + c x^4) - \right. \\
& 6 i a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c d^3 e^2 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \\
& \frac{1}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} 4 i c^2 d^5 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \\
& 6 i a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c d^3 e^2 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \\
& \frac{1}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} 2 i a c d e^4 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \\
& 6 (-1)^{1/4} a^{1/4} c^{7/4} d^5 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \\
& \text{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \text{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + 6 (-1)^{1/4} a^{5/4} c^{3/4} d e^4 \\
& \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \text{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + \\
& 3 c^2 d^6 e (d + e x)^2 \sqrt{a + c x^4} \text{Log}\left[-d^2 + e^2 x^2\right] - 3 a c d^2 e^5 (d + e x)^2 \sqrt{a + c x^4} \text{Log}\left[-d^2 + e^2 x^2\right] - \\
& 3 c^2 d^6 e (d + e x)^2 \sqrt{a + c x^4} \text{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}\right] + \\
& \left. 3 a c d^2 e^5 (d + e x)^2 \sqrt{a + c x^4} \text{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}\right] \right)
\end{aligned}$$

Problem 196: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^3}{(a + c x^4)^{3/2}} dx$$

Optimal (type 4, 298 leaves, 4 steps) :

$$\begin{aligned}
 & -\frac{3 d e^2 x \sqrt{a + c x^4}}{2 a \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} - \frac{a e^3 - c x (d^3 + 3 d^2 e x + 3 d e^2 x^2)}{2 a c \sqrt{a + c x^4}} + \\
 & \frac{3 d e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{3/4} \sqrt{a + c x^4}} + \\
 & \left(\frac{d (\sqrt{c} d^2 - 3 \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{3/4} \sqrt{a + c x^4}} \right) /
 \end{aligned}$$

Result (type 4, 215 leaves) :

$$\begin{aligned}
 & \left(\sqrt{\frac{\frac{i}{2} \sqrt{c}}{\sqrt{a}}} (-a e^3 + c d x (d^2 + 3 d e x + 3 e^2 x^2)) - \right. \\
 & \left. 3 \sqrt{a} \sqrt{c} d e^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticE}\left[\frac{i}{2} \text{ArcSinh}\left[\sqrt{\frac{\frac{i}{2} \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \sqrt{c} d (-\frac{i}{2} \sqrt{c} d^2 + 3 \sqrt{a} e^2) \right. \\
 & \left. \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[\frac{i}{2} \text{ArcSinh}\left[\sqrt{\frac{\frac{i}{2} \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right) / \left(2 a \sqrt{\frac{\frac{i}{2} \sqrt{c}}{\sqrt{a}}} c \sqrt{a + c x^4} \right)
 \end{aligned}$$

Problem 197: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^2}{(a + c x^4)^{3/2}} dx$$

Optimal (type 4, 270 leaves, 4 steps) :

$$\begin{aligned}
 & \frac{x (d + e x)^2}{2 a \sqrt{a + c x^4}} - \frac{e^2 x \sqrt{a + c x^4}}{2 a \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \\
 & \frac{e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{3/4} \sqrt{a + c x^4}} + \\
 & \left((\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \\
 & \left(4 a^{5/4} c^{3/4} \sqrt{a + c x^4} \right)
 \end{aligned}$$

Result (type 4, 188 leaves) :

$$\begin{aligned} & \left(\frac{i}{\sqrt{a}} \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{c} \times (d + e x)^2 - \sqrt{a} e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \right. \\ & \left. \left. \left(-\frac{i}{\sqrt{c}} d^2 + \sqrt{a} e^2\right) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right) \right) / \\ & \left(2 a^{3/2} \left(\frac{i \sqrt{c}}{\sqrt{a}} \right)^{3/2} \sqrt{a + c x^4} \right) \end{aligned}$$

Problem 198: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d + e x}{(a + c x^4)^{3/2}} dx$$

Optimal (type 4, 114 leaves, 3 steps) :

$$\frac{x (d + e x)}{2 a \sqrt{a + c x^4}} + \frac{d \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 90 leaves) :

$$\frac{x (d + e x) - \frac{i d \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{2 a \sqrt{a + c x^4}}$$

Problem 199: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + c x^4)^{3/2}} dx$$

Optimal (type 4, 108 leaves, 2 steps) :

$$\frac{x}{2 a \sqrt{a + c x^4}} + \frac{\left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 102 leaves) :

$$\frac{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x - \frac{i}{\sqrt{a}} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{2 a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a + c x^4}}$$

Problem 200: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d + e x) (a + c x^4)^{3/2}} dx$$

Optimal (type 4, 818 leaves, 14 steps):

$$\begin{aligned} & \frac{e (a e^2 - c d^2 x^2)}{2 a (c d^4 + a e^4) \sqrt{a + c x^4}} + \frac{c d x (d^2 + e^2 x^2)}{2 a (c d^4 + a e^4) \sqrt{a + c x^4}} - \\ & \frac{\sqrt{c} d e^2 x \sqrt{a + c x^4}}{2 a (c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2)} - \frac{e^5 \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{2 (-c d^4 - a e^4)^{3/2}} - \frac{e^5 \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a+c x^4}}\right]}{2 (c d^4 + a e^4)^{3/2}} + \\ & \left(c^{1/4} d e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \\ & \left(2 a^{3/4} (c d^4 + a e^4) \sqrt{a + c x^4} \right) + \\ & \left(c^{1/4} d (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \\ & \left(4 a^{5/4} (c d^4 + a e^4) \sqrt{a + c x^4} \right) + \\ & \left(c^{1/4} d e^4 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \\ & \left(2 a^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) (c d^4 + a e^4) \sqrt{a + c x^4} \right) - \\ & \left(e^4 (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, \right. \right. \\ & \left. \left. 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \left(4 a^{1/4} c^{1/4} d (\sqrt{c} d^2 + \sqrt{a} e^2) (c d^4 + a e^4) \sqrt{a + c x^4} \right) \end{aligned}$$

Result (type 4, 464 leaves):

$$\begin{aligned}
& \frac{1}{2 a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c^{1/4} d (c d^4 + a e^4)^{3/2} \sqrt{a + c x^4}} \\
& \left(-\sqrt{a} c^{3/4} d^2 e^2 \sqrt{c d^4 + a e^4} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \\
& c^{3/4} d^2 (-i \sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{c d^4 + a e^4} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \\
& \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(-2 (-1)^{1/4} a^{5/4} e^4 \sqrt{c d^4 + a e^4} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + c^{1/4} d \left(\sqrt{c d^4 + a e^4} (a e^3 + c d x (d^2 - d e x + e^2 x^2)) + a e^5 \right. \right. \\
& \left. \left. \sqrt{a + c x^4} \operatorname{Log}\left[-d^2 + e^2 x^2\right] - a e^5 \sqrt{a + c x^4} \operatorname{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}\right] \right) \right)
\end{aligned}$$

Problem 201: Result is not expressed in closed-form.

$$\int \frac{x^3 (c + d x)^n}{a + b x^4} dx$$

Optimal (type 5, 349 leaves, 10 steps):

$$\begin{aligned}
& \frac{(c + d x)^{1+n} \operatorname{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/4} (c+d x)}{b^{1/4} c - \sqrt{-\sqrt{-a}} d}]}{4 b^{3/4} \left(b^{1/4} c - \sqrt{-\sqrt{-a}} d\right) (1+n)} - \\
& \frac{(c + d x)^{1+n} \operatorname{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/4} (c+d x)}{b^{1/4} c + \sqrt{-\sqrt{-a}} d}]}{4 b^{3/4} \left(b^{1/4} c + \sqrt{-\sqrt{-a}} d\right) (1+n)} - \\
& \frac{(c + d x)^{1+n} \operatorname{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/4} (c+d x)}{b^{1/4} c - (-a)^{1/4} d}]}{4 b^{3/4} \left(b^{1/4} c - (-a)^{1/4} d\right) (1+n)} - \\
& \frac{(c + d x)^{1+n} \operatorname{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/4} (c+d x)}{b^{1/4} c + (-a)^{1/4} d}]}{4 b^{3/4} \left(b^{1/4} c + (-a)^{1/4} d\right) (1+n)}
\end{aligned}$$

Result (type 7, 526 leaves):

$$\begin{aligned}
& \frac{1}{4 b n} (c + d x)^n \left(c^3 \operatorname{RootSum} [b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \right. \\
& \quad \left. \frac{\operatorname{Hypergeometric2F1} [-n, -n, 1 - n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n}}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] - \right. \\
& \quad \left. 3 c^2 \operatorname{RootSum} [b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \right. \\
& \quad \left. \frac{\operatorname{Hypergeometric2F1} [-n, -n, 1 - n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] + \right. \\
& \quad \left. 3 c \operatorname{RootSum} [b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \right. \\
& \quad \left. \frac{\operatorname{Hypergeometric2F1} [-n, -n, 1 - n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^2}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] - \right. \\
& \quad \left. \operatorname{RootSum} [b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \right. \\
& \quad \left. \frac{\operatorname{Hypergeometric2F1} [-n, -n, 1 - n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^3}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] \right)
\end{aligned}$$

Problem 202: Result is not expressed in closed-form.

$$\int \frac{x^3 (c + d x)^{1+n}}{a + b x^4} dx$$

Optimal (type 5, 349 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1} [1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c - \sqrt{-\sqrt{-a}} d}]}{4 b^{3/4} \left(b^{1/4} c - \sqrt{-\sqrt{-a}} d\right) (2 + n)} - \\
& \quad \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1} [1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c + \sqrt{-\sqrt{-a}} d}]}{4 b^{3/4} \left(b^{1/4} c + \sqrt{-\sqrt{-a}} d\right) (2 + n)} - \\
& \quad \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1} [1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c - (-a)^{1/4} d}]}{4 b^{3/4} \left(b^{1/4} c - (-a)^{1/4} d\right) (2 + n)} - \\
& \quad \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1} [1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c + (-a)^{1/4} d}]}{4 b^{3/4} \left(b^{1/4} c + (-a)^{1/4} d\right) (2 + n)}
\end{aligned}$$

Result (type 7, 691 leaves):

$$\begin{aligned}
& \frac{1}{4 b^2 n (1+n)} \\
& (c+d x)^n \left((b c^4 + a d^4) (1+n) \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \right. \\
& \left. \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n}}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] - \right. \\
& b \left(-4 c n - 4 d n x + 3 c^3 (1+n) \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \right. \\
& \left. \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] - \right. \\
& 3 c^2 (1+n) \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \\
& \left. \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^2}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] + \right. \\
& c \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \\
& \left. \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^3}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] + \right. \\
& c n \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \\
& \left. \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^3}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] \right)
\end{aligned}$$

Problem 203: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(c+d x+e x^2) \sqrt{a+b x^4}} dx$$

Optimal (type 4, 1605 leaves, 16 steps):

$$\begin{aligned}
& - \left(\left(e^2 \operatorname{ArcTan} \left[\sqrt{2} \sqrt{(-b d^4 + 4 b c d^2 e - 2 b c^2 e^2 - 2 a e^4 - b d \sqrt{d^2 - 4 c e}) (d^2 - 2 c e)} (d^2 - 2 c e) \right] x \right) / \right. \\
& \left. \left(e \left(d + \sqrt{d^2 - 4 c e} \right) \sqrt{a + b x^4} \right) \right) / \\
& \left(\sqrt{2} \sqrt{d^2 - 4 c e} \sqrt{(-2 a e^4 - b (d^4 - 4 c d^2 e + 2 c^2 e^2 + d^3 \sqrt{d^2 - 4 c e} - 2 c d e \sqrt{d^2 - 4 c e}))} \right) \Bigg) + \\
& \left(e^2 \operatorname{ArcTan} \left[\sqrt{2} \sqrt{(-b d^4 + 4 b c d^2 e - 2 b c^2 e^2 - 2 a e^4 + b d \sqrt{d^2 - 4 c e}) (d^2 - 2 c e)} (d^2 - 2 c e) \right] x \right) / \\
& \left(e \left(d - \sqrt{d^2 - 4 c e} \right) \sqrt{a + b x^4} \right) \Bigg) / \\
& \left(\sqrt{2} \sqrt{d^2 - 4 c e} \sqrt{(-2 a e^4 - b (d^4 - 4 c d^2 e + 2 c^2 e^2 - d^3 \sqrt{d^2 - 4 c e} + 2 c d e \sqrt{d^2 - 4 c e}))} \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left(e^2 \operatorname{ArcTanh} \left[\left(4 a e^2 + b \left(d - \sqrt{d^2 - 4 c e} \right)^2 x^2 \right) \right] \right. \\
& \quad \left. \left(2 \sqrt{2} \sqrt{\left(b d^4 - 4 b c d^2 e + 2 b c^2 e^2 + 2 a e^4 - b d \sqrt{d^2 - 4 c e} (d^2 - 2 c e) \right) \sqrt{a + b x^4}} \right) \right) / \\
& \quad \left(\sqrt{2} \sqrt{d^2 - 4 c e} \sqrt{b d^4 - 4 b c d^2 e + 2 b c^2 e^2 + 2 a e^4 - b d \sqrt{d^2 - 4 c e} (d^2 - 2 c e)} \right) + \\
& \left(e^2 \operatorname{ArcTanh} \left[\left(4 a e^2 + b \left(d + \sqrt{d^2 - 4 c e} \right)^2 x^2 \right) \right] \right. \\
& \quad \left. \left(2 \sqrt{2} \sqrt{\left(b d^4 - 4 b c d^2 e + 2 b c^2 e^2 + 2 a e^4 + b d \sqrt{d^2 - 4 c e} (d^2 - 2 c e) \right) \sqrt{a + b x^4}} \right) \right) / \\
& \quad \left(\sqrt{2} \sqrt{d^2 - 4 c e} \sqrt{b d^4 - 4 b c d^2 e + 2 b c^2 e^2 + 2 a e^4 + b d \sqrt{d^2 - 4 c e} (d^2 - 2 c e)} \right) + \\
& \left(b^{1/4} e \left(d - \sqrt{d^2 - 4 c e} \right) \left(\sqrt{a} + \sqrt{b} x^2 \right) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right] \right) / \\
& \quad \left(2 a^{1/4} \sqrt{d^2 - 4 c e} \left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e - d \sqrt{d^2 - 4 c e} \right) \right) \sqrt{a + b x^4} \right) - \\
& \left(b^{1/4} e \left(d + \sqrt{d^2 - 4 c e} \right) \left(\sqrt{a} + \sqrt{b} x^2 \right) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right] \right) / \\
& \quad \left(2 a^{1/4} \sqrt{d^2 - 4 c e} \left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \sqrt{a + b x^4} \right) + \\
& \left(e \left(2 \sqrt{a} e^2 - \sqrt{b} \left(d^2 - 2 c e - d \sqrt{d^2 - 4 c e} \right) \right) \left(\sqrt{a} + \sqrt{b} x^2 \right) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \right. \\
& \quad \left. \operatorname{EllipticPi} \left[\frac{\left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e - d \sqrt{d^2 - 4 c e} \right) \right)^2}{4 \sqrt{a} \sqrt{b} e^2 \left(d - \sqrt{d^2 - 4 c e} \right)^2}, 2 \operatorname{ArcTan} \left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right] \right) / \\
& \quad \left(2 a^{1/4} b^{1/4} \sqrt{d^2 - 4 c e} \left(d - \sqrt{d^2 - 4 c e} \right) \left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e - d \sqrt{d^2 - 4 c e} \right) \right) \sqrt{a + b x^4} \right) - \\
& \left(e \left(2 \sqrt{a} e^2 - \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \left(\sqrt{a} + \sqrt{b} x^2 \right) \sqrt{\frac{a + b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \right. \\
& \quad \left. \operatorname{EllipticPi} \left[\frac{\left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right)^2}{4 \sqrt{a} \sqrt{b} e^2 \left(d + \sqrt{d^2 - 4 c e} \right)^2}, 2 \operatorname{ArcTan} \left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right] \right) / \\
& \quad \left(2 a^{1/4} b^{1/4} \sqrt{d^2 - 4 c e} \left(d + \sqrt{d^2 - 4 c e} \right) \left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \sqrt{a + b x^4} \right)
\end{aligned}$$

Result (type 4, 653 leaves) :

$$\begin{aligned}
 & - \left(\left((-1)^{1/4} \sqrt{2} \right) \sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} x \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right. \\
 & \quad \left. \left(\frac{i}{2} \sqrt{a} + \sqrt{b} x^2 \right) \left(b^{1/4} \left(-\sqrt{b} c + (-1)^{1/4} a^{1/4} b^{1/4} d - \frac{i}{2} \sqrt{a} e \right) \right. \right. \\
 & \quad \left. \left. \sqrt{d^2 - 4 c e} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} x \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right], -1] \right. \right. \\
 & \quad \left. \left. + (-1)^{1/4} a^{1/4} \left(-\left(-2 \frac{i}{2} \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \operatorname{EllipticPi}\left[\right. \right. \right. \\
 & \quad \left. \left. \left. \frac{2 (-1)^{3/4} a^{1/4} e - \frac{i}{2} b^{1/4} \left(-d + \sqrt{d^2 - 4 c e} \right)}{2 (-1)^{1/4} a^{1/4} e + b^{1/4} \left(-d + \sqrt{d^2 - 4 c e} \right)}, \operatorname{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} x \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right], -1 \right] \right. \right. \\
 & \quad \left. \left. - \left(2 \frac{i}{2} \sqrt{a} e^2 + \sqrt{b} \left(-d^2 + 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \right. \right. \\
 & \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{\frac{i}{2} \left(2 (-1)^{1/4} a^{1/4} e + b^{1/4} \left(d + \sqrt{d^2 - 4 c e} \right) \right)}{-2 (-1)^{1/4} a^{1/4} e + b^{1/4} \left(d + \sqrt{d^2 - 4 c e} \right)}, \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} x \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right], -1 \right] \right] \right) \right) / \\
 & \quad \left(a^{1/4} \sqrt{d^2 - 4 c e} \left(b c^2 - a e^2 - \frac{i}{2} \sqrt{a} \sqrt{b} (d^2 - 2 c e) \right) \sqrt{\frac{\frac{i}{2} \sqrt{a} + \sqrt{b} x^2}{\left((-1)^{1/4} a^{1/4} - b^{1/4} x \right)^2}} \right. \\
 & \quad \left. \left. \sqrt{a + b x^4} \right) \right)
 \end{aligned}$$

Problem 204: Unable to integrate problem.

$$\int \frac{\sqrt{a x^{23}}}{\sqrt{1 + x^5}} dx$$

Optimal (type 3, 75 leaves, 6 steps) :

$$-\frac{3 \sqrt{a x^{23}} \sqrt{1 + x^5}}{20 x^9} + \frac{\sqrt{a x^{23}} \sqrt{1 + x^5}}{10 x^4} + \frac{3 \sqrt{a x^{23}} \operatorname{ArcSinh}[x^{5/2}]}{20 x^{23/2}}$$

Result (type 8, 21 leaves) :

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}} dx$$

Problem 205: Unable to integrate problem.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}} dx$$

Optimal (type 3, 50 leaves, 5 steps) :

$$\frac{\sqrt{ax^{13}} \sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}} \operatorname{ArcSinh}[x^{5/2}]}{5x^{13/2}}$$

Result (type 8, 21 leaves) :

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}} dx$$

Problem 206: Unable to integrate problem.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}} dx$$

Optimal (type 3, 24 leaves, 4 steps) :

$$\frac{2\sqrt{ax^3} \operatorname{ArcSinh}[x^{5/2}]}{5x^{3/2}}$$

Result (type 8, 21 leaves) :

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}} dx$$

Problem 212: Unable to integrate problem.

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x(1-x^4)} \right) dx$$

Optimal (type 3, 49 leaves, 8 steps) :

$$\frac{\operatorname{ArcTan}[x]}{2} + \frac{\sqrt{ax^6} \operatorname{ArcTan}[x]}{2x^3} + \frac{\operatorname{ArcTanh}[x]}{2} - \frac{\sqrt{ax^6} \operatorname{ArcTanh}[x]}{2x^3}$$

Result (type 8, 35 leaves) :

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x(1-x^4)} \right) dx$$

Problem 213: Unable to integrate problem.

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5} \right) dx$$

Optimal (type 3, 49 leaves, 9 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{ax^6} \text{ArcTan}[x]}{2x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{ax^6} \text{ArcTanh}[x]}{2x^3}$$

Result (type 8, 32 leaves):

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5} \right) dx$$

Problem 216: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 83 leaves, 4 steps):

$$\frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticF}[2\text{ArcTan}[\sqrt{x}], \frac{1}{2}]}{3x^{3/2}\sqrt{1+x^2}}$$

Result (type 4, 77 leaves):

$$\frac{1}{3\sqrt{1+\frac{1}{x^2}}x^{5/2}} - 2\sqrt{ax^3}\sqrt{1+x^2} \left(\sqrt{1+\frac{1}{x^2}}x^{3/2} - (-1)^{1/4} \text{EllipticF}\left[\text{i ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{x}}\right], -1\right] \right)$$

Problem 218: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$\frac{2\sqrt{ax}\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[\frac{\sqrt{ax}}{\sqrt{a}}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}} + \frac{\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{\sqrt{ax}}{\sqrt{a}}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}}$$

Result (type 4, 58 leaves):

$$\frac{1}{\sqrt{x}} 2 (-1)^{3/4} \sqrt{ax} \\ \left(-\text{EllipticE}\left[\text{i ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] + \text{EllipticF}\left[\text{i ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] \right)$$

Problem 219: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 54 leaves, 3 steps):

$$\frac{\sqrt{\frac{a}{x}} \sqrt{x} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\sqrt{x}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}}$$

Result (type 4, 57 leaves):

$$\frac{2 (-1)^{1/4} \sqrt{\frac{a}{x}} \sqrt{1+x^2} \text{EllipticF}\left[\text{i ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{x}}\right], -1\right]}{\sqrt{1+\frac{1}{x^2}} \sqrt{x}}$$

Problem 221: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 159 leaves, 6 steps):

$$\begin{aligned} & -2 \sqrt{\frac{a}{x^3}} x \sqrt{1+x^2} + \frac{2 \sqrt{\frac{a}{x^3}} x^2 \sqrt{1+x^2}}{1+x} - \\ & \frac{2 \sqrt{\frac{a}{x^3}} x^{3/2} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\sqrt{x}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}} + \\ & \frac{\sqrt{\frac{a}{x^3}} x^{3/2} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\sqrt{x}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}} \end{aligned}$$

Result (type 4, 74 leaves):

$$2 \sqrt{\frac{a}{x^3}} x \left(-\sqrt{1+x^2} + (-1)^{3/4} \sqrt{x} \left(-\text{EllipticE}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] + \text{EllipticF}\left[\pm \text{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] \right) \right)$$

Problem 224: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 292 leaves, 5 steps):

$$\begin{aligned} & \frac{(1+\sqrt{3}) \sqrt{ax^3} \sqrt{1+x^3}}{x (1+(1+\sqrt{3}) x)} - \\ & \left(3^{1/4} \sqrt{ax^3} (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3}) x)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{1+(1-\sqrt{3}) x}{1+(1+\sqrt{3}) x}\right], \frac{1}{4} (2+\sqrt{3})\right] \right) / \\ & \left(x \sqrt{\frac{x (1+x)}{(1+(1+\sqrt{3}) x)^2}} \sqrt{1+x^3} - \left((1-\sqrt{3}) \sqrt{ax^3} (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3}) x)^2}} \right. \right. \\ & \left. \left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{1+(1-\sqrt{3}) x}{1+(1+\sqrt{3}) x}\right], \frac{1}{4} (2+\sqrt{3})\right] \right) / \left(2 \times 3^{1/4} x \sqrt{\frac{x (1+x)}{(1+(1+\sqrt{3}) x)^2}} \sqrt{1+x^3} \right) \right) \end{aligned}$$

Result (type 4, 174 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{ax^3} \sqrt{1+x^3}} a x \left(1+x^3 + \frac{1}{\sqrt{6}} (1-(-1)^{2/3}) x^2 \sqrt{\frac{-(-1)^{1/3}+x}{(1+(-1)^{1/3}) x}} \sqrt{\frac{(1+x)(-1+\pm \sqrt{3}+2x)}{x^2}} \right. \\ & \left((1+(-1)^{1/3}) \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3}(1+x)}{(-1+(-1)^{2/3}) x}}\right], 1+(-1)^{2/3}\right] - \right. \\ & \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3}(1+x)}{(-1+(-1)^{2/3}) x}}\right], 1+(-1)^{2/3}\right] \right) \right) \end{aligned}$$

Problem 225: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 260 leaves, 4 steps):

$$\frac{2 \sqrt{ax^2} \sqrt{1+x^3}}{x (1+\sqrt{3}+x)} -$$

$$\left(3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{ax^2} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticE}[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}] \right) /$$

$$\left(x \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) +$$

$$\left(2 \sqrt{2} \sqrt{ax^2} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}] \right) /$$

$$\left(3^{1/4} x \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right)$$

Result (type 4, 134 leaves):

$$- \left(\left(2 a x \sqrt{-(-1)^{1/6} ((-1)^{2/3}+x)} \sqrt{1+(-1)^{1/3} x+(-1)^{2/3} x^2} \right. \right.$$

$$\left. \left. \sqrt{3} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}] + \right. \right.$$

$$\left. \left. (-1)^{5/6} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}] \right) \right) / \left(3^{1/4} \sqrt{ax^2} \sqrt{1+x^3} \right)$$

Problem 227: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 116 leaves, 3 steps):

$$\left(\sqrt{\frac{a}{x}} x (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3})x)^2}} \text{EllipticF}[\text{ArcCos}\left[\frac{1+(1-\sqrt{3})x}{1+(1+\sqrt{3})x}\right], \frac{1}{4} (2+\sqrt{3})] \right) /$$

$$\left(3^{1/4} \sqrt{\frac{x (1+x)}{(1+(1+\sqrt{3})x)^2}} \sqrt{1+x^3} \right)$$

Result (type 4, 106 leaves):

$$\begin{aligned}
& - \frac{1}{3^{1/4} \sqrt{1+x^3}} 2 (-1)^{1/6} \sqrt{-(-1)^{1/6} \left((-1)^{2/3} + \frac{1}{x} \right)} \\
& \sqrt{1 + \frac{(-1)^{2/3}}{x^2} + \frac{(-1)^{1/3}}{x}} \sqrt{\frac{a}{x}} x^2 \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} \left(1 + \frac{1}{x}\right)}}{3^{1/4}}\right], (-1)^{1/3}]
\end{aligned}$$

Problem 229: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 312 leaves, 6 steps):

$$\begin{aligned}
& -2 \sqrt{\frac{a}{x^3}} x \sqrt{1+x^3} + \frac{2 \left(1+\sqrt{3}\right) \sqrt{\frac{a}{x^3}} x^2 \sqrt{1+x^3}}{1+\left(1+\sqrt{3}\right) x} - \left(2 \times 3^{1/4} \sqrt{\frac{a}{x^3}} x^2 (1+x) \right. \\
& \left. \sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right) x\right)^2}} \text{EllipticE}[\text{ArcCos}\left[\frac{1+\left(1-\sqrt{3}\right) x}{1+\left(1+\sqrt{3}\right) x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)] \right) / \\
& \left(\sqrt{\frac{x (1+x)}{\left(1+\left(1+\sqrt{3}\right) x\right)^2}} \sqrt{1+x^3} \right) - \left(\left(1-\sqrt{3}\right) \sqrt{\frac{a}{x^3}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right) x\right)^2}} \right. \\
& \left. \text{EllipticF}[\text{ArcCos}\left[\frac{1+\left(1-\sqrt{3}\right) x}{1+\left(1+\sqrt{3}\right) x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)] \right) / \left(3^{1/4} \sqrt{\frac{x (1+x)}{\left(1+\left(1+\sqrt{3}\right) x\right)^2}} \sqrt{1+x^3} \right)
\end{aligned}$$

Result (type 4, 165 leaves):

$$\begin{aligned}
& - \frac{1}{\sqrt{\frac{a}{x^3}} \sqrt{1+x^3}} \sqrt{\frac{2}{3} \left(-1+(-1)^{2/3}\right) a} \sqrt{\frac{-(-1)^{1/3}+x}{\left(1+(-1)^{1/3}\right) x}} \sqrt{\frac{(1+x) \left(-1+\frac{1}{2} \sqrt{3}+2 x\right)}{x^2}} \\
& \left(\left(1+(-1)^{1/3}\right) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3} (1+x)}{\left(-1+(-1)^{2/3}\right) x}}\right], 1+(-1)^{2/3}] - \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3} (1+x)}{\left(-1+(-1)^{2/3}\right) x}}\right], 1+(-1)^{2/3}] \right)
\end{aligned}$$

Problem 230: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 281 leaves, 5 steps):

$$\begin{aligned} & -\sqrt{\frac{a}{x^4}} x \sqrt{1+x^3} + \frac{\sqrt{\frac{a}{x^4}} x^2 \sqrt{1+x^3}}{1+\sqrt{3}+x} - \\ & \left(3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{\frac{a}{x^4}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticE}[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}] \right) / \\ & \left(2 \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) + \\ & \left(\sqrt{2} \sqrt{\frac{a}{x^4}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}] \right) / \\ & \left(3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3} \right) \end{aligned}$$

Result (type 4, 146 leaves):

$$\begin{aligned} & \frac{1}{3 \sqrt{1+x^3}} \sqrt{\frac{a}{x^4}} x \left(-3 (1+x^3) - 3^{3/4} x \sqrt{-(-1)^{1/6} ((-1)^{2/3}+x)} \right. \\ & \left. \sqrt{1+(-1)^{1/3} x+(-1)^{2/3} x^2} \left(\sqrt{3} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}] + \right. \right. \\ & \left. \left. (-1)^{5/6} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}] \right) \right) \end{aligned}$$

Problem 235: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax}}{\sqrt{dx+ex} \sqrt{fx}} dx$$

Optimal (type 4, 114 leaves, 2 steps):

$$\frac{2 \sqrt{-e^2 + d f} \sqrt{a x} \sqrt{\frac{e (e+f x)}{e^2-d f}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{d+e x}}{\sqrt{-e^2+d f}}\right], 1-\frac{e^2}{d f}\right]}{e \sqrt{f} \sqrt{-\frac{e x}{d}} \sqrt{e+f x}}$$

Result (type 4, 106 leaves) :

$$\begin{aligned} & - \left(\left(2 \pm e \sqrt{a x} \sqrt{1 + \frac{f x}{e}} \right. \right. \\ & \left. \left. \left(\text{EllipticE}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{e x}{d}}\right], \frac{d f}{e^2}\right] - \text{EllipticF}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{e x}{d}}\right], \frac{d f}{e^2}\right] \right) \right) / \\ & \left(f \sqrt{\frac{e x}{d+e x}} \sqrt{d+e x} \sqrt{e+f x} \right) \end{aligned}$$

Problem 261: Result more than twice size of optimal antiderivative.

$$\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x} dx$$

Optimal (type 3, 32 leaves, 6 steps) :

$$2 \sqrt{1-x^2} - 2 \text{ArcTanh}\left[\sqrt{1-x^2}\right] + 2 \text{Log}[x]$$

Result (type 3, 84 leaves) :

$$2 \left(\sqrt{1-x^2} + \text{Log}[-x] + \text{Log}\left[1 - \sqrt{1+x}\right] - \text{Log}\left[2 + \sqrt{1-x} - \sqrt{1+x}\right] - \text{Log}\left[1 + \sqrt{1+x}\right] + \text{Log}\left[2 + \sqrt{1-x} + \sqrt{1+x}\right] \right)$$

Problem 263: Result more than twice size of optimal antiderivative.

$$\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^3} dx$$

Optimal (type 3, 34 leaves, 6 steps) :

$$-\frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^2} + \text{ArcTanh}\left[\sqrt{1-x^2}\right]$$

Result (type 3, 88 leaves) :

$$-\frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^2} - \text{Log}[1 - \sqrt{1+x}] + \\ \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] + \text{Log}[1 + \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 286: Result more than twice size of optimal antiderivative.

$$\int \frac{(-\sqrt{1-x} - \sqrt{1+x}) (\sqrt{1-x} + \sqrt{1+x})}{x} dx$$

Optimal (type 3, 32 leaves, 7 steps):

$$-2\sqrt{1-x^2} + 2\text{ArcTanh}[\sqrt{1-x^2}] - 2\text{Log}[x]$$

Result (type 3, 84 leaves):

$$-2 \left(\sqrt{1-x^2} + \text{Log}[-x] + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}] \right)$$

Problem 288: Result more than twice size of optimal antiderivative.

$$\int \frac{(-\sqrt{1-x} - \sqrt{1+x}) (\sqrt{1-x} + \sqrt{1+x})}{x^3} dx$$

Optimal (type 3, 33 leaves, 7 steps):

$$\frac{1}{x^2} + \frac{\sqrt{1-x^2}}{x^2} - \text{ArcTanh}[\sqrt{1-x^2}]$$

Result (type 3, 85 leaves):

$$\frac{1}{x^2} + \frac{\sqrt{1-x^2}}{x^2} + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 289: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x} + \sqrt{1+x}}{-\sqrt{1-x} + \sqrt{1+x}} dx$$

Optimal (type 3, 28 leaves, 15 steps):

$$\sqrt{1-x^2} - \text{ArcTanh}[\sqrt{1-x^2}] + \text{Log}[x]$$

Result (type 3, 82 leaves):

$$\sqrt{1-x^2} + \text{Log}[-x] + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 291: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 5, 121 leaves, 4 steps):

$$\frac{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1+n)} + \frac{1}{2 d^2 e (1+n)}$$

$$a f^2 \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1}[2, 1+n, 2+n, \frac{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}{d}]$$

Result (type 8, 27 leaves):

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 298: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Optimal (type 3, 225 leaves, 6 steps):

$$\frac{2 a d f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{e} - \frac{a d^2 f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} +$$

$$\frac{a f^2 \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2}}{3 e} + \frac{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{7/2}}{7 e} - \frac{5 a d^{3/2} f^2 \text{ArcTanh}\left[\frac{\sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{\sqrt{d}}\right]}{2 e}$$

Result (type 8, 29 leaves):

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Problem 299: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Optimal (type 3, 183 leaves, 6 steps) :

$$\begin{aligned} & \frac{a f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{e} - \frac{a d f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \\ & \frac{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2}}{5 e} - \frac{3 a \sqrt{d} f^2 \operatorname{ArcTanh} \left[\frac{\sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{\sqrt{d}} \right]}{2 e} \end{aligned}$$

Result (type 8, 29 leaves) :

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Problem 302: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2}} dx$$

Optimal (type 3, 158 leaves, 5 steps) :

$$\begin{aligned} & - \frac{1 + \frac{a f^2}{d^2}}{e \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}} - \frac{a f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 d^2 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{3 a f^2 \operatorname{ArcTanh} \left[\frac{\sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{\sqrt{d}} \right]}{2 d^{5/2} e} \end{aligned}$$

Result (type 8, 29 leaves) :

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2}} dx$$

Problem 303: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Optimal (type 3, 199 leaves, 6 steps) :

$$\begin{aligned} & - \frac{1 + \frac{a f^2}{d^2}}{3 e \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{3/2}} - \frac{2 a f^2}{d^3 e \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}} - \\ & + \frac{a f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 d^3 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)} + \frac{5 a f^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{\sqrt{d}}\right]}{2 d^{7/2} e} \end{aligned}$$

Result (type 8, 29 leaves) :

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Problem 310: Unable to integrate problem.

$$\int \left(\frac{1}{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}} \right)^n dx$$

Optimal (type 5, 166 leaves, 4 steps) :

$$\begin{aligned}
& \frac{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1+n)} + \\
& \left(f^2 (4 a e^2 - b^2 f^2) \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1}[2, \right. \\
& \left. \left. 1+n, 2+n, \frac{2 e \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)}{2 d e - b f^2}] \right) \Bigg/ \left(2 e (2 d e - b f^2)^2 (1+n) \right)
\end{aligned}$$

Result (type 8, 30 leaves):

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 316: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^3} dx$$

Optimal (type 3, 330 leaves, 3 steps):

$$\begin{aligned}
& - \frac{d^2 e - b d f^2 + a e f^2}{(2 d e - b f^2)^2 \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^2} - \\
& - \frac{2 f^2 (4 a e^2 - b^2 f^2)}{(2 d e - b f^2)^3 \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)} - \frac{2 e f^2 (4 a e^2 - b^2 f^2)}{(2 d e - b f^2)^3 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}} \right) \right)} + \\
& - \frac{6 e f^2 (4 a e^2 - b^2 f^2) \log \left[d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right]}{(2 d e - b f^2)^4} - \\
& - \frac{6 e f^2 (4 a e^2 - b^2 f^2) \log \left[b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}} \right) \right]}{(2 d e - b f^2)^4}
\end{aligned}$$

Result (type 3, 665 leaves) :

$$\begin{aligned}
 & \frac{4 e^3 x}{(2 d e - b f^2)^3} - \frac{2 (d^2 e - b d f^2 + a e f^2)^3}{(-2 d e + b f^2)^4 (d^2 + 2 d e x - f^2 (a + b x))^2} - \\
 & \frac{3 (4 a^2 e^3 f^4 + b^2 d f^4 (-d e + b f^2) + a e f^2 (4 d^2 e^2 - 4 b d e f^2 - b^2 f^4))}{(-2 d e + b f^2)^4 (d^2 + 2 d e x - f^2 (a + b x))} - \\
 & \left(2 f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \left(b^3 f^6 x + b e f^2 (-3 d^3 - a d f^2 + d^2 e x - 9 a e f^2 x + 8 d e^2 x^2) + \right. \right. \\
 & \left. \left. b^2 (a f^6 - e f^4 x (d + 2 e x)) - 2 e^2 (3 a^2 f^4 + d^2 e x (3 d + 4 e x) - a d f^2 (5 d + 9 e x)) \right) \right) / \\
 & \left((-2 d e + b f^2)^3 (d^2 + 2 d e x - f^2 (a + b x))^2 \right) - \frac{3 e f^2 (4 a e^2 - b^2 f^2) \operatorname{Log}[d^2 + 2 d e x - f^2 (a + b x)]}{(-2 d e + b f^2)^4} + \\
 & \frac{3 (4 a e^3 f^2 - b^2 e f^4) \operatorname{Log}[d^2 + 2 d e x - f^2 (a + b x)]}{(-2 d e + b f^2)^4} - \\
 & \frac{3 e f^2 (4 a e^2 - b^2 f^2) \operatorname{Log}[b f^2 + 2 e \left(e x + f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right)]}{(-2 d e + b f^2)^4} + \frac{1}{(-2 d e + b f^2)^4} \\
 & 3 e f^2 (4 a e^2 - b^2 f^2) \operatorname{Log}[b^2 f^4 x + 2 d^2 e \left(e x - f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right)] - \\
 & 2 a e f^2 \left(2 d + e x + f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right) + b f^2 \left(d^2 + a f^2 - 2 d e x + 2 d f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right)
 \end{aligned}$$

Problem 317: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Optimal (type 3, 370 leaves, 6 steps) :

$$\begin{aligned}
& \frac{f^2 (2 d e - b f^2) (4 a e^2 - b^2 f^2) \sqrt{d + e x + f} \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}{4 e^4} + \\
& \frac{f^2 (4 a e^2 - b^2 f^2) \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2}}{12 e^3} + \frac{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{7/2}}{7 e} - \\
& \frac{f^2 (2 d e - b f^2)^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f} \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}{16 e^4 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}} \right) \right)} - \frac{1}{16 \sqrt{2} e^{9/2}} \\
& 5 f^2 (2 d e - b f^2)^{3/2} (4 a e^2 - b^2 f^2) \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f} \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}{\sqrt{2 d e - b f^2}} \right]
\end{aligned}$$

Result (type 8, 32 leaves):

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Problem 318: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Optimal (type 3, 302 leaves, 6 steps):

$$\begin{aligned}
& \frac{f^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f} \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}{4 e^3} + \frac{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{5/2}}{5 e} - \\
& \frac{f^2 (2 d e - b f^2) (4 a e^2 - b^2 f^2) \sqrt{d + e x + f} \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}{8 e^3 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}}\right)\right)} - \\
& \frac{3 f^2 \sqrt{2 d e - b f^2} (4 a e^2 - b^2 f^2) \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f} \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}{\sqrt{2 d e - b f^2}}\right]}{8 \sqrt{2} e^{7/2}}
\end{aligned}$$

Result (type 8, 32 leaves):

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Problem 321: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{3/2}} dx$$

Optimal (type 3, 269 leaves, 5 steps):

$$\begin{aligned}
& - \frac{4 (d^2 e - b d f^2 + a e f^2)}{(2 d e - b f^2)^2 \sqrt{d + e x + f} \sqrt{a + b x + \frac{e^2 x^2}{f^2}}} - \frac{f^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f} \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}{(2 d e - b f^2)^2 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}}\right)\right)} + \\
& \frac{3 f^2 (4 a e^2 - b^2 f^2) \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f} \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}{\sqrt{2 d e - b f^2}}\right]}{\sqrt{2} \sqrt{e} (2 d e - b f^2)^{5/2}}
\end{aligned}$$

Result (type 8, 32 leaves):

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{3/2}} dx$$

Problem 322: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Optimal (type 3, 335 leaves, 6 steps) :

$$\begin{aligned} & - \frac{4 (d^2 e - b d f^2 + a e f^2)}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{3/2}} - \\ & - \frac{3 (2 d e - b f^2)^2 \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{3/2}}{(2 d e - b f^2)^3 \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}} - \\ & - \frac{4 f^2 (4 a e^2 - b^2 f^2)}{(2 d e - b f^2)^3 \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}} - \frac{2 e f^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{(2 d e - b f^2)^3 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}}\right)\right)} + \\ & + \frac{5 \sqrt{2} \sqrt{e} f^2 (4 a e^2 - b^2 f^2) \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{\sqrt{2 d e - b f^2}}\right]}{(2 d e - b f^2)^{7/2}} \end{aligned}$$

Result (type 8, 32 leaves) :

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Problem 323: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^2 \left(x + \sqrt{a + x^2}\right)^n dx$$

Optimal (type 3, 164 leaves, 3 steps) :

$$\begin{aligned}
& - \frac{a^5 (x + \sqrt{a + x^2})^{-5+n}}{32 (5-n)} - \frac{5 a^4 (x + \sqrt{a + x^2})^{-3+n}}{32 (3-n)} - \frac{5 a^3 (x + \sqrt{a + x^2})^{-1+n}}{16 (1-n)} + \\
& \frac{5 a^2 (x + \sqrt{a + x^2})^{1+n}}{16 (1+n)} + \frac{5 a (x + \sqrt{a + x^2})^{3+n}}{32 (3+n)} + \frac{(x + \sqrt{a + x^2})^{5+n}}{32 (5+n)}
\end{aligned}$$

Result (type 3, 338 leaves) :

$$\begin{aligned}
& \frac{1}{2} (x + \sqrt{a + x^2})^n \left(- \frac{2 a^2 (x - n \sqrt{a + x^2})}{-1 + n^2} + \right. \\
& \frac{1}{16} \left(\frac{a^5}{(-5 + n) (x + \sqrt{a + x^2})^5} - \frac{3 a^4}{(-3 + n) (x + \sqrt{a + x^2})^3} + \frac{2 a^3}{(-1 + n) (x + \sqrt{a + x^2})} + \right. \\
& \left. \frac{2 a^2 (x + \sqrt{a + x^2})}{1 + n} - \frac{3 a (x + \sqrt{a + x^2})^3}{3 + n} + \frac{(x + \sqrt{a + x^2})^5}{5 + n} \right) + \\
& \left(4 a \sqrt{a + x^2} \left(2 a^3 n + a^2 (-3 + n) n x \left((-3 + n) x - 2 \sqrt{a + x^2} \right) + \right. \right. \\
& \left. \left. 4 (3 - n - 3 n^2 + n^3) x^5 (x + \sqrt{a + x^2}) + a (3 - 4 n + n^2) x^3 \left((3 + 5 n) x + (1 + 3 n) \sqrt{a + x^2} \right) \right) \right) / \\
& \left((-3 + n) (-1 + n) (1 + n) (3 + n) (x + \sqrt{a + x^2})^2 \left(a + x (x + \sqrt{a + x^2}) \right) \right)
\end{aligned}$$

Problem 326: Unable to integrate problem.

$$\int \frac{(x + \sqrt{a + x^2})^n}{a + x^2} dx$$

Optimal (type 5, 59 leaves, 2 steps) :

$$\frac{2 (x + \sqrt{a + x^2})^{1+n} \text{Hypergeometric2F1}[1, \frac{1+n}{2}, \frac{3+n}{2}, -\frac{(x + \sqrt{a + x^2})^2}{a}]}{a (1+n)}$$

Result (type 8, 23 leaves) :

$$\int \frac{(x + \sqrt{a + x^2})^n}{a + x^2} dx$$

Problem 327: Unable to integrate problem.

$$\int \frac{(x + \sqrt{a + x^2})^n}{(a + x^2)^2} dx$$

Optimal (type 5, 59 leaves, 2 steps) :

$$\frac{8 \left(x + \sqrt{a + x^2}\right)^{3+n} \text{Hypergeometric2F1}\left[3, \frac{3+n}{2}, \frac{5+n}{2}, -\frac{\left(x + \sqrt{a + x^2}\right)^2}{a}\right]}{a^3 (3+n)}$$

Result (type 8, 23 leaves) :

$$\int \frac{\left(x + \sqrt{a + x^2}\right)^n}{(a + x^2)^2} dx$$

Problem 328: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^2 \left(x - \sqrt{a + x^2}\right)^n dx$$

Optimal (type 3, 176 leaves, 3 steps) :

$$\begin{aligned} & -\frac{a^5 \left(x - \sqrt{a + x^2}\right)^{-5+n}}{32 (5-n)} - \frac{5 a^4 \left(x - \sqrt{a + x^2}\right)^{-3+n}}{32 (3-n)} - \frac{5 a^3 \left(x - \sqrt{a + x^2}\right)^{-1+n}}{16 (1-n)} + \\ & \frac{5 a^2 \left(x - \sqrt{a + x^2}\right)^{1+n}}{16 (1+n)} + \frac{5 a \left(x - \sqrt{a + x^2}\right)^{3+n}}{32 (3+n)} + \frac{\left(x - \sqrt{a + x^2}\right)^{5+n}}{32 (5+n)} \end{aligned}$$

Result (type 3, 361 leaves) :

$$\begin{aligned} & \frac{1}{2} \left(x - \sqrt{a + x^2}\right)^n \left(-\frac{2 a^2 \left(x + n \sqrt{a + x^2}\right)}{-1 + n^2} + \right. \\ & \frac{1}{16} \left(\frac{a^5}{(-5 + n) \left(x - \sqrt{a + x^2}\right)^5} + \frac{2 a^3}{(-1 + n) \left(x - \sqrt{a + x^2}\right)} + \frac{2 a^2 \left(x - \sqrt{a + x^2}\right)}{1 + n} + \right. \\ & \left. \frac{\left(x - \sqrt{a + x^2}\right)^5}{5 + n} + \frac{3 a^4}{(-3 + n) \left(-x + \sqrt{a + x^2}\right)^3} + \frac{3 a \left(-x + \sqrt{a + x^2}\right)^3}{3 + n} \right) + \\ & \left(4 a \sqrt{a + x^2} \left(2 a^3 n - 4 (3 - n - 3 n^2 + n^3) x^5 \left(-x + \sqrt{a + x^2}\right) + a^2 (-3 + n) n x \right. \right. \\ & \left. \left. \left((-3 + n) x + 2 \sqrt{a + x^2}\right) - a (3 - 4 n + n^2) x^3 \left(-(3 + 5 n) x + (1 + 3 n) \sqrt{a + x^2}\right) \right) \right) \Bigg) \\ & \left. \left((-3 + n) (-1 + n) (1 + n) (3 + n) \left(x - \sqrt{a + x^2}\right)^2 \left(-a + x \left(-x + \sqrt{a + x^2}\right)\right) \right) \right) \end{aligned}$$

Problem 331: Unable to integrate problem.

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{a + x^2} dx$$

Optimal (type 5, 63 leaves, 2 steps) :

$$\frac{2 \left(x - \sqrt{a + x^2}\right)^{1+n} \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\frac{\left(x - \sqrt{a + x^2}\right)^2}{a}\right]}{a (1+n)}$$

Result (type 8, 25 leaves) :

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{a + x^2} dx$$

Problem 332: Unable to integrate problem.

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{(a + x^2)^2} dx$$

Optimal (type 5, 63 leaves, 2 steps) :

$$\frac{8 \left(x - \sqrt{a + x^2}\right)^{3+n} \text{Hypergeometric2F1}\left[3, \frac{3+n}{2}, \frac{5+n}{2}, -\frac{\left(x - \sqrt{a + x^2}\right)^2}{a}\right]}{a^3 (3+n)}$$

Result (type 8, 25 leaves) :

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{(a + x^2)^2} dx$$

Problem 333: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{5/2} \left(x + \sqrt{a + x^2}\right)^n dx$$

Optimal (type 3, 187 leaves, 3 steps) :

$$\begin{aligned} & -\frac{a^6 \left(x + \sqrt{a + x^2}\right)^{-6+n}}{64 (6-n)} - \frac{3 a^5 \left(x + \sqrt{a + x^2}\right)^{-4+n}}{32 (4-n)} - \frac{15 a^4 \left(x + \sqrt{a + x^2}\right)^{-2+n}}{64 (2-n)} + \\ & \frac{5 a^3 \left(x + \sqrt{a + x^2}\right)^n}{16 n} + \frac{15 a^2 \left(x + \sqrt{a + x^2}\right)^{2+n}}{64 (2+n)} + \frac{3 a \left(x + \sqrt{a + x^2}\right)^{4+n}}{32 (4+n)} + \frac{\left(x + \sqrt{a + x^2}\right)^{6+n}}{64 (6+n)} \end{aligned}$$

Result (type 3, 659 leaves) :

$$\begin{aligned}
& \left(\left(x + \sqrt{a + x^2} \right)^{9+n} \left(a + x \left(x + \sqrt{a + x^2} \right) \right) \left(\frac{4 a^3}{n} + \frac{a^6}{(-6+n) \left(x + \sqrt{a + x^2} \right)^6} - \frac{2 a^5}{(-4+n) \left(x + \sqrt{a + x^2} \right)^4} - \right. \right. \\
& \left. \left. \frac{a^4}{(-2+n) \left(x + \sqrt{a + x^2} \right)^2} - \frac{a^2 \left(x + \sqrt{a + x^2} \right)^2}{2+n} - \frac{2 a \left(x + \sqrt{a + x^2} \right)^4}{4+n} + \frac{\left(x + \sqrt{a + x^2} \right)^6}{6+n} \right) \right) / \\
& \left(64 \left(512 x^{10} \left(x + \sqrt{a + x^2} \right) + a^5 \left(10 x + \sqrt{a + x^2} \right) + 256 a x^8 \left(6 x + 5 \sqrt{a + x^2} \right) + \right. \right. \\
& \left. \left. 10 a^4 x^2 \left(17 x + 5 \sqrt{a + x^2} \right) + 16 a^3 x^4 \left(52 x + 25 \sqrt{a + x^2} \right) + 32 a^2 x^6 \left(53 x + 35 \sqrt{a + x^2} \right) \right) + \right. \\
& \left(2 a \sqrt{a + x^2} \left(x + \sqrt{a + x^2} \right)^{4+n} \left(2 a^4 + a^3 (-4+n) x \left((-4+n) x - 2 \sqrt{a + x^2} \right) + \right. \right. \\
& \left. \left. 8 (-4+n) n x^7 \left(x + \sqrt{a + x^2} \right) + 4 a (-4+n) n x^5 \left(4 x + 3 \sqrt{a + x^2} \right) + \right. \right. \\
& \left. \left. a^2 (-4+n) x^3 \left((-4+9n) x + 4 (-1+n) \sqrt{a + x^2} \right) \right) \right) / \\
& \left((-4+n) n (4+n) \left(128 x^8 \left(x + \sqrt{a + x^2} \right) + a^4 \left(8 x + \sqrt{a + x^2} \right) + 64 a x^6 \left(5 x + 4 \sqrt{a + x^2} \right) + \right. \right. \\
& \left. \left. 8 a^3 x^2 \left(11 x + 4 \sqrt{a + x^2} \right) + 16 a^2 x^4 \left(17 x + 10 \sqrt{a + x^2} \right) \right) + \right. \\
& \left(a^2 (a+x^2) \left(x + \sqrt{a + x^2} \right)^n \left(a^2 (-2+n^2) + 2 (-2+n) n x^3 \left(x + \sqrt{a + x^2} \right) + \right. \right. \\
& \left. \left. a (-2+n) x \left((2+3n) x + 2 (1+n) \sqrt{a + x^2} \right) \right) \right) / \left(n (-4+n^2) \left(a + x \left(x + \sqrt{a + x^2} \right) \right)^2 \right)
\end{aligned}$$

Problem 334: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{3/2} \left(x + \sqrt{a + x^2} \right)^n dx$$

Optimal (type 3, 131 leaves, 3 steps):

$$\begin{aligned}
& - \frac{a^4 \left(x + \sqrt{a + x^2} \right)^{-4+n}}{16 (4-n)} - \frac{a^3 \left(x + \sqrt{a + x^2} \right)^{-2+n}}{4 (2-n)} + \\
& \frac{3 a^2 \left(x + \sqrt{a + x^2} \right)^n}{8 n} + \frac{a \left(x + \sqrt{a + x^2} \right)^{2+n}}{4 (2+n)} + \frac{\left(x + \sqrt{a + x^2} \right)^{4+n}}{16 (4+n)}
\end{aligned}$$

Result (type 3, 355 leaves):

$$\frac{1}{n} \sqrt{a+x^2} \left(x + \sqrt{a+x^2} \right)^n \\ \left(\left(\left(x + \sqrt{a+x^2} \right)^4 \left(2a^4 + a^3 (-4+n) x \left((-4+n) x - 2\sqrt{a+x^2} \right) + 8(-4+n) n x^7 \left(x + \sqrt{a+x^2} \right) + \right. \right. \right. \\ \left. \left. \left. 4a(-4+n) n x^5 \left(4x + 3\sqrt{a+x^2} \right) + a^2 (-4+n) x^3 \left((-4+9n) x + 4(-1+n) \sqrt{a+x^2} \right) \right) \right) \Big/ \right. \\ \left. \left((-4+n)(4+n) \left(128x^8 \left(x + \sqrt{a+x^2} \right) + a^4 \left(8x + \sqrt{a+x^2} \right) + 64a x^6 \left(5x + 4\sqrt{a+x^2} \right) + \right. \right. \right. \\ \left. \left. \left. 8a^3 x^2 \left(11x + 4\sqrt{a+x^2} \right) + 16a^2 x^4 \left(17x + 10\sqrt{a+x^2} \right) \right) \right) + \\ \left. \left(a\sqrt{a+x^2} \left(a^2 (-2+n^2) + 2(-2+n) n x^3 \left(x + \sqrt{a+x^2} \right) + \right. \right. \right. \\ \left. \left. \left. a(-2+n) x \left((2+3n)x + 2(1+n)\sqrt{a+x^2} \right) \right) \right) \Big/ \left((-4+n^2) \left(a+x \left(x + \sqrt{a+x^2} \right) \right)^2 \right) \right)$$

Problem 337: Unable to integrate problem.

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{(a+x^2)^{3/2}} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{4 \left(x + \sqrt{a+x^2} \right)^{2+n} \text{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, -\frac{\left(x + \sqrt{a+x^2} \right)^2}{a} \right]}{a^2 (2+n)}$$

Result (type 8, 25 leaves):

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{(a+x^2)^{3/2}} dx$$

Problem 338: Unable to integrate problem.

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{(a+x^2)^{5/2}} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{16 \left(x + \sqrt{a+x^2} \right)^{4+n} \text{Hypergeometric2F1}\left[4, \frac{4+n}{2}, \frac{6+n}{2}, -\frac{\left(x + \sqrt{a+x^2} \right)^2}{a} \right]}{a^4 (4+n)}$$

Result (type 8, 25 leaves):

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{(a+x^2)^{5/2}} dx$$

Problem 339: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{5/2} \left(x - \sqrt{a + x^2} \right)^n dx$$

Optimal (type 3, 201 leaves, 3 steps):

$$\begin{aligned} & \frac{a^6 \left(x - \sqrt{a + x^2} \right)^{-6+n}}{64 (6-n)} + \frac{3 a^5 \left(x - \sqrt{a + x^2} \right)^{-4+n}}{32 (4-n)} + \frac{15 a^4 \left(x - \sqrt{a + x^2} \right)^{-2+n}}{64 (2-n)} - \\ & \frac{5 a^3 \left(x - \sqrt{a + x^2} \right)^n}{16 n} - \frac{15 a^2 \left(x - \sqrt{a + x^2} \right)^{2+n}}{64 (2+n)} - \frac{3 a \left(x - \sqrt{a + x^2} \right)^{4+n}}{32 (4+n)} - \frac{\left(x - \sqrt{a + x^2} \right)^{6+n}}{64 (6+n)} \end{aligned}$$

Result (type 3, 692 leaves):

$$\begin{aligned} & \left(\left(x - \sqrt{a + x^2} \right)^{9+n} \left(a + x \left(x - \sqrt{a + x^2} \right) \right) \left(\frac{4 a^3}{n} + \frac{a^6}{(-6+n) \left(x - \sqrt{a + x^2} \right)^6} - \frac{2 a^5}{(-4+n) \left(x - \sqrt{a + x^2} \right)^4} - \right. \right. \\ & \left. \left. \frac{a^4}{(-2+n) \left(x - \sqrt{a + x^2} \right)^2} - \frac{a^2 \left(x - \sqrt{a + x^2} \right)^2}{2+n} - \frac{2 a \left(x - \sqrt{a + x^2} \right)^4}{4+n} + \frac{\left(x - \sqrt{a + x^2} \right)^6}{6+n} \right) \right) / \\ & \left(64 \left(a^5 \left(-10 x + \sqrt{a + x^2} \right) + 512 x^{10} \left(-x + \sqrt{a + x^2} \right) + 10 a^4 x^2 \left(-17 x + 5 \sqrt{a + x^2} \right) + \right. \right. \\ & \left. \left. 256 a x^8 \left(-6 x + 5 \sqrt{a + x^2} \right) + 16 a^3 x^4 \left(-52 x + 25 \sqrt{a + x^2} \right) + 32 a^2 x^6 \left(-53 x + 35 \sqrt{a + x^2} \right) \right) \right) + \\ & \left(2 a \sqrt{a + x^2} \left(x - \sqrt{a + x^2} \right)^{4+n} \left(-2 a^4 + 8 (-4+n) n x^7 \left(-x + \sqrt{a + x^2} \right) - \right. \right. \\ & \left. \left. a^3 (-4+n) x \left((-4+n) x + 2 \sqrt{a + x^2} \right) + 4 a (-4+n) n x^5 \left(-4 x + 3 \sqrt{a + x^2} \right) + \right. \right. \\ & \left. \left. a^2 (-4+n) x^3 \left((4-9 n) x + 4 (-1+n) \sqrt{a + x^2} \right) \right) \right) / \\ & \left((-4+n) n (4+n) \left(a^4 \left(-8 x + \sqrt{a + x^2} \right) + 128 x^8 \left(-x + \sqrt{a + x^2} \right) + 8 a^3 x^2 \left(-11 x + 4 \sqrt{a + x^2} \right) + \right. \right. \\ & \left. \left. 64 a x^6 \left(-5 x + 4 \sqrt{a + x^2} \right) + 16 a^2 x^4 \left(-17 x + 10 \sqrt{a + x^2} \right) \right) \right) + \\ & \left(a^2 (a + x^2) \left(x - \sqrt{a + x^2} \right)^n \left(-a^2 (-2+n^2) + 2 (-2+n) n x^3 \left(-x + \sqrt{a + x^2} \right) + \right. \right. \\ & \left. \left. a (-2+n) x \left(-(2+3 n) x + 2 (1+n) \sqrt{a + x^2} \right) \right) \right) / \left(n (-4+n^2) \left(a + x \left(x - \sqrt{a + x^2} \right) \right)^2 \right) \end{aligned}$$

Problem 340: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{3/2} \left(x - \sqrt{a + x^2} \right)^n dx$$

Optimal (type 3, 141 leaves, 3 steps):

$$\frac{a^4 \left(x - \sqrt{a + x^2}\right)^{-4+n}}{16 (4 - n)} + \frac{a^3 \left(x - \sqrt{a + x^2}\right)^{-2+n}}{4 (2 - n)} - \frac{3 a^2 \left(x - \sqrt{a + x^2}\right)^n}{8 n} - \frac{a \left(x - \sqrt{a + x^2}\right)^{2+n}}{4 (2 + n)} - \frac{\left(x - \sqrt{a + x^2}\right)^{4+n}}{16 (4 + n)}$$

Result (type 3, 366 leaves) :

$$\begin{aligned} & \frac{1}{n} \left(x - \sqrt{a + x^2} \right)^n \left(\left(\sqrt{a + x^2} \left(x - \sqrt{a + x^2} \right)^4 \right. \right. \\ & \quad \left(-2 a^4 + 8 (-4 + n) n x^7 \left(-x + \sqrt{a + x^2} \right) - a^3 (-4 + n) x \left((-4 + n) x + 2 \sqrt{a + x^2} \right) + \right. \\ & \quad \left. \left. 4 a (-4 + n) n x^5 \left(-4 x + 3 \sqrt{a + x^2} \right) + a^2 (-4 + n) x^3 \left((4 - 9 n) x + 4 (-1 + n) \sqrt{a + x^2} \right) \right) \right) / \\ & \quad \left((-4 + n) (4 + n) \left(a^4 \left(-8 x + \sqrt{a + x^2} \right) + 128 x^8 \left(-x + \sqrt{a + x^2} \right) + 8 a^3 x^2 \left(-11 x + 4 \sqrt{a + x^2} \right) \right) + \right. \\ & \quad \left. \left. 64 a x^6 \left(-5 x + 4 \sqrt{a + x^2} \right) + 16 a^2 x^4 \left(-17 x + 10 \sqrt{a + x^2} \right) \right) + \\ & \quad \left(a (a + x^2) \left(-a^2 (-2 + n^2) + 2 (-2 + n) n x^3 \left(-x + \sqrt{a + x^2} \right) + \right. \right. \\ & \quad \left. \left. a (-2 + n) x \left(- (2 + 3 n) x + 2 (1 + n) \sqrt{a + x^2} \right) \right) \right) / \left((-4 + n^2) \left(a + x \left(x - \sqrt{a + x^2} \right)^2 \right) \right) \end{aligned}$$

Problem 343: Unable to integrate problem.

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^{3/2}} dx$$

Optimal (type 5, 63 leaves, 2 steps) :

$$-\frac{4 \left(x - \sqrt{a + x^2}\right)^{2+n} \text{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, -\frac{\left(x - \sqrt{a + x^2}\right)^2}{a}\right]}{a^2 (2 + n)}$$

Result (type 8, 27 leaves) :

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^{3/2}} dx$$

Problem 344: Unable to integrate problem.

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^{5/2}} dx$$

Optimal (type 5, 63 leaves, 2 steps) :

$$-\frac{16 \left(x - \sqrt{a + x^2}\right)^{4+n} \text{Hypergeometric2F1}\left[4, \frac{4+n}{2}, \frac{6+n}{2}, -\frac{\left(x - \sqrt{a + x^2}\right)^2}{a}\right]}{a^4 (4 + n)}$$

Result (type 8, 27 leaves) :

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{(a + x^2)^{5/2}} dx$$

Problem 345: Unable to integrate problem.

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}\right)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Optimal (type 3, 365 leaves, 4 steps) :

$$\begin{aligned} & \frac{(d^2 - a f^2)^5 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-5+n}}{32 e f^4 (5 - n)} - \frac{5 (d^2 - a f^2)^4 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-3+n}}{32 e f^4 (3 - n)} + \\ & \frac{5 (d^2 - a f^2)^3 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-1+n}}{16 e f^4 (1 - n)} + \frac{5 (d^2 - a f^2)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{1+n}}{16 e f^4 (1 + n)} - \\ & \frac{5 (d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{3+n}}{32 e f^4 (3 + n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{5+n}}{32 e f^4 (5 + n)} \end{aligned}$$

Result (type 8, 58 leaves) :

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}\right)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Problem 346: Unable to integrate problem.

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}\right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Optimal (type 3, 239 leaves, 4 steps) :

$$\frac{\left(d^2 - a f^2\right)^3 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-3+n}}{8 e f^2 (3-n)} - \frac{3 \left(d^2 - a f^2\right)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-1+n}}{8 e f^2 (1-n)} - \\ \frac{3 \left(d^2 - a f^2\right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{1+n}}{8 e f^2 (1+n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{3+n}}{8 e f^2 (3+n)}$$

Result (type 8, 56 leaves):

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}\right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Problem 347: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$\frac{\left(d^2 - a f^2\right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-1+n}}{2 e (1-n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{1+n}}{2 e (1+n)}$$

Result (type 8, 35 leaves):

$$\int \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Problem 348: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Optimal (type 5, 122 leaves, 2 steps):

$$- \left(2 f^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1} \left[1, \frac{1+n}{2}, \frac{3+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^2}{d^2 - a f^2} \right] \right) / (e (d^2 - a f^2) (1+n))$$

Result (type 8, 58 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Problem 349: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^2} dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$- \left(8 f^4 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{3+n} \text{Hypergeometric2F1} \left[3, \frac{3+n}{2}, \frac{5+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^2}{d^2 - a f^2} \right] \right) / (e (d^2 - a f^2)^3 (3+n))$$

Result (type 8, 58 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^2} dx$$

Problem 350: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n dx$$

Optimal (type 3, 107 leaves, 5 steps) :

$$\frac{(d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-1+n}}{2 e (1 - n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1 + n)}$$

Result (type 8, 35 leaves) :

$$\int \left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n dx$$

Problem 351: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Optimal (type 5, 122 leaves, 3 steps) :

$$-\left(2 f^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^2}{d^2 - a f^2} \right] \right) / (e (d^2 - a f^2) (1 + n))$$

Result (type 8, 58 leaves) :

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Problem 352: Unable to integrate problem.

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 297 leaves, 4 steps) :

$$\begin{aligned} & - \frac{\left(d^2 - a f^2 \right)^4 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-4+n}}{16 e f^3 (4 - n)} + \\ & \frac{\left(d^2 - a f^2 \right)^3 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-2+n}}{4 e f^3 (2 - n)} + \frac{3 \left(d^2 - a f^2 \right)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{8 e f^3 n} - \\ & \frac{\left(d^2 - a f^2 \right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n}}{4 e f^3 (2 + n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{4+n}}{16 e f^3 (4 + n)} \end{aligned}$$

Result (type 8, 60 leaves) :

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 353: Unable to integrate problem.

$$\int \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 171 leaves, 4 steps) :

$$\begin{aligned} & - \frac{\left(d^2 - a f^2 \right)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-2+n}}{4 e f (2 - n)} - \\ & \frac{\left(d^2 - a f^2 \right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{2 e f n} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n}}{4 e f (2 + n)} \end{aligned}$$

Result (type 8, 60 leaves) :

$$\int \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 354: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{f \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{e n}$$

Result (type 8, 60 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}} dx$$

Problem 355: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2}} dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$\left\{ 4 f^3 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n} \right.$$

$$\left. \text{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^2}{d^2 - a f^2}\right] \right\} / \left(e (d^2 - a f^2)^2 (2+n) \right)$$

Result (type 8, 60 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2}} dx$$

Problem 356: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{\sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}}} dx$$

Optimal (type 3, 41 leaves, 4 steps) :

$$\frac{f \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{e n}$$

Result (type 8, 60 leaves) :

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{\sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}}} dx$$

Problem 357: Unable to integrate problem.

$$\int \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 327 leaves, 5 steps) :

$$\begin{aligned}
& - \frac{\left(d^2 - a f^2\right)^2 \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-2+n}}{4 e f (2-n) \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} \\
& + \frac{\left(d^2 - a f^2\right) \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{2 e f n \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} \\
& \frac{\sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{2+n}}{4 e f (2+n) \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}}
\end{aligned}$$

Result (type 8, 64 leaves):

$$\int \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Problem 358: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{\sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}} dx$$

Optimal (type 3, 93 leaves, 4 steps):

$$\frac{f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{e n \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}}$$

Result (type 8, 64 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n}{\sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}} dx$$

Problem 359: Unable to integrate problem.

$$\int \frac{\left(d + ex + f \sqrt{a + \frac{2dx}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\left(a g + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}\right)^{3/2}} dx$$

Optimal (type 5, 177 leaves, 4 steps) :

$$\begin{aligned} & \left(4f^3 \sqrt{a + \frac{2dx}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dx}{f^2} + \frac{e^2x^2}{f^2}} \right)^{2+n} \right. \\ & \left. \text{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, \frac{\left(d + ex + f \sqrt{a + \frac{2dx}{f^2} + \frac{e^2x^2}{f^2}}\right)^2}{d^2 - af^2}\right] \right) / \\ & \left(e (d^2 - af^2)^2 g (2+n) \sqrt{a g + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \right) \end{aligned}$$

Result (type 8, 64 leaves) :

$$\int \frac{\left(d + ex + f \sqrt{a + \frac{2dx}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\left(a g + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}\right)^{3/2}} dx$$

Problem 360: Unable to integrate problem.

$$\int \frac{\left(d + ex + f \sqrt{\frac{af^2 + ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2 g + egx(2d+ex)}{f^2}}} dx$$

Optimal (type 3, 93 leaves, 5 steps) :

$$\frac{f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n}{e n \sqrt{a g + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}}$$

Result (type 8, 62 leaves) :

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{\sqrt{\frac{a f^2 g + e g x (2 d + e x)}{f^2}}} dx$$

Problem 361: Unable to integrate problem.

$$\int \frac{1}{(a + b x) \sqrt{c + d x^2} \sqrt{e + f x^2}} dx$$

Optimal (type 4, 191 leaves, 7 steps) :

$$-\frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{b^2 e+a^2 f} \sqrt{c+d x^2}}{\sqrt{b^2 c+a^2 d} \sqrt{e+f x^2}}\right]}{\sqrt{b^2 c+a^2 d} \sqrt{b^2 e+a^2 f}}+\frac{\sqrt{-c} \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \operatorname{EllipticPi}\left[-\frac{b^2 c}{a^2 d}, \operatorname{ArcSin}\left[\frac{\sqrt{d} x}{\sqrt{-c}}\right], \frac{c f}{d e}\right]}{a \sqrt{d} \sqrt{c+d x^2} \sqrt{e+f x^2}}$$

Result (type 8, 32 leaves) :

$$\int \frac{1}{(a + b x) \sqrt{c + d x^2} \sqrt{e + f x^2}} dx$$

Problem 362: Result more than twice size of optimal antiderivative.

$$\int \frac{e-2 f x^2}{e^2+4 d f x^2+4 e f x^2+4 f^2 x^4} dx$$

Optimal (type 3, 81 leaves, 4 steps) :

$$-\frac{\operatorname{Log}\left[e-2 \sqrt{-d} \sqrt{f} x+2 f x^2\right]}{4 \sqrt{-d} \sqrt{f}}+\frac{\operatorname{Log}\left[e+2 \sqrt{-d} \sqrt{f} x+2 f x^2\right]}{4 \sqrt{-d} \sqrt{f}}$$

Result (type 3, 191 leaves) :

$$\begin{aligned} & -\frac{\left(-d-2 e+\sqrt{d} \sqrt{d+2 e}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{d+e-\sqrt{d}} \sqrt{d+2 e}}\right]}{\sqrt{d+e-\sqrt{d}} \sqrt{d+2 e}} \\ & \left.\left.\frac{\left(d+2 e+\sqrt{d} \sqrt{d+2 e}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{d+e+\sqrt{d}} \sqrt{d+2 e}}\right]}{\sqrt{d+e+\sqrt{d}} \sqrt{d+2 e}}\right)\right/ \left(2 \sqrt{2} \sqrt{d} \sqrt{d+2 e} \sqrt{f}\right) \end{aligned}$$

Problem 363: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{e-2 f x^2}{e^2-4 d f x^2+4 e f x^2+4 f^2 x^4} dx$$

Optimal (type 3, 73 leaves, 4 steps) :

$$-\frac{\text{Log}[e - 2 \sqrt{d} \sqrt{f} x + 2 f x^2]}{4 \sqrt{d} \sqrt{f}} + \frac{\text{Log}[e + 2 \sqrt{d} \sqrt{f} x + 2 f x^2]}{4 \sqrt{d} \sqrt{f}}$$

Result (type 3, 233 leaves) :

$$\left(-\frac{\left(-\frac{i}{2} d + 2 \frac{i}{2} e + \sqrt{d} \sqrt{-d + 2 e} \right) \text{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{-d + e - \frac{i}{2} \sqrt{d} \sqrt{-d + 2 e}}} \right]}{\sqrt{-d + e - \frac{i}{2} \sqrt{d} \sqrt{-d + 2 e}}} - \frac{\left(\frac{i}{2} d - 2 \frac{i}{2} e + \sqrt{d} \sqrt{-d + 2 e} \right) \text{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{-d + e + \frac{i}{2} \sqrt{d} \sqrt{-d + 2 e}}} \right]}{\sqrt{-d + e + \frac{i}{2} \sqrt{d} \sqrt{-d + 2 e}}} \right) / \left(2 \sqrt{2} \sqrt{d} \sqrt{-d + 2 e} \sqrt{f} \right)$$

Problem 364: Result is not expressed in closed-form.

$$\int \frac{e - 4 f x^3}{e^2 + 4 d f x^2 + 4 e f x^3 + 4 f^2 x^6} dx$$

Optimal (type 3, 38 leaves, 2 steps) :

$$\frac{\text{ArcTan}\left[\frac{2 \sqrt{d} \sqrt{f} x}{e + 2 f x^3} \right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 7, 87 leaves) :

$$-\frac{1}{4 f} \text{RootSum}\left[e^2 + 4 d f \#1^2 + 4 e f \#1^3 + 4 f^2 \#1^6 \&, \frac{-e \text{Log}[x - \#1] + 4 f \text{Log}[x - \#1] \#1^3}{2 d \#1 + 3 e \#1^2 + 6 f \#1^5} \& \right]$$

Problem 365: Result is not expressed in closed-form.

$$\int \frac{e - 4 f x^3}{e^2 - 4 d f x^2 + 4 e f x^3 + 4 f^2 x^6} dx$$

Optimal (type 3, 38 leaves, 2 steps) :

$$\frac{\text{ArcTanh}\left[\frac{2 \sqrt{d} \sqrt{f} x}{e + 2 f x^3} \right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 7, 87 leaves) :

$$-\frac{1}{4 f} \text{RootSum}\left[e^2 - 4 d f \#1^2 + 4 e f \#1^3 + 4 f^2 \#1^6 \&, \frac{-e \text{Log}[x - \#1] + 4 f \text{Log}[x - \#1] \#1^3}{-2 d \#1 + 3 e \#1^2 + 6 f \#1^5} \& \right]$$

Problem 366: Unable to integrate problem.

$$\int \frac{e - 2 f (-1 + n) x^n}{e^2 + 4 d f x^2 + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 8, 44 leaves):

$$\int \frac{e - 2 f (-1 + n) x^n}{e^2 + 4 d f x^2 + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Problem 367: Unable to integrate problem.

$$\int \frac{e - 2 f (-1 + n) x^n}{e^2 - 4 d f x^2 + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 8, 44 leaves):

$$\int \frac{e - 2 f (-1 + n) x^n}{e^2 - 4 d f x^2 + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Problem 370: Result is not expressed in closed-form.

$$\int \frac{x^2 (3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 + 4dfx^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 85 leaves):

$$\frac{\frac{1}{8f} \text{RootSum}\left[e^2 + 4ef\#1^2 + 4f^2\#1^4 + 4df\#1^6 \&, \frac{3e \text{Log}[x - \#1]\#1 + 2f \text{Log}[x - \#1]\#1^3}{e + 2f\#1^2 + 3d\#1^4} \&\right]}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6}$$

Problem 371: Result is not expressed in closed-form.

$$\int \frac{x^2 (3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 85 leaves):

$$\frac{1}{8 f} \text{RootSum}\left[e^2 + 4 e f \#1^2 + 4 f^2 \#1^4 - 4 d f \#1^6 \&, \frac{3 e \text{Log}[x - \#1] \#1 + 2 f \text{Log}[x - \#1] \#1^3}{e + 2 f \#1^2 - 3 d \#1^4} \&\right]$$

Problem 374: Result is not expressed in closed-form.

$$\int \frac{x (2 e - 2 f x^3)}{e^2 + 4 e f x^3 + 4 d f x^4 + 4 f^2 x^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2 \sqrt{d} \sqrt{f} x^2}{e+2 f x^3}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 7, 86 leaves):

$$-\frac{1}{2 f} \text{RootSum}\left[e^2 + 4 e f \#1^3 + 4 d f \#1^4 + 4 f^2 \#1^6 \&, \frac{-e \text{Log}[x - \#1] + f \text{Log}[x - \#1] \#1^3}{3 e \#1 + 4 d \#1^2 + 6 f \#1^4} \&\right]$$

Problem 375: Result is not expressed in closed-form.

$$\int \frac{x (2 e - 2 f x^3)}{e^2 + 4 e f x^3 - 4 d f x^4 + 4 f^2 x^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 \sqrt{d} \sqrt{f} x^2}{e+2 f x^3}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 7, 86 leaves):

$$-\frac{1}{2 f} \text{RootSum}\left[e^2 + 4 e f \#1^3 - 4 d f \#1^4 + 4 f^2 \#1^6 \&, \frac{-e \text{Log}[x - \#1] + f \text{Log}[x - \#1] \#1^3}{3 e \#1 - 4 d \#1^2 + 6 f \#1^4} \&\right]$$

Problem 380: Unable to integrate problem.

$$\int \frac{x^m (e (1 + m) + 2 f (1 + m - n) x^n)}{e^2 + 4 d f x^{2+2 m} + 4 e f x^n + 4 f^2 x^{2 n}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2 \sqrt{d} \sqrt{f} x^{1+m}}{e+2 f x^n}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 8, 58 leaves):

$$\int \frac{x^m (e (1 + m) + 2 f (1 + m - n) x^n)}{e^2 + 4 d f x^{2+2 m} + 4 e f x^n + 4 f^2 x^{2 n}} dx$$

Problem 381: Unable to integrate problem.

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 - 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 \sqrt{d} \sqrt{f} x^{1+m}}{e+2 f x^n}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 8, 58 leaves):

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 - 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Problem 385: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x (a c + b c x^2 + d \sqrt{a + b x^2})} dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{d \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right]}{\sqrt{a} (a c^2 - d^2)} + \frac{c \text{Log}[x]}{a c^2 - d^2} - \frac{c \text{Log}[d + c \sqrt{a + b x^2}]}{a c^2 - d^2}$$

Result (type 3, 282 leaves):

$$\begin{aligned} & -\frac{1}{2 a c^2 - 2 d^2} \left(c \text{Log}[4] + \left(-2 c + \frac{2 d}{\sqrt{a}} \right) \text{Log}[x] + c \text{Log}[a c^2 - d^2 + b c^2 x^2] - \right. \\ & \left. \frac{2 d \text{Log}[a + \sqrt{a} \sqrt{a + b x^2}]}{\sqrt{a}} + c \text{Log}\left[\frac{(a c^2 - d^2) (a c - \frac{i}{2} \sqrt{b} \sqrt{a c^2 - d^2} x + d \sqrt{a + b x^2})}{\sqrt{b} c d^2 (\frac{i}{2} \sqrt{a c^2 - d^2} + \sqrt{b} c x)}\right] + \right. \\ & \left. c \text{Log}\left[\frac{(a c^2 - d^2) (a c + \frac{i}{2} \sqrt{b} \sqrt{a c^2 - d^2} x + d \sqrt{a + b x^2})}{\sqrt{b} c d^2 (-\frac{i}{2} \sqrt{a c^2 - d^2} + \sqrt{b} c x)}\right]\right) \end{aligned}$$

Problem 386: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 (a c + b c x^2 + d \sqrt{a + b x^2})} dx$$

Optimal (type 3, 151 leaves, 8 steps):

$$-\frac{a c - d \sqrt{a + b x^2}}{2 a (a c^2 - d^2) x^2} - \frac{b d (3 a c^2 - d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right]}{2 a^{3/2} (a c^2 - d^2)^2} - \frac{b c^3 \operatorname{Log}[x]}{(a c^2 - d^2)^2} + \frac{b c^3 \operatorname{Log}[d + c \sqrt{a + b x^2}]}{(a c^2 - d^2)^2}$$

Result (type 3, 430 leaves) :

$$\begin{aligned} & \frac{1}{2 a^{3/2} (-a c^2 + d^2)^2 x^2} \left(-a^{5/2} c^3 + a^{3/2} c d^2 + a^{3/2} c^2 d \sqrt{a + b x^2} - \sqrt{a} d^3 \sqrt{a + b x^2} - \right. \\ & b (2 a^{3/2} c^3 - 3 a c^2 d + d^3) x^2 \operatorname{Log}[x] + a^{3/2} b c^3 x^2 \operatorname{Log}[a c^2 - d^2 + b c^2 x^2] - \\ & 3 a b c^2 d x^2 \operatorname{Log}[a + \sqrt{a} \sqrt{a + b x^2}] + b d^3 x^2 \operatorname{Log}[a + \sqrt{a} \sqrt{a + b x^2}] + \\ & a^{3/2} b c^3 x^2 \operatorname{Log}\left[-\frac{2 (-a c^2 + d^2)^2 (a c - \pm \sqrt{b} \sqrt{a c^2 - d^2} x + d \sqrt{a + b x^2})}{b^{3/2} c^3 d^2 (\pm \sqrt{a c^2 - d^2} + \sqrt{b} c x)}\right] + \\ & \left. a^{3/2} b c^3 x^2 \operatorname{Log}\left[-\frac{2 (-a c^2 + d^2)^2 (a c + \pm \sqrt{b} \sqrt{a c^2 - d^2} x + d \sqrt{a + b x^2})}{b^{3/2} c^3 d^2 (-\pm \sqrt{a c^2 - d^2} + \sqrt{b} c x)}\right]\right) \end{aligned}$$

Problem 393: Unable to integrate problem.

$$\int \frac{1}{x (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 3, 93 leaves, 7 steps) :

$$\frac{2 d \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3 \sqrt{a} (a c^2 - d^2)} + \frac{c \operatorname{Log}[x]}{a c^2 - d^2} - \frac{2 c \operatorname{Log}[d + c \sqrt{a + b x^3}]}{3 (a c^2 - d^2)}$$

Result (type 8, 31 leaves) :

$$\int \frac{1}{x (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Problem 394: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 3, 154 leaves, 8 steps) :

$$-\frac{a c - d \sqrt{a + b x^3}}{3 a (a c^2 - d^2) x^3} - \frac{b d (3 a c^2 - d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3 a^{3/2} (a c^2 - d^2)^2} - \frac{b c^3 \operatorname{Log}[x]}{(a c^2 - d^2)^2} + \frac{2 b c^3 \operatorname{Log}[d + c \sqrt{a + b x^3}]}{3 (a c^2 - d^2)^2}$$

Result (type 6, 596 leaves) :

$$\begin{aligned}
& \frac{1}{9} \left(\left(6 b^2 c^2 d x^3 \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \right. \\
& \quad \left. + \left(\sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(4 a (a c^2 - d^2) \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right. \right. \\
& \quad \left. \left. + b x^3 \left(-2 a c^2 \text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right. \right. \\
& \quad \left. \left. \left. + (-a c^2 + d^2) \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \right) \right) \\
& \quad + \frac{1}{a x^3} \left(- \left(\left(5 b^2 c^2 d (3 a c^2 - d^2) x^6 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3}\right] \right) \right. \\
& \quad \left. \left((a c^2 - d^2) \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right. \right. \\
& \quad \left. \left. \left(5 b c^2 x^3 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3}\right] + (-2 a c^2 + 2 d^2) \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. 2, \frac{7}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3}\right] - a c^2 \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3}\right] \right) \right) \right) \\
& \quad + \frac{1}{(-a c^2 + d^2)^2} \left(-3 (a c^2 - d^2) (a c - d \sqrt{a + b x^3}) - 9 a b c^3 x^3 \text{Log}[x] + \right. \\
& \quad \left. \left. \left. 3 a b c^3 x^3 \text{Log}[a c^2 - d^2 + b c^2 x^3] \right) \right)
\end{aligned}$$

Problem 396: Unable to integrate problem.

$$\int \frac{x}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Optimal (type 6, 304 leaves, 9 steps) :

$$\begin{aligned}
& - \frac{d x^2 \sqrt{1 + \frac{b x^3}{a}} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{2 (a c^2 - d^2) \sqrt{a + b x^3}} - \frac{\text{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3} c^{1/3} (a c^2 - d^2)^{1/3}} - \\
& \frac{\text{Log}\left[\left(a c^2 - d^2\right)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 b^{2/3} c^{1/3} (a c^2 - d^2)^{1/3}} + \frac{\text{Log}\left[\left(a c^2 - d^2\right)^{2/3} - b^{1/3} c^{2/3} \left(a c^2 - d^2\right)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 b^{2/3} c^{1/3} (a c^2 - d^2)^{1/3}}
\end{aligned}$$

Result (type 8, 29 leaves) :

$$\int \frac{x}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Problem 397: Unable to integrate problem.

$$\int \frac{1}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Optimal (type 6, 300 leaves, 9 steps) :

$$\begin{aligned}
 & -\frac{d x \sqrt{1 + \frac{b x^3}{a}} \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{(a c^2 - d^2) \sqrt{a + b x^3}} - \frac{c^{1/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3} (a c^2 - d^2)^{2/3}} + \\
 & \frac{c^{1/3} \log\left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 b^{1/3} (a c^2 - d^2)^{2/3}} - \frac{c^{1/3} \log\left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 b^{1/3} (a c^2 - d^2)^{2/3}}
 \end{aligned}$$

Result (type 8, 27 leaves) :

$$\int \frac{1}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Problem 398: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 6, 319 leaves, 10 steps) :

$$\begin{aligned}
 & -\frac{c}{(a c^2 - d^2) x} + \frac{d \sqrt{1 + \frac{b x^3}{a}} \operatorname{AppellF1}\left[-\frac{1}{3}, \frac{1}{2}, 1, \frac{2}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{(a c^2 - d^2) x \sqrt{a + b x^3}} + \\
 & \frac{b^{1/3} c^{5/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (a c^2 - d^2)^{4/3}} + \frac{b^{1/3} c^{5/3} \log\left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 (a c^2 - d^2)^{4/3}} - \\
 & \frac{b^{1/3} c^{5/3} \log\left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 (a c^2 - d^2)^{4/3}}
 \end{aligned}$$

Result (type 6, 1029 leaves) :

$$\begin{aligned}
& - \frac{c}{a c^2 x - d^2 x} + \frac{d \sqrt{a + b x^3}}{a^2 c^2 x - a d^2 x} + \\
& \left(5 a b c^2 d x^2 \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \Big/ \left(2 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right. \\
& \left. \left(10 a (a c^2 - d^2) \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - 3 b x^3 \left(2 a c^2 \text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) \right. \\
& \left. \left(5 b d^3 x^2 \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \Big/ \left(2 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right. \\
& \left. \left(10 a (a c^2 - d^2) \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(2 a c^2 \text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + \right. \right. \\
& \left. \left. (a c^2 - d^2) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) \right) + \\
& \left(8 b^2 c^2 d x^5 \text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \Big/ \\
& \left(5 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(-16 a (a c^2 - d^2) \text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(2 a c^2 \text{AppellF1} \left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + \right. \right. \\
& \left. \left. (a c^2 - d^2) \text{AppellF1} \left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) \right) - \\
& \frac{b^{1/3} c^{5/3} \text{ArcTan} \left[\frac{-1 + \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}} \right]}{\sqrt{3} (a c^2 - d^2)^{4/3}} + \frac{b^{1/3} c^{5/3} \text{Log} \left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x \right]}{3 (a c^2 - d^2)^{4/3}} - \\
& \frac{b^{1/3} c^{5/3} \text{Log} \left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2 \right]}{6 (a c^2 - d^2)^{4/3}}
\end{aligned}$$

Problem 399: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 6, 324 leaves, 10 steps):

$$\begin{aligned}
& - \frac{c}{2(a c^2 - d^2) x^2} + \frac{d \sqrt{1 + \frac{b x^2}{a}} \operatorname{AppellF1}\left[-\frac{2}{3}, \frac{1}{2}, 1, \frac{1}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{2(a c^2 - d^2) x^2 \sqrt{a + b x^3}} + \\
& \frac{b^{2/3} c^{7/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right] - \frac{b^{2/3} c^{7/3} \operatorname{Log}\left[\left(a c^2 - d^2\right)^{1/3} + b^{1/3} c^{2/3} x\right]}{3(a c^2 - d^2)^{5/3}} +}{\sqrt{3} (a c^2 - d^2)^{5/3}} \\
& \frac{b^{2/3} c^{7/3} \operatorname{Log}\left[\left(a c^2 - d^2\right)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 (a c^2 - d^2)^{5/3}}
\end{aligned}$$

Result (type 6, 1044 leaves):

$$\begin{aligned}
& - \frac{c}{2(a c^2 - d^2) x^2} + \frac{d \sqrt{a + b x^3}}{2 a (a c^2 - d^2) x^2} + \\
& \left(10 a b c^2 d x \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) / \left(\sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right. \\
& \left. \left(8 a (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] - 3 b x^3 \left(2 a c^2 \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] + (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \right) - \\
& \left(2 b d^3 x \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) / \left(\sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right. \\
& \left. \left(8 a (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] - 3 b x^3 \left(2 a c^2 \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] + (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \right) + \\
& \left(7 b^2 c^2 d x^4 \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) / \\
& \left(8 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(14 a (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] - 3 b x^3 \left(2 a c^2 \operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] + (a c^2 - d^2) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \right) - \\
& \frac{b^{2/3} c^{7/3} \operatorname{ArcTan}\left[\frac{-(a c^2 - d^2)^{1/3} + 2 b^{1/3} c^{2/3} x}{\sqrt{3} (a c^2 - d^2)^{1/3}}\right] - \frac{b^{2/3} c^{7/3} \operatorname{Log}\left[\left(a c^2 - d^2\right)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 (a c^2 - d^2)^{5/3}} +}{\sqrt{3} (a c^2 - d^2)^{5/3}} \\
& \frac{b^{2/3} c^{7/3} \operatorname{Log}\left[\left(a c^2 - d^2\right)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 (a c^2 - d^2)^{5/3}}
\end{aligned}$$

Problem 400: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{a c + b c x^n + d \sqrt{a + b x^n}} dx$$

Optimal (type 6, 135 leaves, 4 steps):

$$\begin{aligned} & \frac{d x \sqrt{1 + \frac{b x^n}{a}} \text{AppellF1}\left[\frac{1}{n}, \frac{1}{2}, 1, 1 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2}\right]}{(a c^2 - d^2) \sqrt{a + b x^n}} + \\ & \frac{c x \text{Hypergeometric2F1}\left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b c^2 x^n}{a c^2 - d^2}\right]}{a c^2 - d^2} \end{aligned}$$

Result (type 6, 320 leaves):

$$\begin{aligned} & - \left(\left(2 a d (a c^2 - d^2) (1 + n) \times \text{AppellF1}\left[\frac{1}{n}, \frac{1}{2}, 1, 1 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2}\right] \right) \right) / \\ & \left(\sqrt{a + b x^n} (a c^2 - d^2 + b c^2 x^n) \left(-2 a b c^2 n x^n \text{AppellF1}\left[1 + \frac{1}{n}, \frac{1}{2}, 2, 2 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2}\right] + \right. \right. \\ & (a c^2 - d^2) \left(-b n x^n \text{AppellF1}\left[1 + \frac{1}{n}, \frac{3}{2}, 1, 2 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2}\right] + \right. \\ & \left. \left. \left. 2 a (1 + n) \text{AppellF1}\left[\frac{1}{n}, \frac{1}{2}, 1, 1 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2}\right] \right) \right) \right) + \\ & \frac{c x \text{Hypergeometric2F1}\left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b c^2 x^n}{a c^2 - d^2}\right]}{a c^2 - d^2} \end{aligned}$$

Problem 401: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m}{a c + b c x^n + d \sqrt{a + b x^n}} dx$$

Optimal (type 6, 167 leaves, 4 steps):

$$\begin{aligned} & \frac{d x^{1+m} \sqrt{1 + \frac{b x^n}{a}} \text{AppellF1}\left[\frac{1+m}{n}, \frac{1}{2}, 1, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2}\right]}{(a c^2 - d^2) (1 + m) \sqrt{a + b x^n}} + \\ & \frac{c x^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b c^2 x^n}{a c^2 - d^2}\right]}{(a c^2 - d^2) (1 + m)} \end{aligned}$$

Result (type 6, 373 leaves):

$$\begin{aligned}
& \left(x^{1+m} \left(- \left(\left(2 a d (-a c^2 + d^2)^2 (1 + m + n) \text{AppellF1} \left[\frac{1+m}{n}, \frac{1}{2}, 1, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(\sqrt{a + b x^n} (a c^2 - d^2 + b c^2 x^n) \right. \right. \right. \\
& \quad \left. \left. \left. \left(2 a (a c^2 - d^2) (1 + m + n) \text{AppellF1} \left[\frac{1+m}{n}, \frac{1}{2}, 1, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. b n x^n \left(2 a c^2 \text{AppellF1} \left[\frac{1+m+n}{n}, \frac{1}{2}, 2, 2 + \frac{1+m}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] + (a c^2 - d^2) \right. \right. \right. \\
& \quad \left. \left. \left. \left. \text{AppellF1} \left[\frac{1+m+n}{n}, \frac{3}{2}, 1, 2 + \frac{1+m}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) \right) \right) \right) + \\
& \quad \left. c \text{Hypergeometric2F1} \left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) \right) \left((a c^2 - \right. \\
& \quad \left. d^2) (1 + \right. \\
& \quad \left. m) \right)
\end{aligned}$$

Problem 404: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x} - x^{5/2}} dx$$

Optimal (type 3, 13 leaves, 5 steps):

$$\text{ArcTan}[\sqrt{x}] + \text{ArcTanh}[\sqrt{x}]$$

Result (type 3, 33 leaves):

$$\text{ArcTan}[\sqrt{x}] - \frac{1}{2} \text{Log}[1 - \sqrt{x}] + \frac{1}{2} \text{Log}[1 + \sqrt{x}]$$

Problem 425: Unable to integrate problem.

$$\int \left(a + \frac{b}{x} \right)^m (c + d x)^n dx$$

Optimal (type 6, 80 leaves, 4 steps):

$$\frac{1}{1-m} \left(a + \frac{b}{x} \right)^m x \left(1 + \frac{ax}{b} \right)^{-m} (c + d x)^n \left(1 + \frac{dx}{c} \right)^{-n} \text{AppellF1} [1-m, -m, -n, 2-m, -\frac{ax}{b}, -\frac{dx}{c}]$$

Result (type 8, 19 leaves):

$$\int \left(a + \frac{b}{x} \right)^m (c + d x)^n dx$$

Problem 429: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x} \right)^m}{c + d x} dx$$

Optimal (type 5, 101 leaves, 5 steps) :

$$-\frac{c \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{d (a c - b d) (1 + m)} + \frac{\left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, 1 + \frac{b}{a x}\right]}{a d (1 + m)}$$

Result (type 8, 19 leaves) :

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{c + d x} dx$$

Problem 430: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^2} dx$$

Optimal (type 5, 56 leaves, 3 steps) :

$$-\frac{b \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{(a c - b d)^2 (1 + m)}$$

Result (type 8, 19 leaves) :

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^2} dx$$

Problem 431: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^3} dx$$

Optimal (type 5, 112 leaves, 4 steps) :

$$-\frac{d \left(a + \frac{b}{x}\right)^{1+m}}{2 c (a c - b d) \left(d + \frac{c}{x}\right)^2} - \left(\begin{aligned} & \left(b \left(2 a c - b d (1 + m)\right) \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right] \right) / \\ & \left(2 c (a c - b d)^3 (1 + m)\right) \end{aligned} \right)$$

Result (type 8, 19 leaves) :

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^3} dx$$

Problem 432: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^4} dx$$

Optimal (type 5, 185 leaves, 5 steps) :

$$\begin{aligned} & \frac{d^2 \left(a + \frac{b}{x}\right)^{1+m}}{3 c^2 (a c - b d) \left(d + \frac{c}{x}\right)^3} - \frac{d \left(6 a c - b d (4 + m)\right) \left(a + \frac{b}{x}\right)^{1+m}}{6 c^2 (a c - b d)^2 \left(d + \frac{c}{x}\right)^2} - \\ & \left(b \left(6 a^2 c^2 - 6 a b c d (1 + m) + b^2 d^2 (2 + 3 m + m^2)\right) \left(a + \frac{b}{x}\right)^{1+m} \right. \\ & \left. \text{Hypergeometric2F1}[2, 1 + m, 2 + m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}] \right) / \left(6 c^2 (a c - b d)^4 (1 + m)\right) \end{aligned}$$

Result (type 8, 19 leaves) :

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^4} dx$$

Problem 436: Unable to integrate problem.

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - b x^2}} dx$$

Optimal (type 3, 28 leaves, 3 steps) :

$$\frac{\sqrt{b - \frac{a}{x^2}} x \text{Log}[x]}{\sqrt{a - b x^2}}$$

Result (type 8, 27 leaves) :

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - b x^2}} dx$$

Problem 439: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c + d x)^{3/2}}{\sqrt{a + \frac{b}{x^2}}} dx$$

Optimal (type 4, 406 leaves, 8 steps):

$$\begin{aligned} & \frac{2 c \sqrt{c + d x} (b + a x^2)}{5 a \sqrt{a + \frac{b}{x^2}} x} + \frac{2 (c + d x)^{3/2} (b + a x^2)}{5 a \sqrt{a + \frac{b}{x^2}} x} + \\ & \left(2 \sqrt{b} (a c^2 - 3 b d^2) \sqrt{c + d x} \sqrt{1 + \frac{a x^2}{b}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1 - \frac{\sqrt{-a} x}{\sqrt{b}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{-a} \sqrt{b} d}{a c - \sqrt{-a} \sqrt{b} d}\right] \right) / \\ & \left(5 (-a)^{3/2} d \sqrt{a + \frac{b}{x^2}} x \sqrt{\frac{a (c + d x)}{a c - \sqrt{-a} \sqrt{b} d}} \right) - \\ & \left(2 \sqrt{b} c (a c^2 + b d^2) \sqrt{\frac{a (c + d x)}{a c - \sqrt{-a} \sqrt{b} d}} \sqrt{1 + \frac{a x^2}{b}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1 - \frac{\sqrt{-a} x}{\sqrt{b}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{-a} \sqrt{b} d}{a c - \sqrt{-a} \sqrt{b} d}\right] \right) / \left(5 (-a)^{3/2} d \sqrt{a + \frac{b}{x^2}} x \sqrt{c + d x} \right) \end{aligned}$$

Result (type 4, 540 leaves):

$$\begin{aligned}
& \frac{1}{5 \sqrt{a + \frac{b}{x^2}} x} \\
& \sqrt{c + d x} \left(\frac{2 (2 c + d x) (b + a x^2)}{a} + \left(2 \left(d^2 \sqrt{-c - \frac{i \sqrt{b} d}{\sqrt{a}}} (-3 b^2 d^2 + a^2 c^2 x^2 + a b (c^2 - 3 d^2 x^2)) + \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} \left(-i a^{3/2} c^3 + a \sqrt{b} c^2 d + 3 i \sqrt{a} b c d^2 - 3 b^{3/2} d^3 \right) \sqrt{\frac{d \left(\frac{i \sqrt{b}}{\sqrt{a}} + x \right)}{c + d x}} \sqrt{-\frac{\frac{i \sqrt{b} d}{\sqrt{a}} - d x}{c + d x}} \right. \right. \\
& \left. \left. \left. (c + d x)^{3/2} \text{EllipticE}\left[\frac{i \text{ArcSinh}\left[\frac{\sqrt{-c - \frac{i \sqrt{b} d}{\sqrt{a}}}}{\sqrt{c + d x}}\right]}{\sqrt{a}}, \frac{\sqrt{a} c - i \sqrt{b} d}{\sqrt{a} c + i \sqrt{b} d}\right] - \sqrt{a} \sqrt{b} d \right. \right. \right. \\
& \left. \left. \left. \left(a c^2 + 4 i \sqrt{a} \sqrt{b} c d - 3 b d^2 \right) \sqrt{\frac{d \left(\frac{i \sqrt{b}}{\sqrt{a}} + x \right)}{c + d x}} \sqrt{-\frac{\frac{i \sqrt{b} d}{\sqrt{a}} - d x}{c + d x}} (c + d x)^{3/2} \text{EllipticF}\left[\right. \right. \right. \\
& \left. \left. \left. \left. i \text{ArcSinh}\left[\frac{\sqrt{-c - \frac{i \sqrt{b} d}{\sqrt{a}}}}{\sqrt{c + d x}}\right], \frac{\sqrt{a} c - i \sqrt{b} d}{\sqrt{a} c + i \sqrt{b} d} \right] \right] \right) \right) \right) / \left(a^2 d^2 \sqrt{-c - \frac{i \sqrt{b} d}{\sqrt{a}}} (c + d x) \right)
\end{aligned}$$

Problem 519: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1 + \frac{1}{x^2}} x}{(1 + x^2)^2} dx$$

Optimal (type 2, 9 leaves, 2 steps):

$$\frac{1}{\sqrt{1 + \frac{1}{x^2}}}$$

Result (type 2, 20 leaves):

$$\frac{\sqrt{1 + \frac{1}{x^2}} x^2}{1 + x^2}$$

Problem 520: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1 + \frac{1}{x^2}} \times (1 + x^2)} dx$$

Optimal (type 2, 9 leaves, 2 steps):

$$\frac{1}{\sqrt{1 + \frac{1}{x^2}}}$$

Result (type 2, 20 leaves):

$$\frac{\sqrt{1 + \frac{1}{x^2}} \times x^2}{1 + x^2}$$

Problem 580: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$2 \operatorname{ArcTan}\left[\sqrt{-\frac{x}{1+x}}\right]$$

Result (type 3, 32 leaves):

$$\frac{2 \sqrt{-\frac{x}{1+x}} \sqrt{1+x} \operatorname{ArcSinh}\left[\sqrt{x}\right]}{\sqrt{x}}$$

Problem 581: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$2 \operatorname{ArcTan}\left[\sqrt{\frac{1-x}{1+x}}\right]$$

Result (type 3, 47 leaves):

$$\frac{2 \sqrt{\frac{1-x}{1+x}} \sqrt{1-x^2} \operatorname{ArcSin}\left[\frac{\sqrt{1+x}}{\sqrt{2}}\right]}{-1+x}$$

Problem 582: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{a+b x}{c-b x}}}{a+b x} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTan}\left[\sqrt{\frac{a+b x}{c-b x}}\right]}{b}$$

Result (type 3, 80 leaves):

$$\frac{\frac{1}{2} \sqrt{c-b x} \sqrt{\frac{a+b x}{c-b x}} \operatorname{Log}\left[2 \sqrt{c-b x} \sqrt{a+b x}-\frac{1}{2} (a-c+2 b x)\right]}{b \sqrt{a+b x}}$$

Problem 583: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{a+b x}{c+d x}}}{a+b x} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{\frac{a+b x}{c+d x}}}{\sqrt{b}}\right]}{\sqrt{b} \sqrt{d}}$$

Result (type 3, 89 leaves):

$$\frac{\sqrt{\frac{a+b x}{c+d x}} \sqrt{c+d x} \operatorname{Log}\left[b c+a d+2 b d x+2 \sqrt{b} \sqrt{d} \sqrt{a+b x} \sqrt{c+d x}\right]}{\sqrt{b} \sqrt{d} \sqrt{a+b x}}$$

Problem 602: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x+\sqrt{-3-4 x-x^2}} dx$$

Optimal (type 3, 108 leaves, 10 steps):

$$\begin{aligned} & -\text{ArcTan}\left[\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right] - \sqrt{2} \text{ ArcTan}\left[\frac{1 - \frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right] + \\ & \frac{1}{2} \text{ Log}[3+x] + \frac{1}{2} \text{ Log}\left[\frac{3\sqrt{-1-x} + \sqrt{-1-x}x + x\sqrt{3+x}}{(3+x)^{3/2}}\right] \end{aligned}$$

Result (type 3, 1012 leaves):

$$\begin{aligned}
& \frac{1}{8} \left(4 \operatorname{ArcSin}[2+x] - 4 \sqrt{2} \operatorname{ArcTan}\left[\sqrt{2} (1+x)\right] + \right. \\
& 2 \sqrt{1-2 i \sqrt{2}} \operatorname{ArcTan}\left[\left(60+51 i \sqrt{2}+\left(-16+6 i \sqrt{2}\right) x^4+\right.\right. \\
& \left.\left.54 i \sqrt{1-2 i \sqrt{2}} \sqrt{-3-4 x-x^2}+x\left(68+176 i \sqrt{2}+99 i \sqrt{1-2 i \sqrt{2}} \sqrt{-3-4 x-x^2}\right)+\right.\right. \\
& 2 i x^3 \left(34 \left(i+\sqrt{2}\right)+9 \sqrt{1-2 i \sqrt{2}} \sqrt{-3-4 x-x^2}\right)+ \\
& i x^2 \left(44 i+185 \sqrt{2}+72 \sqrt{1-2 i \sqrt{2}} \sqrt{-3-4 x-x^2}\right)\Big/\left(93 i+150 \sqrt{2}+\right. \\
& \left.20 \left(17 i+22 \sqrt{2}\right) x+\left(493 i+466 \sqrt{2}\right) x^2+16 \left(19 i+13 \sqrt{2}\right) x^3+\left(66 i+32 \sqrt{2}\right) x^4\right]- \\
& \frac{1}{\sqrt{1+2 i \sqrt{2}}} 2 i \left(-i+2 \sqrt{2}\right) \operatorname{ArcTan}\left[\left(-60+51 i \sqrt{2}+2 \left(8+3 i \sqrt{2}\right) x^4+\right.\right. \\
& \left.\left.54 i \sqrt{1+2 i \sqrt{2}} \sqrt{-3-4 x-x^2}+2 x^3 \left(34+34 i \sqrt{2}+9 i \sqrt{1+2 i \sqrt{2}} \sqrt{-3-4 x-x^2}\right)+\right.\right. \\
& x^2 \left(44+185 i \sqrt{2}+72 i \sqrt{1+2 i \sqrt{2}} \sqrt{-3-4 x-x^2}\right)+ \\
& i x \left(68 i+176 \sqrt{2}+99 \sqrt{1+2 i \sqrt{2}} \sqrt{-3-4 x-x^2}\right)\Big/\Big. \\
& \left(-93 i+150 \sqrt{2}+20 \left(-17 i+22 \sqrt{2}\right) x+\left(-493 i+466 \sqrt{2}\right) x^2+\right. \\
& \left.16 \left(-19 i+13 \sqrt{2}\right) x^3+\left(-66 i+32 \sqrt{2}\right) x^4\right]+ \\
& 2 \operatorname{Log}\left[3+4 x+2 x^2\right]+\frac{\left(-i+2 \sqrt{2}\right) \operatorname{Log}\left[4 \left(3+4 x+2 x^2\right)^2\right]}{\sqrt{1+2 i \sqrt{2}}}+ \\
& \frac{\left(i+2 \sqrt{2}\right) \operatorname{Log}\left[4 \left(3+4 x+2 x^2\right)^2\right]}{\sqrt{1-2 i \sqrt{2}}}- \\
& \frac{1}{\sqrt{1-2 i \sqrt{2}}}\left(\frac{1}{\left(i+2 \sqrt{2}\right)}\right. \\
& \operatorname{Log}\left[\left(3+4 x+2 x^2\right)\left(3+6 i \sqrt{2}+\left(2+2 i \sqrt{2}\right) x^2-2 \sqrt{2-4 i \sqrt{2}} \sqrt{-3-4 x-x^2}\right)+\right. \\
& \left.x\left(4+8 i \sqrt{2}-2 \sqrt{2-4 i \sqrt{2}} \sqrt{-3-4 x-x^2}\right)\right]\Big)- \\
& \frac{1}{\sqrt{1+2 i \sqrt{2}}}\left(-i+2 \sqrt{2}\right) \operatorname{Log}\left[\left(3+4 x+2 x^2\right)\left(3-6 i \sqrt{2}+\left(2-2 i \sqrt{2}\right) x^2-\right.\right. \\
& \left.\left.2 \sqrt{2+4 i \sqrt{2}} \sqrt{-3-4 x-x^2}-2 x\left(-2+4 i \sqrt{2}+\sqrt{2+4 i \sqrt{2}} \sqrt{-3-4 x-x^2}\right)\right)\right]
\end{aligned}$$

Problem 603: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(x + \sqrt{-3 - 4x - x^2})^2} dx$$

Optimal (type 3, 87 leaves, 5 steps):

$$\frac{\frac{1 - \frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}}}}{+ \frac{\text{ArcTan}\left[\frac{1 - \frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right]}{\sqrt{2}}}$$

Result (type 3, 881 leaves):

$$\begin{aligned}
& \frac{1}{16} \left(\frac{8(3+x)}{3+4x+2x^2} + \frac{8(3+2x)\sqrt{-3-4x-x^2}}{3+4x+2x^2} + \right. \\
& 4\sqrt{2} \operatorname{ArcTan}[\sqrt{2}(1+x)] - \frac{1}{\sqrt{1+2\sqrt{2}}} 2\sqrt{2} \operatorname{ArcTan}[\\
& ((2+x)(3(5+4\sqrt{2})+16(2+\sqrt{2})x+2(9+2\sqrt{2})x^2)) / (12\sqrt{2}+8\sqrt{2}) \\
& x^3 - 9\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2} + x(40\sqrt{2}-5\sqrt{2}-12\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2}) + \\
& x^2(36\sqrt{2}-6\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2}) \Big)] + \frac{1}{\sqrt{1-2\sqrt{2}}} \\
& 2(2\sqrt{2}) \operatorname{ArcTanh}[((2+x)(3(5\sqrt{2}+4\sqrt{2})+16(2\sqrt{2})x+2(9\sqrt{2})x^2)) / \\
& (-5(8\sqrt{2})x + (-8\sqrt{2}+6\sqrt{2})x^3 - 12\sqrt{1-2\sqrt{2}}x\sqrt{-3-4x-x^2} + \\
& x^2(-36\sqrt{2}-6\sqrt{1-2\sqrt{2}}\sqrt{-3-4x-x^2}) - \\
& 3(4\sqrt{2}+2\sqrt{2}+3\sqrt{1-2\sqrt{2}}\sqrt{-3-4x-x^2}) \Big)] - \\
& \frac{(-2\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1+2\sqrt{2}}} - \frac{(2\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1-2\sqrt{2}}} + \\
& \frac{1}{\sqrt{1-2\sqrt{2}}} \\
& (2\sqrt{2}) \operatorname{Log}[(3+4x+2x^2) \left(3+6\sqrt{2} + (2+2\sqrt{2})x^2 - \right. \\
& \left. 2\sqrt{2-4\sqrt{2}}\sqrt{-3-4x-x^2} + x(4+8\sqrt{2}-2\sqrt{2-4\sqrt{2}}\sqrt{-3-4x-x^2}) \right)] + \\
& \frac{1}{\sqrt{1+2\sqrt{2}}} (-2\sqrt{2}) \operatorname{Log}[(3+4x+2x^2) \left(3-6\sqrt{2} + (2-2\sqrt{2})x^2 - \right. \\
& \left. 2\sqrt{2+4\sqrt{2}}\sqrt{-3-4x-x^2} - 2x(-2+4\sqrt{2}+\sqrt{2+4\sqrt{2}}\sqrt{-3-4x-x^2}) \right)]
\end{aligned}$$

Problem 604: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(x+\sqrt{-3-4x-x^2})^3} dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$-\frac{13 - \frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18 \left(1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2 \left(2 - \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9 \left(1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - \frac{\frac{1}{2} \frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{2\sqrt{2}}$$

Result (type 3, 914 leaves) :

$$\begin{aligned} & \frac{1}{32} \left(\frac{8(-3+2x)}{(3+4x+2x^2)^2} - \frac{8(2+3x)}{3+4x+2x^2} - \right. \\ & \frac{8\sqrt{-3-4x-x^2}(15+26x+22x^2+8x^3)}{(3+4x+2x^2)^2} - 12\sqrt{2}\operatorname{ArcTan}[\sqrt{2}(1+x)] + \frac{1}{\sqrt{1+2\sqrt{2}}} \\ & 6(2+\sqrt{2})\operatorname{ArcTan}\left[\left((2+x)\left(3\left(5+4\sqrt{2}\right)+16\left(2+\sqrt{2}\right)x+2\left(9+2\sqrt{2}\right)x^2\right)\right)\right] / \\ & \left(12\sqrt{2}+6\sqrt{2}+\left(8\sqrt{2}+6\sqrt{2}\right)x^3-9\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2}+\right. \\ & x\left(40\sqrt{2}-5\sqrt{2}-12\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2}\right)+ \\ & \left.x^2\left(36\sqrt{2}+8\sqrt{2}-6\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right] - \frac{1}{\sqrt{1-2\sqrt{2}}} \\ & 6(2\sqrt{2})\operatorname{ArcTanh}\left[\left((2+x)\left(3\left(5\sqrt{2}+4\sqrt{2}\right)+16\left(2\sqrt{2}\right)x+2\left(9\sqrt{2}+2\sqrt{2}\right)x^2\right)\right)\right] / \\ & \left(-5\left(8\sqrt{2}\right)x+\left(-8\sqrt{2}+6\sqrt{2}\right)x^3-12\sqrt{1-2\sqrt{2}}x\sqrt{-3-4x-x^2}+x^2\left(-36\sqrt{2}-6\sqrt{1-2\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right] + \\ & \frac{3\left(-2\sqrt{2}\right)\operatorname{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1+2\sqrt{2}}} + \frac{3\left(2\sqrt{2}\right)\operatorname{Log}\left[4\left(3+4x+2x^2\right)^2\right]}{\sqrt{1-2\sqrt{2}}} - \\ & \frac{1}{\sqrt{1-2\sqrt{2}}} \\ & 3\left(2\sqrt{2}\right)\operatorname{Log}\left[\left(3+4x+2x^2\right)\left(3+6\sqrt{2}+\left(2+2\sqrt{2}\right)x^2-\right.\right. \\ & \left.2\sqrt{2-4\sqrt{2}}\sqrt{-3-4x-x^2}+x\left(4+8\sqrt{2}-2\sqrt{2-4\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)] - \\ & \frac{1}{\sqrt{1+2\sqrt{2}}} 3\left(-2\sqrt{2}\right)\operatorname{Log}\left[\left(3+4x+2x^2\right)\left(3-6\sqrt{2}+\left(2-2\sqrt{2}\right)x^2-\right.\right. \\ & \left.2\sqrt{2+4\sqrt{2}}\sqrt{-3-4x-x^2}-2x\left(-2+4\sqrt{2}+\sqrt{2+4\sqrt{2}}\sqrt{-3-4x-x^2}\right)\right)] \end{aligned}$$

Problem 607: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 102 leaves, 7 steps) :

$$\begin{aligned} & \frac{2}{35} (13 - 3(-1+x)^2) \sqrt{3 - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \\ & \frac{1}{7} (3 - 2(-1+x)^2 - (-1+x)^4)^{3/2} (-1+x) + \\ & \frac{16}{5} \sqrt{3} \text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}] - \frac{176}{35} \sqrt{3} \text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}] \end{aligned}$$

Result (type 4, 278 leaves) :

$$\left(\begin{aligned} & 896 - 1056x + 352x^2 + 848x^3 - 1420x^4 + 1152x^5 - 602x^6 + 206x^7 - 45x^8 + 5x^9 + \frac{1}{\sqrt{-\frac{\frac{i}{2}(-2+x)}{(-i+\sqrt{3})x}}} \\ & 112i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-\frac{i}{2}+\sqrt{3}}] - \\ & 304i\sqrt{2}\sqrt{-\frac{\frac{i}{2}(-2+x)}{(-\frac{i}{2}+\sqrt{3})x}}x^2\sqrt{\frac{4-2x+x^2}{x^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-\frac{i}{2}+\sqrt{3}}] \end{aligned} \right) / \\ \left(35\sqrt{-x(-8+8x-4x^2+x^3)} \right)$$

Problem 608: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{8x - 8x^2 + 4x^3 - x^4} dx$$

Optimal (type 4, 62 leaves, 6 steps) :

$$\frac{1}{3} \sqrt{3 - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \frac{2 \text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}]}{\sqrt{3}} - \frac{4 \text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 256 leaves) :

$$\begin{aligned}
& - \left(\left(-16 + 24x - 24x^2 + 14x^3 - 5x^4 + x^5 - \frac{1}{\sqrt{-\frac{\frac{i}{2}(-2+x)}{(-\frac{i}{2}+\sqrt{3})x}}} \right. \right. \\
& \quad \left. \left. 2i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-\frac{i}{2}+\sqrt{3}}\right] + \right. \right. \\
& \quad \left. \left. 8i\sqrt{2}\sqrt{-\frac{\frac{i}{2}(-2+x)}{(-\frac{i}{2}+\sqrt{3})x}}x^2\sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-\frac{i}{2}+\sqrt{3}}\right] \right) / \right. \\
& \quad \left. \left(3\sqrt{-x(-8+8x-4x^2+x^3)} \right) \right)
\end{aligned}$$

Problem 609: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 17 leaves, 3 steps):

$$\begin{aligned}
& - \frac{\operatorname{EllipticF}\left[\operatorname{ArcSin}[1-x], -\frac{1}{3}\right]}{\sqrt{3}}
\end{aligned}$$

Result (type 4, 156 leaves):

$$\begin{aligned}
& \left(\sqrt{-\frac{i}{2}+\sqrt{3}+\frac{4i}{x}} \sqrt{-\frac{\frac{i}{2}(-2+x)}{(-\frac{i}{2}+\sqrt{3})x}} x \right. \\
& \quad \left. \left(-4+x-\frac{i}{2}\sqrt{3}x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{-\frac{i}{2}+\sqrt{3}}\right] \right) / \\
& \quad \left(\sqrt{2} \sqrt{\frac{i}{2}+\sqrt{3}-\frac{4i}{x}} \sqrt{-x(-8+8x-4x^2+x^3)} \right)
\end{aligned}$$

Problem 610: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{3/2}} dx$$

Optimal (type 4, 73 leaves, 6 steps):

$$\frac{\left(5 + (-1+x)^2\right) (-1+x)}{24 \sqrt{3-2(-1+x)^2 - (-1+x)^4}} + \frac{\text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}]}{8 \sqrt{3}} - \frac{\text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{4 \sqrt{3}}$$

Result (type 4, 261 leaves):

$$\frac{1}{24(-2+x)x}\sqrt{-x(-8+8x-4x^2+x^3)}$$

$$\begin{aligned} & \left(\frac{1}{\sqrt{\frac{4-2x+x^2}{x^2}}} \sqrt{2} \left(-\frac{i}{2} + \sqrt{3} \right) \sqrt{-\frac{i(-2+x)}{\left(-\frac{i}{2} + \sqrt{3} \right) x}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - \frac{4i}{x}}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-\frac{i}{2} + \sqrt{3}} \right] - \right. \\ & \left. \frac{1}{4-2x+x^2} \left(2+x^2 - \right. \right. \\ & \left. \left. 4\frac{i}{2}\sqrt{2} \sqrt{-\frac{i(-2+x)}{\left(-\frac{i}{2} + \sqrt{3} \right) x}} x^2 \sqrt{\frac{4-2x+x^2}{x^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - \frac{4i}{x}}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-\frac{i}{2} + \sqrt{3}} \right] \right) \right) \end{aligned}$$

Problem 611: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 109 leaves, 7 steps):

$$\begin{aligned} & \frac{\left(5 + (-1+x)^2\right) (-1+x)}{72 \left(3 - 2(-1+x)^2 - (-1+x)^4\right)^{3/2}} + \frac{\left(26 + 7(-1+x)^2\right) (-1+x)}{432 \sqrt{3 - 2(-1+x)^2 - (-1+x)^4}} + \\ & \frac{7 \text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}]}{144 \sqrt{3}} - \frac{11 \text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{144 \sqrt{3}} \end{aligned}$$

Result (type 4, 298 leaves) :

$$\left(\frac{1}{\sqrt{\frac{i(-2+x)}{(-i+\sqrt{3})x}}} 7 \pm \sqrt{2} (-2+x)x^2 \sqrt{\frac{4-2x+x^2}{x^2}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{\frac{i+\sqrt{3}-4i}{\sqrt{2}3^{1/4}}}}{x}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}] + \right. \\ \left(36 - 232x + 274x^2 - 226x^3 + 115x^4 - 37x^5 + 7x^6 - 19 \pm \sqrt{2} \sqrt{-\frac{\frac{i(-2+x)}{(-i+\sqrt{3})x}}{x^3}} \right. \\ \left. \left. \sqrt{\frac{4-2x+x^2}{x^2}} (-8+8x-4x^2+x^3) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{i+\sqrt{3}-4i}{\sqrt{2}3^{1/4}}}}{x}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}] \right) \middle/ \right. \\ \left. (-8+8x-4x^2+x^3) \right)$$

Problem 612: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int ((2-x) \times (4-2x+x^2))^{3/2} dx$$

Optimal (type 4, 102 leaves, 7 steps) :

$$\frac{2}{35} (13 - 3 (-1+x)^2) \sqrt{3 - 2 (-1+x)^2 - (-1+x)^4} (-1+x) + \\ \frac{1}{7} (3 - 2 (-1+x)^2 - (-1+x)^4)^{3/2} (-1+x) + \\ \frac{16}{5} \sqrt{3} \text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}] - \frac{176}{35} \sqrt{3} \text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]$$

Result (type 4, 278 leaves) :

$$\begin{aligned}
& \left(\sqrt{-x(-8 + 8x - 4x^2 + x^3)} \right. \\
& \left. \left(\sqrt{\frac{4 - 2x + x^2}{x^2}} (-224 + 152x + 44x^2 - 228x^3 + 230x^4 - 116x^5 + 35x^6 - 5x^7) + \right. \right. \\
& \left. \left. 112\sqrt{2} \left(-\frac{i}{2} + \sqrt{3}\right) \sqrt{-\frac{\frac{i}{2}(-2+x)}{(-\frac{i}{2} + \sqrt{3})x}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - \frac{4i}{x}}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{-\frac{i}{2} + \sqrt{3}}] + \right. \right. \\
& \left. \left. 304\frac{i}{2}\sqrt{2} \sqrt{-\frac{\frac{i}{2}(-2+x)}{(-\frac{i}{2} + \sqrt{3})x}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - \frac{4i}{x}}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{-\frac{i}{2} + \sqrt{3}}] \right) \right) / \\
& \left(35(-2+x)x \sqrt{\frac{4 - 2x + x^2}{x^2}} \right)
\end{aligned}$$

Problem 613: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{(2-x)x(4-2x+x^2)} dx$$

Optimal (type 4, 62 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x) + \\
& \frac{2\text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}]}{\sqrt{3}} - \frac{4\text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{\sqrt{3}}
\end{aligned}$$

Result (type 4, 256 leaves):

$$\begin{aligned} & \left(\sqrt{-x(-8 + 8x - 4x^2 + x^3)} \right) \left(\sqrt{\frac{4 - 2x + x^2}{x^2}} (-4 + 4x - 3x^2 + x^3) + \right. \\ & 2\sqrt{2} \left(-\frac{i}{2} + \sqrt{3} \right) \sqrt{-\frac{\frac{i}{2}(-2+x)}{(-\frac{i}{2} + \sqrt{3})x}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - \frac{4i}{x}}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{-\frac{i}{2} + \sqrt{3}}] + \\ & \left. 8i\sqrt{2} \sqrt{-\frac{\frac{i}{2}(-2+x)}{(-\frac{i}{2} + \sqrt{3})x}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - \frac{4i}{x}}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{-\frac{i}{2} + \sqrt{3}}] \right) / \\ & \left(3(-2+x)x\sqrt{\frac{4 - 2x + x^2}{x^2}} \right) \end{aligned}$$

Problem 614: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}} dx$$

Optimal (type 4, 17 leaves, 3 steps):

$$\frac{\text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 100 leaves):

$$\begin{aligned} & - \left(\left((-1)^{1/3} (-2+x)^2 \sqrt{\frac{x(-1+\frac{i}{2}\sqrt{3}+x)}{(-2+x)^2}} \sqrt{\frac{-2+x-(-1)^{1/3}x}{-2+x}} \right. \right. \\ & \left. \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{-\frac{(-1)^{2/3}x}{-2+x}}\right], (-1)^{2/3}] \right) / \left(\sqrt{-x(-8+8x-4x^2+x^3)} \right) \right) \end{aligned}$$

Problem 615: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}} dx$$

Optimal (type 4, 73 leaves, 6 steps) :

$$\frac{\left(5 + (-1+x)^2\right) (-1+x)}{24 \sqrt{3 - 2 (-1+x)^2 - (-1+x)^4}} + \frac{\text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}]}{8 \sqrt{3}} - \frac{\text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{4 \sqrt{3}}$$

Result (type 4, 298 leaves) :

$$\begin{aligned} & \left(-2+x \right)^2 x \left(4 - 2x + x^2 \right) \\ & \left(2 (-1+x) x - 3 (4 - 2x + x^2) - \frac{3x (4 - 2x + x^2)}{-2+x} - 4 (2-x) \sqrt{\frac{4 - 2x + x^2}{(-2+x)^2}} x \sqrt{\frac{4 - 2x + x^2}{(-2+x)^2}} - \right. \\ & \quad \left. \sqrt{2} \left(\frac{i}{2} + \sqrt{3}\right) \sqrt{\frac{\frac{i}{2} x}{\left(\frac{i}{2} + \sqrt{3}\right) (-2+x)}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3}} - \frac{4i}{-2+x}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{\frac{i}{2} + \sqrt{3}}] + \right. \\ & \quad \left. 4i\sqrt{2} \sqrt{\frac{\frac{i}{2} x}{\left(\frac{i}{2} + \sqrt{3}\right) (-2+x)}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3}} - \frac{4i}{-2+x}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{\frac{i}{2} + \sqrt{3}}] \right) / \\ & \quad \left(96 (-x (-8 + 8x - 4x^2 + x^3))^{3/2} \right) \end{aligned}$$

Problem 616: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left((2-x) \times (4-2x+x^2)\right)^{5/2}} dx$$

Optimal (type 4, 109 leaves, 7 steps) :

$$\begin{aligned} & \frac{\left(5 + (-1+x)^2\right) (-1+x)}{72 \left(3 - 2 (-1+x)^2 - (-1+x)^4\right)^{3/2}} + \frac{\left(26 + 7 (-1+x)^2\right) (-1+x)}{432 \sqrt{3 - 2 (-1+x)^2 - (-1+x)^4}} + \\ & \quad \frac{7 \text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}]}{144 \sqrt{3}} - \frac{11 \text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{144 \sqrt{3}} \end{aligned}$$

Result (type 4, 327 leaves) :

$$\begin{aligned}
 & \left(-2 + x \right)^3 x^2 \left(4 - 2x + x^2 \right)^2 \\
 & - \frac{7x \left(4 - 2x + x^2 \right)}{-2 + x} + \frac{36 + 216x - 622x^2 + 670x^3 - 445x^4 + 187x^5 - 49x^6 + 7x^7}{\left(-2 + x \right)^2 x \left(4 - 2x + x^2 \right)} + \\
 & \frac{7 \pm \sqrt{2} x \sqrt{\frac{4 - 2x + x^2}{(-2+x)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3}} - \frac{4i}{-2+x}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{i + \sqrt{3}}\right]}{\sqrt{\frac{i x}{\left(i + \sqrt{3}\right) (-2+x)}}} - \\
 & 19 \pm \sqrt{2} (-2+x) \sqrt{\frac{\pm x}{\left(\pm i + \sqrt{3}\right) (-2+x)}} \sqrt{\frac{4 - 2x + x^2}{(-2+x)^2}} \\
 & \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3}} - \frac{4i}{-2+x}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{i + \sqrt{3}}\right] \Bigg) \Bigg) / \left(432 \left(-x \left(-8 + 8x - 4x^2 + x^3 \right) \right)^{5/2} \right)
 \end{aligned}$$

Problem 617: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^{3/2} dx$$

Optimal (type 4, 730 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{7} \left(\frac{c}{d} + x \right) (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)^{3/2} + \frac{1}{35 d^2} \\
& 2 c \left(\frac{c}{d} + x \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \left(7 c^3 + 20 a d^2 - 3 c d^2 \left(\frac{c}{d} + x \right)^2 \right) - \\
& \frac{16 c^3 (c^3 + 8 a d^2) \left(\frac{c}{d} + x \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}}{35 d^2 \sqrt{c^3 + 4 a d^2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)} + \\
& \left(\frac{16 c^{13/4} (c^3 + 4 a d^2)^{3/4} (c^3 + 8 a d^2)}{\sqrt{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \right. \\
& \left. \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] \right) / \\
& \left(35 d^5 \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) + \left(8 c^{7/4} (c^3 + 4 a d^2)^{3/4} \right. \\
& \left. \left(\sqrt{c^3 + 4 a d^2} (c^3 + 5 a d^2) - c^{3/2} (c^3 + 8 a d^2) \right) \sqrt{\frac{d^2 (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)}{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \right. \\
& \left. \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] \right) / \\
& \left(35 d^5 \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right)
\end{aligned}$$

Result (type 4, 10468 leaves):

$$\begin{aligned}
& \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \\
& \left(\frac{4 c^2 (2 c^3 + 15 a d^2)}{35 d^3} - \frac{4 c (c^3 - 15 a d^2) x}{35 d^2} + \frac{2 c^3 x^2}{35 d} + \frac{34 c^2 x^3}{35} + \frac{5}{7} c d x^4 + \frac{d^2 x^5}{7} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{35 d^3} 16 c^2 \left(2 a c^3 d \left(\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \quad \left. \sqrt{\left(\left(\left(\left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \right. \\
& \quad \left. \left(\left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) \right) \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \\
& \quad \sqrt{\left(\left(\left(\left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-c - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \left(\left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-c - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right) \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) \\
& \quad \sqrt{\left(\left(\left(\left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \left(\left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right) \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \left(\left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right) \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\left[\frac{-c + \sqrt{c^2 + 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right] \left(-\frac{-c + \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} + x \right) \right) \right], \\
& \left(\left(\frac{-c + \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} - \frac{-c - \sqrt{c^2 + 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \right. \\
& \quad \left. \left(\frac{-c - \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} - \frac{-c + \sqrt{c^2 + 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \right) / \\
& \quad \left(\left(\frac{-c - \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} - \frac{-c - \sqrt{c^2 + 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \right. \\
& \quad \left. \left(\frac{-c + \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} - \frac{-c + \sqrt{c^2 + 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \right) / \\
& \quad \left(\left(-\frac{-c - \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} + \frac{-c + \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \right. \\
& \quad \left. \left(-\frac{-c + \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} + \frac{-c + \sqrt{c^2 + 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \right) / \\
& \quad \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) + \\
& 40 a^2 d^3 \left(\frac{-c - \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} - \frac{-c + \sqrt{c^2 + 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \\
& \sqrt{\left(\left(\left(\left(-\frac{-c + \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} + \frac{-c + \sqrt{c^2 + 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \right. \right.} \\
& \quad \left. \left. \left(-\frac{-c - \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} + x \right) \right) \right) / \\
& \quad \left(\left(-\frac{-c - \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} + \frac{-c + \sqrt{c^2 + 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \right. \\
& \quad \left. \left(-\frac{-c + \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} + x \right) \right) \right) \left(-\frac{-c + \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} + x \right)^2 \\
& \sqrt{\left(\left(\left(\left(-\frac{-c - \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} + \frac{-c + \sqrt{c^2 - 2 \cdot i \cdot \sqrt{a} \cdot \sqrt{c} \cdot d}}{d} \right) \right. \right.} \\
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \Bigg) \Bigg/ \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \left. \left. \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \\
& \sqrt{\left(\left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right.} \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \Bigg/ \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right.} \\
& \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \Bigg) } \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right.} \right. \\
& \left. \left. \left. \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \Bigg/ \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right.} \\
& \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right], \\
& \left(\left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \Bigg/ \\
& \left(\left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \Bigg] \Bigg] \Bigg/ \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) - \\
& \left(8 c^5 \left(\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \right.} \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \\
& \left. \sqrt{\left(\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \right.} \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \\
& \left. \sqrt{\left(\left(\left(\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right.} \\
& \left. \left(\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - d x \right) \right) \right) / \\
& \left. \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} \right) \text{EllipticF}[\text{ArcSin}[\right. \right. \\
& \left. \left. \left(\left(\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right. \right. \\
& \left. \left. \left(\left(\sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 i \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 i \sqrt{a} \sqrt{c} d} - \right. \right. \right. \right. \\
& \left. \left. \left. \left. d x \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2} \right] + \frac{1}{d} \\
& 2 \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \operatorname{EllipticPi} \left[\frac{-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d}}{-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d}}, \operatorname{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - d x \right)} \right] \right] \\
& \left. \frac{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2} \right] \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) - \\
& \left(64 a c^2 d^2 \left(\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \right. \\
& \left. \sqrt{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) /} \right. \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \right.} \\
& \quad \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \quad \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \\
& \quad \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + dx \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - dx \right) \right) \right) \\
& \quad \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \operatorname{EllipticF}[\operatorname{ArcSin}[\right. \right. \\
& \quad \left. \left. \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + dx \right) \right) / \right. \right. \\
& \quad \left. \left. \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \right. \right. \right. \\
& \quad \left. \left. \left. dx \right) \right) \right)] , \frac{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2} \right] + \frac{1}{d} \\
& \quad 2 \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \operatorname{EllipticPi}\left[\frac{\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d}}{\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d}}, \operatorname{ArcSin}[\right. \right. \\
& \quad \left. \left. \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + dx \right) \right) / \right. \right. \\
& \quad \left. \left. \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \right. \right. \right. \\
& \quad \left. \left. \left. dx \right) \right) \right)] , \frac{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2} \right] \right) / \\
& \quad \left(\left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-c + \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} \right) \\
& \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} - \\
& \frac{1}{\sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} c^4 d \left(\left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right. \\
& \left. \left(-\frac{-c - \sqrt{c^2 + 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) + \\
& 2 \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} \right) \\
& \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \\
& \sqrt{\left(\left(\sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \right.} \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \\
& \sqrt{\left(\left(\sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \right.} \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \\
& \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{Im} \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right.} \\
& \left. \left(\left(\sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{Im} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{Im} \sqrt{a} \sqrt{c} d} - d x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \operatorname{EllipticE} \left[\right. \right. \\
& \quad \operatorname{ArcSin} \left[\sqrt{ \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. } \right. \\
& \quad \left. \left. \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) / \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right) \right) \right] , \\
& \quad \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \right) / \left(2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) + \\
& \quad \left(d \left(\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) - \frac{1}{d} \left(-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \right. \\
& \quad \left. \left. \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \right) \\
& \quad \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{ \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. } \right. \right. \\
& \quad \left. \left. \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) / \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right) \right) \right] , \\
& \quad \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \right) / \\
& \quad \left(2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) - \\
& \quad \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \text{EllipticPi} \left[\frac{\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \right. \\
& \text{ArcSin} \left[\sqrt{ \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. } \right. \\
& \left. \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) / \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \\
& \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right) \right) \right], \\
& \left. \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \right) / \\
& \left. \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \right] - \\
& \frac{1}{\sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} 8 a c d^3 \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right. \\
& \left. \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) + \right. \\
& 2 \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \right. \\
& \sqrt{ \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right. } \\
& \left. \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. } \\
& \left. \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \right.} \\
& \quad \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \quad \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) \\
& \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + dx \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - dx \right) \right) \right) \\
& \left(\left(d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \text{EllipticE}[\right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + dx \right) \right) / \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - dx \right) \right) \right) \right], \right. \\
& \quad \left. \left(\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right) / \left(2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) + \right. \\
& \quad \left. \left(d \left(\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) - \frac{1}{d} \left(-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \right) \right] \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + dx \right) \right) / \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - dx \right) \right) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \Bigg| \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \\
& \left. \left(2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) - \right. \\
& \left. \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \right. \right. \\
& \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \text{EllipticPi} \left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \right. \right. \\
& \left. \left. \text{ArcSin} \left[\sqrt{ \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) \right] \Bigg| \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \right. \\
& \left. \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right) \right) \right], \right. \\
& \left. \left. \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \right) \Bigg| \frac{\left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)}{\left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)} \right) \right) \right)
\end{aligned}$$

Problem 618: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} dx$$

Optimal (type 4, 622 leaves, 5 steps):

$$\begin{aligned}
& \frac{1}{3} \left(\frac{c}{d} + x \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} - \frac{2 c^2 \left(\frac{c}{d} + x \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}}{3 \sqrt{c^3 + 4 a d^2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)} + \\
& \left(2 c^{9/4} (c^3 + 4 a d^2)^{3/4} \sqrt{\frac{d^2 (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)}{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \right. \\
& \left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] \right) / \\
& \left(3 d^3 \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) + \\
& \left(c^{3/4} (c^3 + 4 a d^2)^{1/4} \left(c^3 + 4 a d^2 - c^{3/2} \sqrt{c^3 + 4 a d^2} \right) \sqrt{\frac{d^2 (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)}{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \right. \\
& \left. \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] \right) / \\
& \left(3 d^3 \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right)
\end{aligned}$$

Result (type 4, 5218 leaves):

$$\begin{aligned}
& \left(\frac{c}{3 d} + \frac{x}{3} \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} + \\
& \frac{1}{3 d} 2 c \left(\left(8 a d \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \sqrt{\left(\left(\left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right.} \right. \\
& \left. \left. \left. \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \quad \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \\
& \sqrt{\left(\left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right.} \\
& \quad \left. \left. \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \quad \left. \left. \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \\
& \sqrt{\left(\left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right.} \\
& \quad \left. \left. \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \quad \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right.} \right. \\
& \quad \left. \left. \left. \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \quad \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right], \\
& \left(\left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \quad \left. \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) / \\
& \left(\left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)] \Bigg) / \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) - \\
& \left(8 c^2 \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \right. \\
& \left. \sqrt{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) /} \right. \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right] \\
& \left. \sqrt{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) /} \right. \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right] \\
& \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right.} \\
& \left. \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \text{EllipticF}[\text{ArcSin}\left[\right. \right. \right. \\
& \left. \left. \left. \sqrt{\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right. \right. \\
& \left. \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \right. \right. \right. \\
& \left. \left. \left. \left. d x \right) \right) \right] , \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] + \frac{1}{d} \right. \\
& \left. 2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \text{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \text{ArcSin}\left[\right. \right. \\
& \left. \left. \sqrt{\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right. \right. \\
& \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \right. \right. \right. \\
& \left. \left. \left. d x \right) \right) \right] , \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \right) / \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) - \\
& \frac{1}{\sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} c d \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right. \\
& \left. \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \\
& \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \\
& \sqrt{\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \right.} \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right. \right.} \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \right. \right.} \\
& \left. \left. \left. \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) \\
& \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right.} \\
& \left. \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - d x \right) \right) \right) \\
& \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \operatorname{EllipticE}[\right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + d x \right) \right) / \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - d x \right) \right) \right) \right], \right. \\
& \left. \left. \left. \left. \left. \left. \left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2 \right] \right) / \left(2 \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2 \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(d \left(\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \right. \right. \\
& \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) - \frac{1}{d} \left(-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \\
& \left(\left. \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \right. \\
& \left. \left. \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + dx \right) \right) / \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \\
& \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - dx \right) \right) \right], \\
& \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2 \right] / \right. \\
& \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2 \right) \right) / \\
& \left(2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) - \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right. \right. \\
& \left. \left. -\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \text{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}} \right], \\
& \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \right. \\
& \left. \left. \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + dx \right) \right) / \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \\
& \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - dx \right) \right) \right], \\
& \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2 \right] / \right.
\end{aligned}$$

$$\left(\left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right)$$

Problem 619: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} dx$$

Optimal (type 4, 227 leaves, 2 steps) :

$$\begin{aligned} & \left(c^3 + 4 a d^2 \right)^{1/4} \sqrt{\frac{d^2 (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)}{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \\ & \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] \Bigg) \\ & \left(2 c^{1/4} d \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) \end{aligned}$$

Result (type 4, 822 leaves) :

$$\begin{aligned}
& \left(2 \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - dx \right) \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + dx \right) \right. \\
& \quad \left. \sqrt{\left(- \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \left(c - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + dx \right) \right) / \right. \right.} \\
& \quad \left. \left. \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - dx \right) \right) \right) \right) \right) \\
& \quad \sqrt{\left(- \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \left(c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + dx \right) \right) / \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \right. \right.} \\
& \quad \left. \left. \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - dx \right) \right) \right) } \operatorname{EllipticF} [\\
& \quad \operatorname{ArcSin} \left[\sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + dx \right) \right) / \right. \right.} \\
& \quad \left. \left. \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - dx \right) \right) \right) \right], \\
& \quad \left. \left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2 \right] / \left(d \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right. \\
& \quad \left. \left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2 \right) \right) / \left(d \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + dx \right) \right) / \right. \right.} \\
& \quad \left. \left. \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - dx \right) \right) \right) \right) \\
& \quad \sqrt{4 a c + x^2 (2 c + d x)^2}
\end{aligned}$$

Problem 620: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)^{3/2}} dx$$

Optimal (type 4, 674 leaves, 5 steps):

$$\begin{aligned}
& - \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4 a d^2 - c d^2 \left(\frac{c}{d} + x\right)^2\right)}{8 a c \left(c^3 + 4 a d^2\right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} - \\
& \frac{d^2 \left(\frac{c}{d} + x\right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}}{8 a \left(c^3 + 4 a d^2\right)^{3/2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}}\right)} + \left(c^{1/4} \sqrt{\frac{d^2 \left(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4\right)}{\left(c^3 + 4 a d^2\right) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}}\right)^2}} \right. \\
& \left. \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c + d x}{c^{1/4} \left(c^3 + 4 a d^2\right)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}}\right)\right] \right) / \\
& \left(8 a d \left(c^3 + 4 a d^2\right)^{1/4} \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) + \\
& \left(c^3 + 4 a d^2 - c^{3/2} \sqrt{c^3 + 4 a d^2} \right) \sqrt{\frac{d^2 \left(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4\right)}{\left(c^3 + 4 a d^2\right) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}}\right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \\
& \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c + d x}{c^{1/4} \left(c^3 + 4 a d^2\right)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}}\right)\right] / \\
& \left(16 a c^{5/4} d \left(c^3 + 4 a d^2\right)^{3/4} \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right)
\end{aligned}$$

Result (type 4, 5276 leaves) :

$$\begin{aligned}
& \frac{4 a c d + 2 c^3 x + 4 a d^2 x + 3 c^2 d x^2 + c d^2 x^3}{8 a c \left(c^3 + 4 a d^2\right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} + \\
& \frac{1}{8 a c \left(c^3 + 4 a d^2\right)} d \left(\left(8 a d \left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \sqrt{\left(\left(-\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \right.} \right. \\
& \left. \left. \left(-\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \quad \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \\
& \sqrt{\left(\left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right.} \\
& \quad \left. \left. \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \quad \left. \left. \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \\
& \sqrt{\left(\left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right.} \\
& \quad \left. \left. \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \quad \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right.} \right. \\
& \quad \left. \left. \left. \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) / \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \quad \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right], \\
& \left(\left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \quad \left. \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) / \\
& \left(\left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)] \Bigg) / \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) - \\
& \left(8 c^2 \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \right. \\
& \left. \sqrt{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) /} \right. \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right. \\
& \left. \sqrt{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) /} \right. \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right.} \\
& \left. \left. \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \text{EllipticF}[\text{ArcSin}\left[\right. \right. \right. \\
& \left. \left. \left. \sqrt{\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right. \right. \\
& \left. \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \right. \right. \right. \\
& \left. \left. \left. \left. d x \right) \right) \right] , \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] + \frac{1}{d} \right. \\
& \left. 2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \text{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \text{ArcSin}\left[\right. \right. \\
& \left. \left. \sqrt{\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right. \right. \\
& \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \right. \right. \right. \\
& \left. \left. \left. d x \right) \right) \right] , \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \right] / \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) - \\
& \frac{1}{\sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} c d \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right. \\
& \left. \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \\
& \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \\
& \sqrt{\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \right.} \\
& \left. \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right. \right.} \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) / \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \right. \right.} \\
& \left. \left. \left. \frac{-c + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right) \right) \\
& \sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + d x \right) \right) / \right.} \\
& \left. \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - d x \right) \right) \right) \\
& \left(d \left(-\frac{-c - \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d}}{d} \right) \operatorname{EllipticE}[\right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + d x \right) \right) / \left(\left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - d x \right) \right) \right) \right], \right. \\
& \left. \left. \left. \left. \left. \left. \left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2 \right] \right) / \left(2 \sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right) + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left(\sqrt{c^2 - 2 \operatorname{i} \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \operatorname{i} \sqrt{a} \sqrt{c} d} \right)^2 \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(d \left(\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) - \frac{1}{d} \left(-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \\
& \quad \left(\left. \left. \left. \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \right. \\
& \quad \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \left. \right) / \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \\
& \quad \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right) \right) \right], \\
& \left. \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2 \right] \right) / \\
& \left. \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2 \right) \right) \\
& \left(2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) - \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right. \right. \\
& \quad \left. \left. \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \text{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}} \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \right. \right. \right. \right. \\
& \quad \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \left. \right) / \left(\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \right. \right. \\
& \quad \left. \left. \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right) \right) \right], \\
& \left. \left. \left. \left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2 \right] \right) /
\end{aligned}$$

$$\left(\left(-\frac{c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \right)$$

Problem 621: Result more than twice size of optimal antiderivative.

$$\int \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} dx$$

Optimal (type 4, 663 leaves, 5 steps):

$$\frac{1}{3} \left(\frac{d}{4e} + x \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} - \frac{2 d^2 \left(\frac{d}{4e} + x \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}}{\sqrt{5 d^4 + 256 a e^3} \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)} +$$

$$\left(d^2 (5 d^4 + 256 a e^3)^{3/4} \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)^2}} \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right.$$

$$\left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right] \right) /$$

$$\left(8 \sqrt{2} e^2 \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) +$$

$$\left((5 d^4 + 256 a e^3)^{1/4} \left(5 d^4 + 256 a e^3 - 3 d^2 \sqrt{5 d^4 + 256 a e^3} \right) \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)^2}} \right.$$

$$\left. \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right] \right) /$$

$$\left(48 \sqrt{2} e^2 \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right)$$

Result (type 4, 7543 leaves):

$$\left(\frac{d}{12e} + \frac{x}{3} \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} +$$

$$\begin{aligned}
& \frac{1}{24 e} \left(2 d^4 \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \left. \sqrt{\left(\left(\left(\left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \right. \\
& \quad \left. \left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \\
& \quad \sqrt{\left(\left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \right. \\
& \quad \left. \left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \\
& \quad \sqrt{\left(\left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \right. \\
& \quad \left. \left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \right. \\
& \quad \left. \left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \right] / \left(\left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \Bigg] , \\
& \left(\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \left. \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) / \\
& \quad \left(\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \left. \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) / \\
& \quad \left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \left. \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) \\
& \quad \left(\sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) + \\
& \quad \left(256 a e^3 \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \left. \sqrt{\left(\left(\left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right.} \\
& \quad \left. \left. \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) / \\
& \quad \left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \left. \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \Bigg) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \\
& \quad \sqrt{\left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right.} \\
& \quad \left. \left. \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \Bigg) \Bigg/ \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \right. \right. \\
& \left. \left. -\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \Bigg) \\
& \sqrt{\left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right.} \\
& \left. \left(-\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \Bigg/ \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \right. \right. \\
& \left. \left. -\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \Bigg) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right.} \right. \\
& \left. \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \Bigg/ \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \right. \right. \\
& \left. \left. -\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \Bigg] , \\
& \left(\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \Bigg/ \\
& \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \Bigg] \Bigg/ \\
& \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) - \\
& \left(12 d^3 e \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \left. \left(- \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(- \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) / \right.} \\
& \left. \left. \left. e \left(- \frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \\
& \left. \left. \left(- \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) / \\
& \left. \sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(- \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) / \right.} \\
& \left. \left. \left. e \left(- \frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \\
& \left. \left. \left(- \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) / \\
& \sqrt{\left(\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right.} \\
& \left. \left. 4 e x \right) \right) / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right. \\
& \left. \left. \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \right) \left(-\frac{1}{4 e} \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right) \right.} \right. \\
& \quad \left. \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right] / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right. \\
& \quad \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \Bigg], \\
& \frac{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2}{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2} + \frac{1}{2 e} \\
& \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \text{ EllipticPi}\left[\frac{-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e}}{\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e}}, \right. \\
& \quad \left. \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right) \right.} \right. \right. \\
& \quad \left. \left. \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right] / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right. \\
& \quad \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \Bigg], \\
& \frac{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2}{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2} \Bigg] / \\
& \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \\
& \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \\
& \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}} \\
& \frac{24}{d^2} \\
& \frac{e^2}{\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^3} \\
& \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) + \\
& \frac{1}{2} \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \\
& \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \\
& \sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) / \right.} \\
& \left. \left(e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \\
& \left. \left. \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \\
& \sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) / \right.} \\
& \left. \left. \left(e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \\
& \left. \left. \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \\
& \sqrt{\left(\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right) / \right.}
\end{aligned}$$

$$\begin{aligned}
& \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \Bigg) \Bigg/ \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \right. \right. \\
& \left. \left. \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \\
& \left(2e \left(-\frac{d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \text{EllipticE}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right.} \right. \\
& \left. \left. \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) \Bigg/ \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \right. \right. \\
& \left. \left. \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \Bigg] , \\
& \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2 \Bigg/ \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \right. \\
& \left. \left. \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2 \Bigg] \Bigg/ \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \right. \\
& \left. 2e \left(\frac{1}{4e} \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \right. \right. \\
& \left. \left. - \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{1}{4e} \left(-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \right. \\
& \left. \left. - \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right.} \right. \\
& \left. \left. \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) \Bigg/ \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \right. \right. \\
& \left. \left. \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \Bigg]
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3} - 4 e x} \right) \Bigg) \Bigg], \\
& \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 \Big/ \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \right. \\
& \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 \Big] \Big/ \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \right. \\
& \left. \left(- \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) - \\
& \left(\left(- \frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \right. \right. \\
& \left. \left. \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \left. - \frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right. \\
& \left. \text{EllipticPi} \left[\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e}, \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right], \text{ArcSin} \left[\right. \right. \\
& \left. \left. - \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right] \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right. \right. \right.} \\
& \left. \left. \left. \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right) \Big/ \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right. \right. \\
& \left. \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \Bigg) \Bigg], \\
& \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 \Big/ \\
& \left. \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 \right] \Bigg)
\end{aligned}$$

$$\left(\left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right)$$

Problem 622: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}} dx$$

Optimal (type 4, 235 leaves, 2 steps) :

$$\begin{aligned} & \left(5d^4 + 256ae^3 \right)^{1/4} \sqrt{\frac{e(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)}{(5d^4 + 256ae^3) \left(1 + \frac{16e^2(\frac{d}{4e} + x)^2}{\sqrt{5d^4 + 256ae^3}} \right)^2}} \left(1 + \frac{16e^2(\frac{d}{4e} + x)^2}{\sqrt{5d^4 + 256ae^3}} \right) \\ & \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d + 4ex}{(5d^4 + 256ae^3)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3d^2}{\sqrt{5d^4 + 256ae^3}} \right) \right] / \\ & \left(\sqrt{2}e\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) \end{aligned}$$

Result (type 4, 1065 leaves) :

$$\begin{aligned} & - \left(\left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \left(d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right. \\ & \left. \sqrt{\left(- \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) \right) / \right.} \\ & \left. \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \\ & \left. \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right) \\ & \sqrt{\left(3d^2 - 2\sqrt{d^4 - 64ae^3} - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} + } \end{aligned}$$

$$\begin{aligned}
& d \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) + \\
& 4 e \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) x \Bigg) / \\
& \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right. \\
& \left. \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \Bigg) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right.} \right. \\
& \left. \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right) / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right. \\
& \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \Bigg], \\
& \left. \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 \right] / \\
& \left. \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 \right) \Bigg) / \\
& \left(2 e \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right. \\
& \left. \sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) / \right.} \right. \\
& \left. \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right. \\
& \left. \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \Bigg) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \Bigg)
\end{aligned}$$

Problem 623: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)^{3/2}} dx$$

Optimal (type 4, 748 leaves, 5 steps) :

$$\frac{4 e \left(\frac{d}{4 e} + x\right) \left(13 d^4 - 256 a e^3 - 48 d^2 e^2 \left(\frac{d}{4 e} + x\right)^2\right)}{(5 d^8 - 64 a d^4 e^3 - 16384 a^2 e^6) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}} +$$

$$\frac{384 d^2 e^2 \left(\frac{d}{4 e} + x\right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}}{(d^4 - 64 a e^3) (5 d^4 + 256 a e^3)^{3/2} \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right)} -$$

$$\left\{ 12 \sqrt{2} d^2 \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right)^2}} \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right)$$

$$\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}}\right)\right] \Bigg|$$

$$\left((d^4 - 64 a e^3) (5 d^4 + 256 a e^3)^{1/4} \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) -$$

$$\left\{ 2 \sqrt{2} \left(5 d^4 + 256 a e^3 - 3 d^2 \sqrt{5 d^4 + 256 a e^3}\right) \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right)^2}} \right.$$

$$\left. \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x\right)^2}{\sqrt{5 d^4 + 256 a e^3}}\right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}}\right)\right]\right\}$$

$$\left((d^4 - 64 a e^3) (5 d^4 + 256 a e^3)^{3/4} \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right)$$

Result (type 4, 7629 leaves) :

$$\frac{2 (-5 d^5 + 128 a d e^3 - 8 d^4 e x + 512 a e^4 x + 72 d^3 e^2 x^2 + 96 d^2 e^3 x^3)}{(-d^4 + 64 a e^3) (5 d^4 + 256 a e^3) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}} - \frac{1}{(d^4 - 64 a e^3) (5 d^4 + 256 a e^3)}$$

$$\begin{aligned}
& 8 e \left(2 d^4 \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \left. \sqrt{\left(\left(\left(\left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) / \right. \\
& \quad \left. \left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \\
& \quad \sqrt{\left(\left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) / \left(\left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \\
& \quad \sqrt{\left(\left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) / \left(\left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \right.} \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) / \left(\left(\left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \\
& \left(\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \left. \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) / \\
& \left(\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \left. \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) / \\
& \left(\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \left. \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \left. \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) + \\
& 256 a e^3 \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \\
& \sqrt{\left(\left(\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right.} \\
& \left. \left. \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) / \\
& \left(\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \left. \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \\
& \sqrt{\left(\left(\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right.}
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \Bigg) \Bigg/ \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \right. \right. \\
& \left. \left. \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \Bigg) \\
& \sqrt{\left(\left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \right.} \\
& \left. \left. \left(-\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \Bigg/ \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \right. \right.} \\
& \left. \left. \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \Bigg) } \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \right.} \right. \\
& \left. \left. \left. \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \Bigg/ \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \right. \right.} \\
& \left. \left. \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \right] , \\
& \left(\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \Bigg/ \\
& \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right)] \Bigg) \Bigg/ \\
& \left(\left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) - \\
& \left(12 d^3 e \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \left. \left(- \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \right. \\
& \left. \sqrt{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(- \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) /} \right. \\
& \left. \left(e \left(- \frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \\
& \left. \left. \left(- \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right. \\
& \left. \sqrt{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(- \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) /} \right. \\
& \left. \left(e \left(- \frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \\
& \left. \left. \left(- \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right. \\
& \left. \sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right.} \\
& \left. \left. 4 e x \right) \right) / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right. \\
& \left. \left. \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \right) \left(- \frac{1}{4 e} \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right) \right.\right. \\
& \quad \left.\left. \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x\right)\right) / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right.\right. \\
& \quad \left.\left.\sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x\right)\right)\right], \\
& \quad \frac{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2}{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2} + \frac{1}{2 e} \\
& \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \text{ EllipticPi}\left[\frac{-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e}}{\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e}}, \right. \\
& \quad \left. \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right) \right.\right. \right. \\
& \quad \left.\left. \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x\right)\right) / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right.\right. \right. \\
& \quad \left.\left. \left.\sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x\right)\right)\right], \\
& \quad \left. \frac{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2}{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2}\right] / \\
& \quad \left(\left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \left. \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \left. \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}} 24 d^2 e^2 \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \\
& \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) + \\
& \frac{1}{2} \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \\
& \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \\
& \sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) / \right.} \\
& \left. \left(e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \\
& \left. \left. \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \\
& \sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) / \right.} \\
& \left. \left(e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \right. \\
& \left. \left. \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \right) \right) \\
& \sqrt{\left(\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(d + \right. \right. \right.} \\
& \left. \left. \left. \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right) / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right.} \\
& \left. \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(2 e \left(- \frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \quad \text{EllipticE}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right.} \right. \\
& \quad \left. \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right] / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right. \\
& \quad \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right)] , \\
& \quad \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 / \\
& \quad \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2] \Big) / \\
& \quad \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \right) + \left(2 e \left(\frac{1}{4 e} \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \right) \right. \right. \\
& \quad \left. \left(- \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) - \right. \\
& \quad \left. \frac{1}{4 e} \left(-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \right) \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \right. \right. \\
& \quad \left. \left. \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) \Big) \text{EllipticF}[\\
& \quad \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right.} \right. \\
& \quad \left. \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right] / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right. \\
& \quad \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right)] ,
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 / \\
& \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2] \Bigg) / \\
& \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \right. \right. \\
& \left. \left. \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) - \\
& \left(-\frac{d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \right. \\
& \left. \left. \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right. \\
& \left. - \text{EllipticPi} \left[\frac{-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e}}{\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e}}, \right. \right. \\
& \left. \left. \text{ArcSin} \left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 / \right.} \right. \right. \\
& \left. \left. \left. \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right) / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \right. \right. \right. \\
& \left. \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \right] , \\
& \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 / \\
& \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2] \Bigg)
\end{aligned}$$

$$\left(\left[-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right] \right) \right)$$

Problem 624: Result more than twice size of optimal antiderivative.

$$\int (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 452 leaves, 8 steps) :

$$\begin{aligned} & -\frac{16 (7 + 2 a) (1 - \sqrt{4 + a}) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1 + x)}{35 \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \\ & \frac{2}{35} \left(13 + 5 a - 3 (-1 + x)^2\right) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4} (-1 + x) + \\ & \frac{1}{7} \left(3 + a - 2 (-1 + x)^2 - (-1 + x)^4\right)^{3/2} (-1 + x) + \left(16 (7 + 2 a) (1 - \sqrt{4 + a}) \right. \\ & \left. \sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]\right) / \\ & \left(35 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}\right) + \left(4 (3 + a) (16 + 5 a) \sqrt{1 + \sqrt{4 + a}} \right. \\ & \left. \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]\right) / \\ & \left(35 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}\right) \end{aligned}$$

Result (type 4, 6287 leaves) :

$$\begin{aligned} & \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \left(\frac{1}{7} (-4 - 3 a) + \frac{1}{35} (-32 + 15 a) x + \frac{14 x^2}{5} - \frac{66 x^3}{35} + \frac{5 x^4}{7} - \frac{x^5}{7}\right) + \\ & \frac{4}{35} \left(\left(40 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2\right.\right. \\ & \left.\left.\left.- 20 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) \sqrt{1 + \sqrt{4 + a}}\right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)}/ \\
& \left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right) \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)}/\right. \\
& \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right)/ \\
& \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right]\Bigg) \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \left(46a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)}/ \\
& \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right.\right.} \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right], \\
& \quad \left.\left.\left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\right) / \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\right]\right] / \\
& \quad \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4}\right) + \\
& \quad \left(10a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2\right. \\
& \quad \left.\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right.\right.} \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) \right. \\
& \quad \left.\sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}}\right. \\
& \quad \left.\sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}}\right. \\
& \quad \left.\text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right.\right.} \right. \right. \\
& \quad \left.\left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right)\right], \right. \\
& \quad \left.\left.\left.\left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\right)\right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)] \Bigg) \Bigg) \Bigg) \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(112 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right)], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \Bigg) \Bigg] \\
& \left(\sqrt{-1 - \sqrt{4+a}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(32a \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \right. \\
& \left. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right. \right.} \right. \\
& \left. \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} + 2\sqrt{-1 - \sqrt{4+a}} \text{EllipticPi}[\\
& \frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right. \right. \right. \right.} \right. \\
& \left. \left. \left. \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \right) \right) \right) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \left. \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] \Bigg) \\
& \left(\sqrt{-1 - \sqrt{4+a}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& 28 \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right) \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) \right) / \right.} \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \right) \\
& \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right)] , \\
& \quad \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right) / \left(2 \sqrt{-1 - \sqrt{4+a}} \right) + \\
& \quad \left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}[\\
& \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \text{ArcSin}[\right. \right. \\
& \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 8a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right. \\
& \left. \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\text{ArcSin}[\right. \\
& \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \\
& \left. \sqrt{-1 + \sqrt{4 + a}} \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \text{ArcSin}[\right. \right. \\
& \left. \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right)] , \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 625: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \, dx$$

Optimal (type 4, 397 leaves, 7 steps):

$$\begin{aligned}
& -\frac{2 \left(1 - \sqrt{4+a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{3 \sqrt{3+a-2 (-1+x)^2 - (-1+x)^4}} + \frac{1}{3} \sqrt{3+a-2 (-1+x)^2 - (-1+x)^4} (-1+x) + \\
& \left(2 \left(1 - \sqrt{4+a}\right) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]\right) / \\
& \left(3 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2 (-1+x)^2 - (-1+x)^4}\right) + \\
& \left(2 (3+a) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]\right) / \\
& \left(3 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2 (-1+x)^2 - (-1+x)^4}\right)
\end{aligned}$$

Result (type 4, 3470 leaves):

$$\begin{aligned}
& \left(-\frac{1}{3} + \frac{x}{3}\right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} + \\
& \frac{2}{3} \left(\left(4 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \right. \right. \\
& \left. \left. \sqrt{\left(\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) /} \right. \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right.\right.} \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right], \\
& \quad \left.\left.\left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\right) / \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\right]\right] / \\
& \quad \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4}\right) + \\
& \quad \left(2a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2\right. \\
& \quad \left.\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right.\right.} \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) / \right. \\
& \quad \left.\left.\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)\right)\right. \\
& \quad \left.\left.\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right] / \\
& \quad \left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)\right. \\
& \quad \left.\left.\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right] / \\
& \quad \left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)\right. \\
& \quad \left.\left.\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) / \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right.\right.} \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right), \\
& \quad \left.\left.\left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\right]\right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)] \Bigg) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(4 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right) \Bigg) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) /} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \quad \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2] + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2, \\
& \quad \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \Bigg) \Bigg] \Bigg) \Bigg) \Bigg) \\
& \left(\sqrt{-1 - \sqrt{4+a}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \Bigg/ \right.} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right) \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \right) \Bigg/ \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) \right) \Bigg/ \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \right) \\
& \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \Bigg/ \right.} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right)] , \\
& \quad \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right) \Bigg/ \left(2 \sqrt{-1 - \sqrt{4+a}} \right) + \right. \\
& \quad \left. \left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \text{EllipticF} \left[\right. \\
& \text{ArcSin} \left[\sqrt{ \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) } \right] / \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2}] / \left(2 \sqrt{-1 - \sqrt{4+a}} \left(-\sqrt{-1 - \sqrt{4+a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4+a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}} , \right. \right. \\
& \text{ArcSin} \left[\sqrt{ \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) } \right] / \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right], \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2}] / \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) \right)
\end{aligned}$$

Problem 626: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 144 leaves, 3 steps) :

$$\begin{aligned}
& \frac{\sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1 - \sqrt{4+a}} \right) \text{EllipticF} \left[\text{ArcTan} \left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}} \right] , -\frac{2\sqrt{4+a}}{1 - \sqrt{4+a}} \right]}{\sqrt{\frac{1 + \frac{(-1+x)^2}{1 - \sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1 + \sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}}
\end{aligned}$$

Result (type 4, 540 leaves) :

$$\begin{aligned}
& \left(2 \left(1 + \sqrt{-1 - \sqrt{4 + a}} \right) - x \right) \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(1 + \sqrt{-1 + \sqrt{4 + a}} \right) - x}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} \right) - x}} \\
& \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} \right) + x}{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} \right) - x}} \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} \right) - x}} \right], \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} \right) - x}} \right. \\
& \left. \sqrt{a - x (-8 + 8x - 4x^2 + x^3)} \right)
\end{aligned}$$

Problem 627: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^{3/2}} dx$$

Optimal (type 4, 437 leaves, 7 steps):

$$\begin{aligned}
& \frac{\left(5 + a + (-1 + x)^2\right) (-1 + x)}{2 (12 + 7 a + a^2) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} - \frac{\left(1 - \sqrt{4 + a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1 + x)}{2 (3 + a) (4 + a) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \\
& \left(\left(1 - \sqrt{4 + a}\right) \sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}[\text{ArcTan}\left[\frac{-1 + x}{\sqrt{1 + \sqrt{4 + a}}}\right], -\frac{2 \sqrt{4 + a}}{1 - \sqrt{4 + a}}] \right) / \\
& \left(2 (3 + a) (4 + a) \sqrt{\frac{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\sqrt{4+a}}}{\frac{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}{1-\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4} \right) + \\
& \frac{\sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}[\text{ArcTan}\left[\frac{-1 + x}{\sqrt{1 + \sqrt{4 + a}}}\right], -\frac{2 \sqrt{4 + a}}{1 - \sqrt{4 + a}}]}{2 (4 + a) \sqrt{\frac{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\sqrt{4+a}}}{\frac{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}{1-\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}}
\end{aligned}$$

Result (type 4, 3526 leaves) :

$$\begin{aligned}
& \frac{(6 + a - 8 x - a x + 3 x^2 - x^3) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4}}{2 (3 + a) (4 + a) (-a - 8 x + 8 x^2 - 4 x^3 + x^4)} + \\
& \frac{1}{2 (3 + a) (4 + a)} \left(\left(4 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \right. \\
& \left. \left. \sqrt{\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) /} \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) / \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) /} \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right],
\end{aligned}$$

$$\begin{aligned}
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)] \Bigg) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(2a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{ \left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) } \right) \\
& \sqrt{ \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \sqrt{ \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{ \left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right] , \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)] \Bigg) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(4 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) } \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \quad \left. \left. \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}}
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \\
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \\
& \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) /} \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \\
& \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right) /} \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) / \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\\
& \text{ArcSin}[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) /} \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right],
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} \Big] \Bigg) \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right. \\
& \left. \sqrt{-1 + \sqrt{4 + a}}\right) \Bigg) + \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}},\right.\right. \\
& \left.\left.\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)}\right]\right)\right. \\
& \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right)\right], \\
& \left.\left.\left.\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} \Big] \Bigg/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\right)\right)
\end{aligned}$$

Problem 628: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 517 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(5 + a + (-1 + x)^2\right) (-1 + x)}{6 (12 + 7 a + a^2) \left(3 + a - 2 (-1 + x)^2 - (-1 + x)^4\right)^{3/2}} + \\
& \frac{\left(104 + 47 a + 5 a^2 + 4 (7 + 2 a) (-1 + x)^2\right) (-1 + x)}{12 (3 + a)^2 (4 + a)^2 \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} - \\
& \frac{(7 + 2 a) \left(1 - \sqrt{4 + a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1 + x)}{3 (3 + a)^2 (4 + a)^2 \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \left(\frac{(7 + 2 a) \left(1 - \sqrt{4 + a}\right)}{\sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right)} \text{EllipticE} \left[\text{ArcTan} \left[\frac{-1 + x}{\sqrt{1 + \sqrt{4 + a}}}\right], -\frac{2 \sqrt{4 + a}}{1 - \sqrt{4 + a}} \right] \right) / \\
& \left(3 (3 + a)^2 (4 + a)^2 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4} \right) + \\
& \left((16 + 5 a) \sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF} \left[\text{ArcTan} \left[\frac{-1 + x}{\sqrt{1 + \sqrt{4 + a}}}\right], -\frac{2 \sqrt{4 + a}}{1 - \sqrt{4 + a}} \right] \right) / \\
& \left(12 (3 + a) (4 + a)^2 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4} \right)
\end{aligned}$$

Result (type 4, 6386 leaves):

$$\begin{aligned}
& \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4} \left(\frac{-6 - a + 8 x + a x - 3 x^2 + x^3}{6 (3 + a) (4 + a) (-a - 8 x + 8 x^2 - 4 x^3 + x^4)^2} + \right. \\
& (132 + 55 a + 5 a^2 - 188 x - 71 a x - 5 a^2 x + 84 x^2 + 24 a x^2 - 28 x^3 - 8 a x^3) / \\
& \left. \left(12 (3 + a)^2 (4 + a)^2 (-a - 8 x + 8 x^2 - 4 x^3 + x^4) \right) \right) + \\
& \frac{1}{12 (3 + a)^2 (4 + a)^2} \left(40 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)}\right. \\
& \quad \left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right], \\
& \quad \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\right] / \\
& \quad \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\right] / \\
& \left(46 a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2\right. \\
& \quad \left.\sqrt{\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)}\right. \\
& \quad \left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) / \\
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)}/\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \quad \left.\left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right)/ \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right]\Bigg] \\
& \quad \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \quad \left(10a^2\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \quad \left.\sqrt{\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)}/\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \quad \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \quad \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)}/\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \quad \left.\left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right)/ \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right]\Bigg]
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(112 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) } \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \quad \left. \left. \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right] / \\
& \quad \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(32 a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) } \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \right. \\
& \left. \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \right. \\
& \left. \left. \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}}
\end{aligned}$$

$$\begin{aligned}
& 28 \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) /} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \right.} \\
& \quad \left. \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right) \\
& \quad \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left. \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right)], \right. \\
& \quad \left. \left. \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \right. \\
& \quad \left(\left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \right. \\
& \quad \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\text{ArcSin}[\right. \\
& \quad \left. \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right)], \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin} \right. \right. \\
& \left. \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/ \right. \right. \right. \\
& \left. \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \right], \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Big/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 8a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right. \\
& \left. \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/ \right. \right. \right. \\
& \left. \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \right. \\
& \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \right. \\
& \left. \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \right. \right. \right. \\
& \left. \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right. \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE} \left[\text{ArcSin} \right. \right. \\
& \left. \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/ \right. \right. \right. \\
& \left. \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \right],
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \quad \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\text{ArcSin}[\\
& \quad \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/ \\
& \quad \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)]], \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \quad \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}[\right. \right. \\
& \quad \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/ \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right)]], \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \Bigg)
\end{aligned}$$

Problem 629: Result more than twice size of optimal antiderivative.

$$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 558 leaves, 14 steps):

$$\begin{aligned}
& \frac{3}{16} (4+a) \left(1 + (-1+x)^2\right) \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} + \\
& \frac{1}{8} \left(1 + (-1+x)^2\right) \left(3+a - 2(-1+x)^2 - (-1+x)^4\right)^{3/2} - \\
& \frac{16 (7+2a) \left(1 - \sqrt{4+a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{35 \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} + \\
& \frac{2}{35} \left(13+5a - 3(-1+x)^2\right) \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \\
& \frac{1}{7} \left(3+a - 2(-1+x)^2 - (-1+x)^4\right)^{3/2} (-1+x) + \\
& \frac{3}{16} (4+a)^2 \operatorname{ArcTan}\left[\frac{1 + (-1+x)^2}{\sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}}\right] + \left(16 (7+2a) \left(1 - \sqrt{4+a}\right) \right. \\
& \left. \sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]\right) / \\
& \left(35 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} + \left(4 (3+a) (16+5a) \sqrt{1 + \sqrt{4+a}} \right. \right. \\
& \left. \left. \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]\right) / \\
& \left(35 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}\right)
\end{aligned}$$

Result (type 4, 7235 leaves):

$$\begin{aligned}
& \left(\left(\frac{1}{56} (52+11a) - \frac{1}{280} (116+55a)x + \frac{1}{80} (-36+25a)x^2 + \frac{74x^3}{35} - \frac{43x^4}{28} + \frac{17x^5}{28} - \frac{x^6}{8}\right) \right. \\
& \left.\left(a - x(-8+8x-4x^2+x^3)\right)^{3/2}\right) / (a+8x-8x^2+4x^3-x^4) + \\
& \frac{1}{280 (a+8x-8x^2+4x^3-x^4)^{3/2}} (a - x(-8+8x-4x^2+x^3))^{3/2} \\
& \left.- \left(2080 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)^2 \right. \right. \\
& \left.\left.\sqrt{\left(\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x\right)\right)}\right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right.} \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right.} \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right.} \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right], \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)] \Big) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \left(208a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right.} \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \\
& \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right.} \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right.} \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right.} \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right], \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)] \Big) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(110a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right.\right. \\
& \quad \left.\left.\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\middle/\right.}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \quad \left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\middle/ \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right]\Big) \\
& \quad \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \quad \left(6944\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \quad \sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right)\right) \\
& \quad \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right)\right) \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\Bigg) \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\Bigg) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\middle/\right.}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right)\right)\right],
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \operatorname{EllipticPi}\left[\\
& \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \operatorname{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \right.} \right. \\
& \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \right. \\
& \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right) \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(2704a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right) \right. \\
& \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)\right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \right.} \right. \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)\right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) \right.} \right. \right. \right. \\
& \left. \left. \left. \left.\left(-1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right]\right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \operatorname{EllipticPi}\left[
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \operatorname{ArcSin} \left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \left. \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \Bigg] \Bigg] \Bigg] \Bigg] \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(210a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/} \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \right.} \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right) \Bigg] \\
& \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right.} \right. \right. \\
& \left. \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right] , \right. \\
& \left. \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \right] , \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] + 2\sqrt{-1 - \sqrt{4 + a}} \operatorname{EllipticPi} [\\
& \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \operatorname{ArcSin} \left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \right. \\
& \left. \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \Bigg) \Bigg) \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& 896 \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \right. \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right.} \right. \\
& \quad \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \right. \\
& \quad \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \right], \\
& \quad \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \quad \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticF} \\
& \text{ArcSin} \left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right] / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \Bigg) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \right. \right. \\
& \left. \left. \text{ArcSin} \left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right] \right] / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \Bigg) / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 256a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right. \\
& \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \\
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Bigg) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right. \Bigg) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)]
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}\left[\right. \right. \\
& \quad \left. \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \quad \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \quad \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \quad \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \quad \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\\
& \quad \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \quad \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \quad \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \quad \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \right. \\
& \quad \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \quad \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \quad \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg)
\end{aligned}$$

Problem 630: Result more than twice size of optimal antiderivative.

$$\int x \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} dx$$

Optimal (type 4, 466 leaves, 12 steps):

$$\begin{aligned} & \frac{1}{4} \left(1 + (-1+x)^2 \right) \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} - \frac{2 \left(1 - \sqrt{4+a} \right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) (-1+x)}{3 \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} + \\ & \frac{1}{3} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \frac{1}{4} (4+a) \operatorname{ArcTan} \left[\frac{1 + (-1+x)^2}{\sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} \right] + \\ & \left(2 \left(1 - \sqrt{4+a} \right) \sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right] \right) / \\ & \left(3 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} \right) + \\ & \left(2 (3+a) \sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right] \right) / \\ & \left(3 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} \right) \end{aligned}$$

Result (type 4, 4389 leaves):

$$\begin{aligned} & \left(\frac{1}{6} - \frac{x}{6} + \frac{x^2}{4} \right) \sqrt{a - x (-8 + 8x - 4x^2 + x^3)} + \frac{1}{6 \sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\ & \sqrt{a - x (-8 + 8x - 4x^2 + x^3)} \left(- \left(\left(8 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right)^2 \right. \right. \\ & \left. \left. \sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \right. \\ & \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \right) \\ & \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right. \right.} \\ & \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) \right) / \right.} \\ & \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \quad \left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right) \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right]\Big) \\
& \quad \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \quad \left(2a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \quad \left.\sqrt{\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \quad \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right/\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right. \\
& \quad \left.\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)} \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \quad \left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right) \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right] \\
& \quad \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \quad \left(40\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \quad \left.\sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right)\right) \\
& \quad \sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right)\right/\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)}
\end{aligned}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \Big) \Big) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big) \Big/} \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \Big) \Big) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \left. \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} \right. , \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \quad \left. \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Big) \Big] \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(6a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \\
& \quad \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \\
& \quad \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \right.}
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right] / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \quad \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& 4 \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) /} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) / \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right) /} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}\left[\right. \right. \\
& \left. \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(\left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \right. \\
& \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\right. \\
& \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \right. \right. \\
& \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right)
\end{aligned}$$

Problem 631: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 179 leaves, 7 steps) :

$$\begin{aligned} & \frac{1}{2} \operatorname{ArcTan} \left[\frac{1 + (-1+x)^2}{\sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} \right] + \\ & \frac{\sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} \end{aligned}$$

Result (type 4, 865 leaves) :

$$\begin{aligned}
& \left(2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right\} \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + 2 \sqrt{-1 - \sqrt{4 + a}} \\
& \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \text{ArcSin} \left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right\} / \left(\sqrt{-1 - \sqrt{4 + a}} \right. \\
& \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a - x (-8 + 8x - 4x^2 + x^3)} \right)
\end{aligned}$$

Problem 632: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^{3/2}} dx$$

Optimal (type 4, 474 leaves, 10 steps):

$$\begin{aligned} & \frac{1 + (-1+x)^2}{2 (4+a) \sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(5+a+(-1+x)^2\right) (-1+x)}{2 (12+7a+a^2) \sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \\ & \frac{\left(1-\sqrt{4+a}\right) \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{2 (3+a) (4+a) \sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \\ & \left(\left(1-\sqrt{4+a}\right) \sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right] \right) / \\ & \left(2 (3+a) (4+a) \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2-(-1+x)^4} \right) + \\ & \frac{\sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{2 (4+a) \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \end{aligned}$$

Result (type 4, 3593 leaves):

$$\begin{aligned} & \left((-a - 2x + ax - ax^2 - x^3) (a + 8x - 8x^2 + 4x^3 - x^4)^2 \right) / \\ & \left(2 (3+a) (4+a) (-a - 8x + 8x^2 - 4x^3 + x^4) (a - x (-8 + 8x - 4x^2 + x^3))^{3/2} \right) + \\ & \frac{1}{2 (3+a) (4+a) (a - x (-8 + 8x - 4x^2 + x^3))^{3/2}} \\ & (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} \left(\left(4 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right)^2 \right. \\ & \left. \sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \right. \\ & \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \right) \\ & \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) \right) \right) } \end{aligned}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \Bigg) \Bigg) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Bigg) /} \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Bigg) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)} \right], \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Bigg) / \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \Bigg) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \Bigg] \Bigg) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(2a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)} / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right.} \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right) /} \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)} \right], \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Bigg], \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \Bigg) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \Bigg] \Bigg) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(4 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)} / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \middle/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right.} \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \middle/ \right.} \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right) \left(\left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \middle/ \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right.} \\
& \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \middle/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \right] \right) \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \middle/ \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)\right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right.\right.} \\
& \quad \left.\left.\left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)\right) / \right.} \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right)} \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right. \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right]\right), \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}}\right) + \\
& \left(\left(-\left(-1 - \sqrt{-1 - \sqrt{4 + a}}\right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) + \right. \right. \\
& \quad \left.\left.\left(-1 + \sqrt{-1 - \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)\right) \text{EllipticF}[\text{ArcSin}[\right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right. \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right]\right), \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \quad \left.\left.\left(\sqrt{-1 + \sqrt{4 + a}}\right)\right) + \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}[\right. \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right. \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right]\right),
\end{aligned}$$

$$\left(\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right) \right)$$

Problem 633: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 591 leaves, 12 steps):

$$\begin{aligned}
& \frac{1 + (-1+x)^2}{6(4+a) \left(3+a-2(-1+x)^2 - (-1+x)^4\right)^{3/2}} + \\
& \frac{1 + (-1+x)^2}{3(4+a)^2 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6(12+7a+a^2) \left(3+a-2(-1+x)^2 - (-1+x)^4\right)^{3/2}} + \\
& \frac{\left(104+47a+5a^2+4(7+2a)(-1+x)^2\right)(-1+x)}{12(3+a)^2(4+a)^2 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} - \\
& \frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3(3+a)^2(4+a)^2 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \left(\frac{(7+2a)\left(1-\sqrt{4+a}\right)}{\sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE} \left[\text{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}, -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right] \right]} \right) / \\
& \left(3(3+a)^2(4+a)^2 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} \right) + \\
& \left((16+5a)\sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF} \left[\text{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}, -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right] \right] \right) / \\
& \left(12(3+a)(4+a)^2 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} \right)
\end{aligned}$$

Result (type 4, 6452 leaves):

$$\left(\left(a + 8x - 8x^2 + 4x^3 - x^4 \right)^3 \left(\frac{a + 2x - ax + ax^2 + x^3}{6 \cdot (3+a) \cdot (4+a) \cdot (-a - 8x + 8x^2 - 4x^3 + x^4)^2} + \right. \right.$$

$$\left. \left. \left(60 + 7a - 3a^2 - 116x - 23ax + 3a^2x + 48x^2 - 4a^2x^2 - 28x^3 - 8ax^3 \right) \right) \right)$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)}\right.\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \quad \left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right]\Big) \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)]\Big)\Big] \\
& \quad \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \quad \left(10a^2\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \quad \left.\sqrt{\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \quad \left.\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right/\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right. \right. \\
& \quad \left.\left.\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right.} \\
& \quad \left.\left.\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)}\right.\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \quad \left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\Big) \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)]\Big) \\
& \quad \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \quad \left(112\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \quad \left.\sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right. \right. \\
& \quad \left.\left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right)\right)\right) \\
& \quad \left.\sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right)\right/\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)}\right)
\end{aligned}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \Big) \Big) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big) \Big/} \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \Big) \Big) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \left. \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} \right. , \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \quad \left. \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Big) \Big] \Big/ \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(32a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right] / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \quad \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg) \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
28 & \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) /} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right.} \right. \\
& \quad \left. \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right) /} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}\left[\right. \right. \\
& \left. \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(\left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\\
& \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \right. \\
& \left. \left. \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \right. \\
& \left. \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] \right], \\
& \left. \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 8a \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right. \\
& \quad \left. \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right) \right. \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) \right) / \right.} \\
& \quad \left. \left. \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \\
& \quad \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left. \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right] , \right. \\
& \quad \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] \right) / \left(2 \sqrt{-1 - \sqrt{4+a}} \right) + \\
& \quad \left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \right. \\
& \quad \left. \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \text{EllipticF}[\right. \\
& \quad \left. \text{ArcSin}[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \right. \\
& \quad \left. \left. \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right] ,
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \Bigg) \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} \right. \right. \\
& \text{ArcSin} \left[\sqrt{ \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] }, \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \Bigg) \Bigg/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right] \Bigg)
\end{aligned}$$

Problem 634: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 585 leaves, 15 steps):

$$\begin{aligned}
& \frac{3}{8} (4+a) \left(1 + (-1+x)^2\right) \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} + \\
& \frac{1}{4} \left(1 + (-1+x)^2\right) \left(3+a - 2(-1+x)^2 - (-1+x)^4\right)^{3/2} + \\
& \frac{4 (140 + 111 a + 21 a^2)}{315} \left(1 - \sqrt{4+a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x) \\
& \frac{315 \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}}{+} \\
& \frac{2}{315} \left(2 (80 + 27 a) + 3 (20 + 7 a) (-1+x)^2\right) \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \\
& \frac{1}{63} \left(15 + 7 (-1+x)^2\right) \left(3+a - 2(-1+x)^2 - (-1+x)^4\right)^{3/2} (-1+x) + \\
& \frac{3}{8} (4+a)^2 \operatorname{ArcTan}\left[\frac{1 + (-1+x)^2}{\sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}}\right] - \left(4 (140 + 111 a + 21 a^2) \left(1 - \sqrt{4+a}\right)\right. \\
& \left.\sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]\right)/ \\
& \left(315 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}\right) + \left(4 (3+a) (100 + 33 a)\right. \\
& \left.\sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]\right)/ \\
& \left(315 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}\right)
\end{aligned}$$

Result (type 4, 8500 leaves):

$$\begin{aligned}
& \left(\left(\frac{1}{252} (404 + 107 a) + \frac{(460 + 81 a) x}{1260} - \frac{1}{360} (100 + 39 a) x^2 + \frac{1}{315} (-80 + 77 a) x^3 + \frac{71 x^4}{42} - \right.\right. \\
& \left.\left.\frac{163 x^5}{126} + \frac{19 x^6}{36} - \frac{x^7}{9}\right) (a - x (-8 + 8 x - 4 x^2 + x^3))^{3/2}\right) / (a + 8 x - 8 x^2 + 4 x^3 - x^4) + \\
& \frac{1}{1260 (a + 8 x - 8 x^2 + 4 x^3 - x^4)^{3/2}} (a - x (-8 + 8 x - 4 x^2 + x^3))^{3/2} \\
& \left(-\left(16160 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)^2\right.\right. \\
& \left.\left.\sqrt{\left(\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x\right)\right)}\right) /
\right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right.} \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right] , \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) / \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right] \Big) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \left(5200a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \right. \\
& \left. \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right] , \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) / \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right] \Big) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \left(162a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \quad \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right.}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \quad \left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right) \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\right] \\
& \quad \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \left(21280\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \quad \left.\sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right.}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right)\right) \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right/\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right. \right. \\
& \quad \left.\left.\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)\right)\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \left(-1-\sqrt{-1-\sqrt{4+a}}\right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right.}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\right)\right]\right],
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \operatorname{EllipticPi}\left[\\
& \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \operatorname{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)\right.}\right. \\
& \left.\left. -1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right] / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right. \\
& \left.\left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(8016a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right.} / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)\right)\right.} / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right. \\
& \left.\left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)\right)\right.} / \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right.}\right. \right. \\
& \left.\left.\left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)\right)\right] / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right. \\
& \left.\left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \operatorname{EllipticPi}\left[
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \operatorname{ArcSin} \left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \left. \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \Bigg] \Bigg] \Bigg] \Bigg] \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(546a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \Big/} \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \Big/ \right.} \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right) \Bigg] \\
& \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right.} \right. \right. \\
& \left. \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + 2\sqrt{-1 - \sqrt{4 + a}} \operatorname{EllipticPi} [\right. \\
& \left. \left. \left. \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \operatorname{ArcSin} \left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \Big/ \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right] \right] \right]
\end{aligned}$$

$$\begin{aligned}
& \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg) \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& 2240 \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \\
& \quad \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right) \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left. \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right)] , \right. \\
& \quad \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \quad \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticF} \\
& \text{ArcSin} \left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right] / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \Bigg) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \right. \right. \\
& \text{ArcSin} \left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right] / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \Bigg) / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg) + \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \frac{1776a}{1776a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right. \\
& \left. \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) } \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}\left[\right. \right. \\
& \left. \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(\left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\\
& \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} , \right. \right. \\
& \left. \left. \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \right. \\
& \left. \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 336a^2 \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \right. \\
& \quad \left. \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right) \right. \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) \right) / \right.} \\
& \quad \left. \left. \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right) \right) \right) \\
& \quad \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left. \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right] , \right. \\
& \quad \left. \left. \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2 \right] \right) / \left(2 \sqrt{-1 - \sqrt{4+a}} \right) + \\
& \quad \left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \right. \\
& \quad \left. \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \text{EllipticF}[\\
& \quad \text{ArcSin}[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right) / \right.} \\
& \quad \left. \left. \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right] ,
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} \right], \right. \\
& \text{ArcSin} \left[\sqrt{ \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) } \right], \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \Bigg)
\end{aligned}$$

Problem 635: Result more than twice size of optimal antiderivative.

$$\int x^2 \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} dx$$

Optimal (type 4, 485 leaves, 13 steps):

$$\begin{aligned}
& \frac{1}{2} \left(1 + (-1+x)^2\right) \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} + \frac{2(8+3a) \left(1 - \sqrt{4+a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{15 \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} + \\
& \frac{1}{15} \left(7 + 3(-1+x)^2\right) \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \\
& \frac{1}{2} (4+a) \operatorname{ArcTan}\left[\frac{1 + (-1+x)^2}{\sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}}\right] - \\
& \left(2(8+3a) \left(1 - \sqrt{4+a}\right) \sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}}\right]\right.\right. \\
& \left.\left. - \frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]\right) \Bigg/ \left(15 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}\right) + \\
& \left(8(3+a) \sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]\right) \Bigg/ \\
& \left(15 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}\right)
\end{aligned}$$

Result (type 4, 5647 leaves):

$$\begin{aligned}
& \left(\frac{1}{3} + \frac{x}{15} - \frac{x^2}{10} + \frac{x^3}{5}\right) \sqrt{a - x(-8 + 8x - 4x^2 + x^3)} + \frac{1}{15 \sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& \sqrt{a - x(-8 + 8x - 4x^2 + x^3)} \left(-\left(40 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)\right)^2\right. \\
& \left.\sqrt{\left(\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x\right)\right) \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)\right)}\right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x\right)\right) \left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x\right)\right)\right)} \\
& \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)\right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x\right)\right)\right)} \\
& \left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)\right) \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x\right)\right)}\right]\right]
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)] , \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)] \Big) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \left(2a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) /} \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right.} \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right] , \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) / \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)] \Big) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(56 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right.} \right. \\
& \left. \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \left. \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \quad \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right] / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \quad \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(6a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \quad \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \right. \\
& \quad \left. \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \right. \\
& \quad \left. \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \quad \left. \left. \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x\right)\right)}\right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1+\sqrt{-1-\sqrt{4+a}} - x\right)\right)\right], \\
& \quad \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2}] + 2\sqrt{-1-\sqrt{4+a}} \text{EllipticPi}[\\
& \quad \frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \right.\right. \\
& \quad \left.\left.-1+\sqrt{-1-\sqrt{4+a}} + x\right)\right]\Big/\left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \right. \\
& \quad \left.\left(1+\sqrt{-1-\sqrt{4+a}} - x\right)\right)\Big)] \cdot \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2}] \Bigg] \\
& \Bigg(\sqrt{-1-\sqrt{4+a}} \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \sqrt{a+8x-8x^2+4x^3-x^4} \Bigg) + \\
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} \\
16 & \left(\left(-1+\sqrt{-1-\sqrt{4+a}} + x\right) \left(-1-\sqrt{-1+\sqrt{4+a}} + x\right) \left(-1+\sqrt{-1+\sqrt{4+a}} + x\right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x\right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x\right)\right)}\right/ \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1+\sqrt{-1-\sqrt{4+a}} - x\right)\right) \\
& \quad \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}} + x\right)\right)\right/\left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \right.} \\
& \quad \left.\left(-1-\sqrt{-1-\sqrt{4+a}} + x\right)\right) \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}} + x\right)\right)\right/\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \right.} \\
& \quad \left.\left(-1+\sqrt{-1-\sqrt{4+a}} - x\right)\right) \Bigg) \\
& \left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x\right) \right) \Bigg) \\
& \left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1+\sqrt{4+a}} + x\right) \right) \Bigg) \\
& \left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \text{EllipticE}[\text{ArcSin}[\right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(\left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \right. \\
& \quad \left. \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\right. \\
& \text{ArcSin}[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \quad \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi}[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \text{ArcSin}[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg) + \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 6a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right. \\
& \quad \left. \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right)
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) /} \\
& \quad \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \\
& \quad \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \quad \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\\
& \quad \text{ArcSin}[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left. \left(\sqrt{-1 + \sqrt{4+a}} \right) \right) + \left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \right. \right. \\ & \operatorname{ArcSin} \left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \right)} \right] / \\ & \left. \left(\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right) \right) \right], \\ & \left. \left. \left. \left(\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}}} \right)^2 \right] \right) / \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) \end{aligned}$$

Problem 636: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 388 leaves, 11 steps):

$$\begin{aligned} & \frac{\left(1 - \sqrt{4+a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{\sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} + \operatorname{ArcTan} \left[\frac{1 + (-1+x)^2}{\sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} \right] - \\ & \left(\left(1 - \sqrt{4+a}\right) \sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right] \right) / \\ & \left(\sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} \right) + \\ & \frac{\sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1 + \sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{\sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} \end{aligned}$$

Result (type 4, 1247 leaves):

$$\frac{1}{\sqrt{a - x (-8 + 8x - 4x^2 + x^3)}}$$

$$\begin{aligned}
& \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \\
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \\
& \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \text{EllipticE}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2 \right] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right],
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \Bigg) \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \right. \\
& \left. \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} \right. \right. \\
& \text{ArcSin} \left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \\
& \left. \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg) \Bigg/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 637: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{(a + 8x - 8x^2 + 4x^3 - x^4)^{3/2}} dx$$

Optimal (type 4, 311 leaves, 10 steps):

$$\begin{aligned}
& \frac{1 + (-1 + x)^2}{(4 + a) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \\
& \frac{(4 + a) \left(2 + (-1 + x)^2 \right) (-1 + x)}{2 (12 + 7 a + a^2) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} - \frac{\left(1 - \sqrt{4 + a} \right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) (-1 + x)}{2 (3 + a) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \\
& \left(\left(1 - \sqrt{4 + a} \right) \sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \text{EllipticE} \left[\text{ArcTan} \left[\frac{-1 + x}{\sqrt{1 + \sqrt{4 + a}}} \right], -\frac{2 \sqrt{4 + a}}{1 - \sqrt{4 + a}} \right] \right) / \\
& \left(2 (3 + a) \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4} \right)
\end{aligned}$$

Result (type 4, 2941 leaves):

$$\begin{aligned}
& \left((-a - 8x - ax + 6x^2 + ax^2 - 4x^3 - ax^3) (a + 8x - 8x^2 + 4x^3 - x^4)^2 \right) / \\
& \left(2 (3 + a) (4 + a) (-a - 8x + 8x^2 - 4x^3 + x^4) (a - x (-8 + 8x - 4x^2 + x^3))^3 \right)^{1/2} -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2 (3 + a) (a - x (-8 + 8 x - 4 x^2 + x^3))^{3/2}} \\
& \left(a + 8 x - 8 x^2 + 4 x^3 - x^4 \right)^{3/2} \left(2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)] , \\
& \left(\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) / \\
& \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right] \right) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4} \right) - \\
& \left(4 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right)} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right. \right. \right. \\
& \quad \left. \left. \left. -1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right] / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \quad \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right)] , \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg) \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) / \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \\
& \quad \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \quad \left. \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) \right) / \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE}[\text{ArcSin}\left[\right. \right. \\
& \left. \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \right) + \\
& \left(\left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF}[\text{ArcSin}\left[\right. \\
& \left. \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\right. \right. \\
& \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right] \right) + \left(\left. \left. \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right. \right. \\
& \left. \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right] , \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right)
\end{aligned}$$

Problem 638: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 582 leaves, 13 steps):

$$\begin{aligned} & \frac{1 + (-1+x)^2}{3 (4+a) \left(3+a-2 (-1+x)^2-(-1+x)^4\right)^{3/2}} + \\ & \frac{2 \left(1+(-1+x)^2\right)}{3 (4+a)^2 \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}} + \frac{(4+a) \left(2+(-1+x)^2\right) (-1+x)}{6 (12+7 a+a^2) \left(3+a-2 (-1+x)^2-(-1+x)^4\right)^{3/2}} + \\ & \frac{\left(29+7 a+(13+3 a) (-1+x)^2\right) (-1+x)}{12 (3+a)^2 (4+a) \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}} - \\ & \frac{(13+3 a) \left(1-\sqrt{4+a}\right) \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{12 (3+a)^2 (4+a) \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}} + \left(\frac{(13+3 a) \left(1-\sqrt{4+a}\right)}{\sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)} \text{EllipticE}[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}] \right) / \\ & \left(12 (3+a)^2 (4+a) \sqrt{\frac{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}{\sqrt{3+a-2 (-1+x)^2-(-1+x)^4}}} + \right. \\ & \left. \frac{\sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}]}{12 (12+7 a+a^2) \sqrt{\frac{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}{\sqrt{3+a-2 (-1+x)^2-(-1+x)^4}}}} \right) \end{aligned}$$

Result (type 4, 5812 leaves):

$$\begin{aligned} & \left((a + 8x - 8x^2 + 4x^3 - x^4)^3 \left(\frac{a + 8x + ax - 6x^2 - ax^2 + 4x^3 + ax^3}{6 (3+a) (4+a) (-a - 8x + 8x^2 - 4x^3 + x^4)^2} + \right. \right. \\ & \left. \left. (24 - 14 a - 6 a^2 - 128 x - 36 a x + 84 x^2 + 27 a x^2 + a^2 x^2 - 52 x^3 - 25 a x^3 - 3 a^2 x^3) \right) \right) / \\ & (a - x (-8 + 8x - 4x^2 + x^3))^5 - \frac{1}{12 (3+a)^2 (4+a) (a - x (-8 + 8x - 4x^2 + x^3))^5} \\ & (a + 8x - 8x^2 + 4x^3 - x^4)^{5/2} \left(20 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right.\right. \\
& \quad \left.\left.\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\middle/\right.}\right. \\
& \quad \left.\left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \left(\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\middle/ \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\left]\right)\middle/ \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \left(4a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right.\right. \\
& \quad \left.\left.\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right)\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)\right)\middle/\right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\right)\middle/\right.}\right. \\
& \quad \left.\left.\left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\right], \\
& \left(\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)\right)\middle/ \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\left]\right)\middle/ \\
& \left(\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\right)\left]\right) \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)-
\end{aligned}$$

$$\begin{aligned}
& \left(52 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right) \\
& \quad \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \\
& \quad \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) \right) / \right.} \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) \right) \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \right.} \right. \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \quad \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}[\\
& \quad \frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right.} \right. \\
& \quad \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \quad \left. \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] \right) / \\
& \quad \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \\
& \quad \left(12a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right.} \\
& \quad \left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right) \\
& \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)\right) / \right.} \\
& \quad \left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \right. \right. \\
& \quad \left.\left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) \sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)\right) / \right.} \\
& \quad \left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)\right)\right) \left(\left(-1 - \sqrt{-1 - \sqrt{4 + a}}\right) \right. \\
& \quad \left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \right.} \right. \\
& \quad \left.\left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\right. \\
& \quad \left.\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2, \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \right.} \right. \\
& \quad \left.\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)\right) / \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \right. \\
& \quad \left.\left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)\right)\right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] \right] \Bigg) \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4}\right) + \\
& \frac{1}{\sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4}} \\
& 13 \left(\left(\sqrt{-1 + \sqrt{-1 - \sqrt{4 + a}}} + x\right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right) + \right. \\
& \quad \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x\right)\right) / \right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1+\sqrt{-1-\sqrt{4+a}} - x\right)\right)\right) \\
& \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}} + x\right)\right) / \left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \right.} \\
& \quad \left.\left(-1-\sqrt{-1-\sqrt{4+a}} + x\right)\right)\right) \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}} + x\right)\right) / \right.} \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x\right)\right)\right) \\
& \left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left.\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x\right)\right) / \right. \\
& \quad \left.\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1+\sqrt{-1-\sqrt{4+a}} - x\right)\right)], \\
& \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2}] / \left(2 \sqrt{-1-\sqrt{4+a}}\right) + \\
& \left(\left(-\left(-1-\sqrt{-1-\sqrt{4+a}}\right) \left(-2-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) + \right. \right. \\
& \quad \left.\left(-1+\sqrt{-1-\sqrt{4+a}}\right) \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)\right) \text{EllipticF}[\\
& \quad \text{ArcSin}\left[\sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x\right)\right) / \right. \\
& \quad \left.\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1+\sqrt{-1-\sqrt{4+a}} - x\right)\right)], \\
& \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2}] / \left(2 \sqrt{-1-\sqrt{4+a}} \left(-\sqrt{-1-\sqrt{4+a}} + \right. \right. \\
& \quad \left.\left.\sqrt{-1+\sqrt{4+a}}\right)\right) + \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin}\left[\sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x\right)\right)\right)} / \right. \\
& \quad \left.\left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1+\sqrt{-1-\sqrt{4+a}} - x\right)\right)\right], \\
& \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2}] / \left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)\right] + \\
& \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} 3a \left(\begin{aligned} & \left(-1+\sqrt{-1-\sqrt{4+a}} + x\right) \left(-1-\sqrt{-1+\sqrt{4+a}} + x\right) \\ & \left(-1+\sqrt{-1+\sqrt{4+a}} + x\right) + \end{aligned} \right. \\
& \left. 2 \left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x\right)^2 \right. \\
& \sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x\right)\right)\right)} / \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1+\sqrt{-1-\sqrt{4+a}} - x\right)\right) \\
& \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}} + x\right)\right)\right)} / \left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \right. \\
& \quad \left.\left(-1-\sqrt{-1-\sqrt{4+a}} + x\right)\right) \sqrt{\left(\left(\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}} + x\right)\right)\right)} / \\
& \quad \left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x\right)\right) \\
& \left(\begin{aligned} & \left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \text{EllipticE}[\text{ArcSin}[\right. \\
& \quad \left. \sqrt{\left(\left(\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x\right)\right)\right)} / \right. \\
& \quad \left. \left(\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right) \left(1+\sqrt{-1-\sqrt{4+a}} - x\right)\right)\right], \\
& \frac{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}}\right)^2}] / \left(2\sqrt{-1-\sqrt{4+a}}\right) +
\end{aligned}$$

$$\begin{aligned}
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \right. \\
& \quad \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticF} \left[\right. \\
& \quad \text{ArcSin} \left[\sqrt{ \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) } \right] / \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \right. \right. \\
& \quad \left. \left. \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \right. \\
& \quad \text{ArcSin} \left[\sqrt{ \left(\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \right) } \right] / \\
& \quad \left. \left(\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right) \right) \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right)
\end{aligned}$$

Problem 639: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8 + 8x - x^3 + 8x^4}} dx$$

Optimal (type 4, 129 leaves, 4 steps):

$$\begin{aligned}
& - \left(x^2 \sqrt{\frac{261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4}{\left(87 + \frac{\sqrt{29} (4+x)^2}{x^2} \right)^2}} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2} \right) \right. \\
& \quad \left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{4+x}{\sqrt{3} 29^{1/4} x} \right], \frac{1}{58} \left(29 + \sqrt{29} \right) \right] \right) / \left(8 \sqrt{3} 29^{1/4} \sqrt{8 + 8x - x^3 + 8x^4} \right)
\end{aligned}$$

Result (type 4, 927 leaves) :

$$\begin{aligned}
 & - \left(\left(2 \text{EllipticF} \left[\right. \right. \right. \\
 & \quad \left. \left. \left. \text{ArcSin} \left[\sqrt{\left(\left(x - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] \right) \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) / \left(\left(x - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \right. \right. \right. \\
 & \quad \left. \left. \left. \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) \right], \\
 & \quad \left(\left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3 \right] \right) \\
 & \quad \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) / \\
 & \quad \left(\left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3 \right] \right) \\
 & \quad \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right] \\
 & \quad \left(x - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right)^2 \\
 & \quad \sqrt{\left(\left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \\
 & \quad \left(x - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3 \right] \right) / \left(\left(x - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \right. \right. \\
 & \quad \left. \left. \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3 \right] \right) \right) \\
 & \quad \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \\
 & \quad \sqrt{\left(\left(x - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] \right) \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \right. \right. \\
 & \quad \left. \left. \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \left(x - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \\
 & \quad \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) / \\
 & \quad \left(\left(x - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right)^2 \right. \\
 & \quad \left. \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right)^2 \right) \right) / \\
 & \quad \left(\sqrt{8 + 8 x - x^3 + 8 x^4} \left(-\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1 \right] + \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] \right) \right. \\
 & \quad \left. \left(\text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2 \right] - \text{Root} \left[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right)
 \end{aligned}$$

Problem 640: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 + 8 x - x^3 + 8 x^4)^{3/2}} dx$$

Optimal (type 4, 431 leaves, 10 steps) :

$$\begin{aligned}
& - \frac{\left(66 - \left(1 + \frac{4}{x}\right)^2\right) x^2}{1008 \sqrt{8 + 8x - x^3 + 8x^4}} + \frac{\left(216 - 7 \left(1 + \frac{4}{x}\right)^2\right) \left(1 + \frac{4}{x}\right) x^2}{12528 \sqrt{8 + 8x - x^3 + 8x^4}} + \\
& \frac{7 \left(261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4\right) \left(1 + \frac{4}{x}\right) x^2}{432 \sqrt{29} \sqrt{8 + 8x - x^3 + 8x^4} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)} - \\
& \left(7 x^2 \sqrt{\frac{261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4}{\left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)^2}} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)\right. \\
& \left. \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{4+x}{\sqrt{3} 29^{1/4} x}\right], \frac{1}{58} \left(29 + \sqrt{29}\right)\right]\right) / \left(144 \sqrt{3} 29^{3/4} \sqrt{8 + 8x - x^3 + 8x^4}\right) + \\
& \left(\left(14 - 5 \sqrt{29}\right) x^2 \sqrt{\frac{261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4}{\left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)^2}} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)\right. \\
& \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{4+x}{\sqrt{3} 29^{1/4} x}\right], \frac{1}{58} \left(29 + \sqrt{29}\right)\right]\right) / \left(576 \sqrt{3} 29^{3/4} \sqrt{8 + 8x - x^3 + 8x^4}\right)
\end{aligned}$$

Result (type 4, 4865 leaves):

$$\begin{aligned}
& \frac{544 + 1539x - 1146x^2 + 784x^3}{21924 \sqrt{8 + 8x - x^3 + 8x^4}} + \\
& \frac{1}{6264} \left(\left(28 \left(x - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] \right)^2 \left(-\text{EllipticF}[\text{ArcSin}[\right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{\left(\left(x - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1] \right) \left(\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4] \right) \right) / \left(\left(x - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] \right) \right. \right. \right. \\
& \left. \left. \left. \left. \left(\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4] \right) \right) \right) \right), \\
& - \left(\left(\left(\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3] \right) \right. \right. \\
& \left. \left. \left(\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4] \right) \right) / \\
& \left(\left(-\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3] \right) \right. \right. \\
& \left. \left. \left(\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4] \right) \right) \right) \\
& \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] + \text{EllipticPi}\left[\left(-\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1] + \right. \right. \\
& \left. \left. \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4] \right) / \\
& \left(-\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] + \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4] \right), \\
& \text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1] \right) \left(\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4] \right) \right) / \left(\left(x - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] \right) \right. \right. \right. \\
& \left. \left. \left. \left. \left(\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4] \right) \right) \right) \right], \\
& - \left(\left(\left(\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3] \right) \right. \right. \\
& \left. \left. \left(\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 4] \right) \right) / \\
& \left(\left(-\text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4 \&, 3] \right) \right)
\end{aligned}$$

Problem 641: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1 + 4x + 4x^2 + 4x^4}} dx$$

Optimal (type 4, 108 leaves, 3 steps):

$$-\left(\left(\sqrt{5} + \left(1 + \frac{1}{x} \right)^2 \right) \sqrt{\frac{5 - 2 \left(1 + \frac{1}{x} \right)^2 + \left(1 + \frac{1}{x} \right)^4}{\left(\sqrt{5} + \left(1 + \frac{1}{x} \right)^2 \right)^2}} x^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{1 + \frac{1}{x}}{5^{1/4}} \right], \frac{1}{10} \left(5 + \sqrt{5} \right) \right] \right) \\ \left(2 \times 5^{1/4} \sqrt{1 + 4x + 4x^2 + 4x^4} \right)$$

Result (type 4, 249 leaves):

$$\left(2 - \frac{i}{2} \right) \sqrt{-\frac{1}{10} + \frac{i}{5}} \sqrt{\frac{\left(2i + \sqrt{-1 - 2i} - \sqrt{-1 + 2i} \right) \left(-i + \sqrt{-1 - 2i} - 2x \right)}{\left(-2i + \sqrt{-1 - 2i} + \sqrt{-1 + 2i} \right) \left(i + \sqrt{-1 - 2i} + 2x \right)}} \left(1 + 2x + 2i x^2 \right) \\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{\left(2i + \sqrt{-1 - 2i} + \sqrt{-1 + 2i} \right) \left(-i + \sqrt{-1 + 2i} + 2x \right)}{\sqrt{-1 + 2i} \left(i + \sqrt{-1 - 2i} + 2x \right)}}}{\sqrt{2}} \right], \frac{1}{2} \left(5 - \sqrt{5} \right) \right] \\ \left(\sqrt{\frac{\left(1 + 2i \right) \left(\left(-1 + i \right) + \sqrt{-1 - 2i} \right) \left(1 + 2x + 2i x^2 \right)}{\left(i + \sqrt{-1 - 2i} + 2x \right)^2}} \sqrt{1 + 4x + 4x^2 + 4x^4} \right)$$

Problem 642: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(1 + 4x + 4x^2 + 4x^4 \right)^{3/2}} dx$$

Optimal (type 4, 367 leaves, 9 steps):

$$\begin{aligned}
& - \frac{\left(3 - \left(1 + \frac{1}{x}\right)^2\right) x^2}{\sqrt{1 + 4 x + 4 x^2 + 4 x^4}} + \frac{\left(13 - 9 \left(1 + \frac{1}{x}\right)^2\right) \left(1 + \frac{1}{x}\right) x^2}{10 \sqrt{1 + 4 x + 4 x^2 + 4 x^4}} + \frac{9 \left(5 - 2 \left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4\right) \left(1 + \frac{1}{x}\right) x^2}{10 \left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right) \sqrt{1 + 4 x + 4 x^2 + 4 x^4}} - \\
& \left(\frac{9 \left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right)}{\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right)^2} x^2 \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{1 + \frac{1}{x}}{5^{1/4}}\right], \frac{1}{10} \left(5 + \sqrt{5}\right)\right] \right) / \\
& \left(2 \times 5^{3/4} \sqrt{1 + 4 x + 4 x^2 + 4 x^4} \right) + \left(3 \left(3 - \sqrt{5}\right) \left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right) \sqrt{\frac{5 - 2 \left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4}{\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right)^2}} \right. \\
& \left. x^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{1 + \frac{1}{x}}{5^{1/4}}\right], \frac{1}{10} \left(5 + \sqrt{5}\right)\right] \right) / \left(4 \times 5^{3/4} \sqrt{1 + 4 x + 4 x^2 + 4 x^4} \right)
\end{aligned}$$

Result (type 4, 3334 leaves):

$$\begin{aligned}
& \frac{19 + 42 x - 16 x^2 + 36 x^3}{10 \sqrt{1 + 4 x + 4 x^2 + 4 x^4}} - \\
& \frac{3}{5} \left(- \left(\left(2 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right]\right) \left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right] + \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right)\right) / \right. \right. \right. \\
& \left. \left. \left. \left(\left(x - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right]\right) \left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] + \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right)\right)\right], \right. \\
& \left(\left(\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right] - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 3\right]\right) \\
& \left(\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right) / \\
& \left(\left(\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 3\right]\right) \\
& \left(\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right] - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right) \right] \\
& \left(x - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right]\right)^2 \\
& \sqrt{\left(\left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] + \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right]\right) \\
& \left(x - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 3\right]\right) / \left(\left(x - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right]\right) \\
& \left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] + \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 3\right]\right)\right)} \\
& \left(\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right) \\
& \sqrt{\left(\left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] + \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right]\right) \\
& \left(x - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right) / \left(\left(x - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right]\right) \\
& \left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] + \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right)\right)} \\
& \sqrt{\left(\left(x - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right]\right) \left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right] + \right. \right. \\
& \left. \left. \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right) / \left(\left(x - \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right]\right) \right. \\
& \left. \left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] + \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right)\right)\right) / \\
& \left(\sqrt{1 + 4 x + 4 x^2 + 4 x^4} \left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 1\right] + \right. \right. \\
& \left. \left. \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right]\right) \\
& \left. \left(-\text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 2\right] + \text{Root}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, 4\right]\right)\right) +
\end{aligned}$$

$$\left(\frac{\left(-\text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 1 \right] - \text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 2 \right] + \text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 3 \right] - \text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 4 \right] \right) \left(\text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 2 \right] + \text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 4 \right] \right)}{\left(-\text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 2 \right] - \text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 1 \right] + \text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 3 \right] - \text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 4 \right] \right) \left(\text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 2 \right] + \text{Root}\left[1+4 \# 1+4 \# 1^2+4 \# 1^4 \&, 4 \right] \right)} \right)$$

Problem 643: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}} dx$$

Optimal (type 4, 126 leaves, 4 steps) :

$$-\left(\left(\sqrt{517} + \left(3 + \frac{4}{x} \right)^2 \right) \sqrt{\frac{517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4}{\left(\sqrt{517} + \left(3 + \frac{4}{x} \right)^2 \right)^2}} x^2 \right. \\ \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{4+3x}{517^{1/4}x} \right], \frac{517+19\sqrt{517}}{1034} \right] \right) / \left(8 \times 517^{1/4} \sqrt{8+24x+8x^2-15x^3+8x^4} \right)$$

Result (type 4, 1148 leaves) :

$$\begin{aligned}
& - \left(\left(2 \operatorname{EllipticF} \left[\right. \right. \right. \\
& \quad \operatorname{ArcSin} \left[\sqrt{ \left(\left(x - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] \right) \left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + \right. \right. \right. \\
& \quad \left. \left. \left. 8 \#1^4 \&, 2] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4] \right) \right) / \\
& \quad \left(\left(x - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] \right) \left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - \right. \right. \\
& \quad \left. \left. 15 \#1^3 + 8 \#1^4 \&, 1] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4] \right) \right) \right) \left. \right], \\
& \quad \left(\left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3] \right) \right. \\
& \quad \left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3] \right) / \\
& \quad \left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4] \right) \left. \right] \\
& \quad \left(x - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] \right)^2 \\
& \quad \sqrt{ \left(\left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] \right) \right. \right. \\
& \quad \left(x - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3] \right) / \\
& \quad \left(\left(x - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] \right) \left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, \right. \right. \\
& \quad \left. \left. 1] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3] \right) \right) \\
& \quad \left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4] \right) \\
& \quad \sqrt{ \left(\left(x - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] \right) \right. \right. \\
& \quad \left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] \right) \left(x - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4] \right) \left(\operatorname{Root} [\right. \right. \\
& \quad \left. \left. 8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4] \right) \right) / \\
& \quad \left(\left(x - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] \right)^2 \left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - \right. \right. \\
& \quad \left. \left. 15 \#1^3 + 8 \#1^4 \&, 1] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4] \right)^2 \right) \right) \left. \right) / \\
& \quad \left(\sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4} \left(-\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \right. \right. \\
& \quad \left. \left. \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] \right) \right. \\
& \quad \left(\operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \operatorname{Root} [8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4] \right) \left. \right) \left. \right)
\end{aligned}$$

Problem 644: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4)^{3/2}} dx$$

Optimal (type 4, 434 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\left(172 - 7 \left(3 + \frac{4}{x}\right)^2\right) x^2}{208 \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}} + \frac{\left(50896 - 2455 \left(3 + \frac{4}{x}\right)^2\right) \left(3 + \frac{4}{x}\right) x^2}{322608 \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}} + \\
& \frac{2455 \left(517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right) \left(3 + \frac{4}{x}\right) x^2}{322608 \left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}} - \\
& \left(2455 \left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{\frac{517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4}{\left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right)^2}} x^2 \text{EllipticE}\right. \\
& \left. 2 \text{ArcTan}\left[\frac{4 + 3 x}{517^{1/4} x}\right], \frac{517 + 19 \sqrt{517}}{1034}\right] \Bigg/ \left(624 \times 517^{3/4} \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}\right) + \\
& \left(\left(4910 - 203 \sqrt{517}\right) \left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{\frac{517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4}{\left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right)^2}} x^2\right. \\
& \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{4 + 3 x}{517^{1/4} x}\right], \frac{517 + 19 \sqrt{517}}{1034}\right]\right) \Bigg/ \left(2496 \times 517^{3/4} \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}\right)
\end{aligned}$$

Result (type 4, 6019 leaves):

$$\begin{aligned}
& \frac{72888 + 89033 x - 94314 x^2 + 39280 x^3}{80652 \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}} + \\
& \frac{1}{161304} \left(\left(147300 \left(x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] \right)^2 \right. \right. \\
& \left. \left. \left(-\text{EllipticF}[\text{ArcSin}[\sqrt((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) \\
& (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + \\
& 8 \#1^4 \&, 4])) / ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \\
& (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - \\
& 15 \#1^3 + 8 \#1^4 \&, 4])))] , -((\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] \\
& - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (\text{Root}[8 + 24 \#1 + \\
& 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \\
& ((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - \\
& 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, \\
& 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])))] \\
& \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \text{EllipticPi}[\\
& (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, \\
& 4]) / (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \\
& \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]), \\
& \text{ArcSin}[\sqrt((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) (\text{Root}[8 + 24 \#1 + \\
& 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \\
& ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - \\
& 15 \#1^3 + 8 \#1^4 \&, 4]))]
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \right. \right.} \\
& \quad \left. \left((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \right), \\
& - \left(\left(\left(\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) \right) / \right. \\
& \quad \left. \left((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \right], \\
& (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) - \\
& \quad \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \\
& \quad \left((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) - \right. \\
& \quad \left. \left[2] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4] \right) \right) - \\
& (\text{EllipticPi}[(-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) / (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]), \text{ArcSin}[\\
& \quad \sqrt{\left((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \right. \right.} \\
& \quad \left. \left((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \right), \\
& - \left(\left(\left(\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \right. \right. \\
& \quad \left. \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) \right) / \right. \right. \\
& \quad \left. \left((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \right], \\
& (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3] - \\
& \quad \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \Big)
\end{aligned}$$

Problem 645: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 + 24x + 8x^2 - 15x^3 + 8x^4)^{5/2}} dx$$

Optimal (type 4, 577 leaves, 12 steps):

$$\begin{aligned}
& - \frac{\left(124415 - 6308 \left(3 + \frac{4}{x}\right)^2\right) x^2}{97344 \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} - \\
& \frac{\left(64489 - 1399 \left(3 + \frac{4}{x}\right)^2\right) x^2}{624 \left(517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right) \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} + \\
& \frac{\left(18932921731 - 1086525994 \left(3 + \frac{4}{x}\right)^2\right) \left(3 + \frac{4}{x}\right) x^2}{78056941248 \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} + \\
& \frac{\left(11921698 - 359497 \left(3 + \frac{4}{x}\right)^2\right) \left(3 + \frac{4}{x}\right) x^2}{483912 \left(517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right) \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} + \\
& \frac{543262997 \left(517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right) \left(3 + \frac{4}{x}\right) x^2}{39028470624 \left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} - \\
& \left(543262997 \left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{\frac{517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4}{\left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right)^2}} x^2 \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{4 + 3x}{517^{1/4} x}\right], \right.\right. \\
& \left.\left. \frac{517 + 19 \sqrt{517}}{1034}\right] \right) / \left(75490272 \times 517^{3/4} \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}\right) + \\
& \left(4346103976 - 175318963 \sqrt{517}\right) \left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{\frac{517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4}{\left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right)^2}} \\
& x^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{4 + 3x}{517^{1/4} x}\right], \frac{517 + 19 \sqrt{517}}{1034}\right] \right) / \\
& \left(1207844352 \times 517^{3/4} \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}\right)
\end{aligned}$$

Result (type 4, 6084 leaves):

$$\begin{aligned}
& \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4} \left(\frac{72888 + 89033x - 94314x^2 + 39280x^3}{241956 (8 + 24x + 8x^2 - 15x^3 + 8x^4)^2} + \right. \\
& (65072399400 + 77274145879x - 83050578336x^2 + 34768831808x^3) / \\
& \left. \left(39028470624 (8 + 24x + 8x^2 - 15x^3 + 8x^4)\right) \right) + \\
& \frac{1}{78056941248} \left(\left(130383119280 (x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2])\right)^2 (-\text{EllipticF}[\right. \right. \\
& \left. \left. \text{ArcSin}[\sqrt((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) / \right. \\
& \left((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right), \\
& - \left(((\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \right. \\
& \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / ((-\text{Root}[8 + 24 \#1 + \right. \\
& \left. 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) \right. \\
& \left. (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \\
& \text{EllipticPi}[(-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + \right. \\
& \left. 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) / (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, \right. \\
& \left. 2] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]), \\
& \text{ArcSin}[\sqrt(((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) (\text{Root}[8 + 24 \#1 + \right. \\
& \left. 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \right. \\
& \left((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right), \\
& - \left(((\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \right. \\
& \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \right. \\
& \left((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \right. \\
& \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \\
& \left(-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 2]) \right) \\
& \sqrt(((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 2]) (x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3])) / \\
& \left((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3])) \right) \\
& (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) \\
& \sqrt(((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) \\
& (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \right. \\
& \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \\
& \left((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right)) \\
& \sqrt(((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 2]) (x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \\
& \left((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - \right. \\
& \left. 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) / \\
& \left(\sqrt(8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4) (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \right. \\
& \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \\
& (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \right. \\
& \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) \right) + \\
& (103049936174 \text{EllipticF}[\text{ArcSin}[\sqrt(((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) \\
& (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \right. \\
& \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) \right) /
\end{aligned}$$

$$\left(-\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2 \right] - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3 \right] - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4 \right]) / (-\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1 \right] + \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2 \right] + \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3 \right] + \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4 \right])) \right)$$

Problem 646: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} dx$$

Optimal (type 4, 130 leaves, 4 steps) :

$$-\left(\left(\sqrt{\frac{613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4}{\left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right)^2}} \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right)x^2 \right. \right. \\ \left. \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{6-x}{613^{1/4} x}\right], \frac{613 + 91 \sqrt{613}}{1226}\right]\right) / \left(12 \times 613^{1/4} \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}\right)\right)$$

Result (type 4, 826 leaves) :

$$\begin{aligned}
& - \left(\left(2 \operatorname{EllipticF} \left[\right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{ArcSin} \left[\sqrt{ \left(\left(x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] \right) \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) } / \right. \right. \right. \\
& \quad \left. \left. \left. \left(\left(x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] \right) \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) \right], \right. \right. \\
& \quad \left. \left. \left. \left(\left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) \right) / \right. \right. \\
& \quad \left. \left. \left. \left(\left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right] \right) \right. \right. \right. \\
& \quad \left. \left. \left. \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) \right] \right) \\
& \quad \left(\sqrt{\frac{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right]}{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right]}} \right. \\
& \quad \left(x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] \right)^2 \\
& \quad \left(\sqrt{\frac{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right]}{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right]}} \right. \\
& \quad \left. \left(\sqrt{\frac{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right]}{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right]}} \right) \right) / \\
& \quad \left(\sqrt{\left(\left(9 - 6 x - 44 x^2 + 15 x^3 + 3 x^4 \right) \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right] \right) \right.} \right. \\
& \quad \left. \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) \right)
\end{aligned}$$

Problem 647: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(9 - 6 x - 44 x^2 + 15 x^3 + 3 x^4)^{3/2}} dx$$

Optimal (type 4, 444 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\left(176 - 23 \left(1 - \frac{6}{x}\right)^2\right) x^2}{51759 \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \frac{\left(45401 - 3722 \left(1 - \frac{6}{x}\right)^2\right) \left(1 - \frac{6}{x}\right) x^2}{31728267 \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \\
& \frac{3722 \left(613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4\right) \left(1 - \frac{6}{x}\right) x^2}{31728267 \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right) \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \\
& \left. \frac{3722 \sqrt{\frac{613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4}{\left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right)^2}} \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right) x^2 \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{6-x}{613^{1/4} x}\right], \frac{613 + 91 \sqrt{613}}{1226}\right]\right) / \left(51759 \times 613^{3/4} \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}\right) - \\
& \left. \left(7444 - 145 \sqrt{613}\right) \sqrt{\frac{613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4}{\left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right)^2}} \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right) x^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{6-x}{613^{1/4} x}\right], \frac{613 + 91 \sqrt{613}}{1226}\right]\right) / \left(207036 \times 613^{3/4} \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}\right)
\end{aligned}$$

Result (type 4, 5428 leaves):

$$\begin{aligned}
& - \frac{2 (-106926 - 592639 x + 232005 x^2 + 44664 x^3)}{10576089 \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \\
& \frac{1}{3525363} \left(\left(148880 \left(x - \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2\right]\right)^2 \right. \right. \\
& \left. \left. \left(-\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1\right]\right)\right.\right.\right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2\right] - \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. 3 \#1^4 \&, 4\right]\right) / \left(\left(x - \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2\right]\right) \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1\right] - \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. 3 \#1^4 \&, 4\right]\right)\right), - \left(\left(\left(\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2\right] - \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3\right]\right) \left(\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. 15 \#1^3 + 3 \#1^4 \&, 1\right] - \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4\right]\right)\right) / \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(\left(-\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1\right] + \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. 15 \#1^3 + 3 \#1^4 \&, 3\right]\right) \left(\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2\right]\right) - \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4\right]\right)\right) \right. \right. \right. \right. \right. \\
& \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2\right] + \text{EllipticPi}\left[\right. \left. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(-\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1\right] + \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4\right]\right) / \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(-\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2\right] + \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4\right]\right), \right. \right. \right. \right. \right. \\
& \text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1\right]\right)\right.\right.\right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left(\text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2\right] - \text{Root}\left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4\right]\right)\right)\right) / \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right), \\
& - \left(\left(\left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) / \\
& \left(\left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) \right) \\
& \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \\
& \sqrt[2]{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \\
& \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \\
& \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \\
& \sqrt{\left(\left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) / \right.} \\
& \left. \left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) \right) \Bigg) / \\
& \left(\sqrt{9 - 6 x - 44 x^2 + 15 x^3 + 3 x^4} \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right. \\
& \left. \sqrt{\left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right)} \right) - \\
& \left(54294 \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] \right) \right.} \right. \right. \\
& \left. \left. \left. \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) / \right. \right. \right. \\
& \left. \left. \left. \left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) \right], \right. \right. \\
& \left. \left. \left. \left(\left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \right. \right. \right. \\
& \left. \left. \left. \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) / \right. \right. \right. \\
& \left. \left. \left. \left(\left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) \right] \right]
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \\
& (x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2])^2 \\
& \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \\
& (x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4])^2 \\
& \left(\frac{\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]}{\sqrt{9 - 6 x - 44 x^2 + 15 x^3 + 3 x^4}} \right) \\
& \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \\
& \sqrt{\left(\left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right)} \\
& \frac{1}{\sqrt{9 - 6 x - 44 x^2 + 15 x^3 + 3 x^4}} 29776 \left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] \right) \right. \\
& \left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \\
& \left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) + \\
& \left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right)^2 \\
& \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \\
& \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \\
& \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \\
& \sqrt{\left(\left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) / \right.} \\
& \left. \left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) \right) \\
& \sqrt{\left(\left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \right) / \left(\left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \right)} \\
& \left(\left(\text{EllipticE}[\text{ArcSin}[\sqrt{\left(\left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) / \left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] \right) \right) \right) \right) \right) / \left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] \right) \right) \right)
\end{aligned}$$

Problem 660: Unable to integrate problem.

$$\int \frac{x - \sqrt{x^6}}{x(1 - x^4)} dx$$

Optimal (type 3, 45 leaves, 9 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 26 leaves):

$$\int \frac{x - \sqrt{x^6}}{x(1 - x^4)} dx$$

Problem 661: Unable to integrate problem.

$$\int \frac{1 - \frac{\sqrt{x^6}}{x}}{1 - x^4} dx$$

Optimal (type 3, 45 leaves, 9 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 26 leaves):

$$\int \frac{1 - \frac{\sqrt{x^6}}{x}}{1 - x^4} dx$$

Problem 662: Unable to integrate problem.

$$\int \frac{x - \sqrt{x^6}}{x - x^5} dx$$

Optimal (type 3, 45 leaves, 10 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 23 leaves):

$$\int \frac{x - \sqrt{x^6}}{x - x^5} dx$$

Problem 663: Unable to integrate problem.

$$\int \frac{x}{x + \sqrt{x^6}} dx$$

Optimal (type 3, 45 leaves, 11 steps) :

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 15 leaves) :

$$\int \frac{x}{x + \sqrt{x^6}} dx$$

Problem 664: Unable to integrate problem.

$$\int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx$$

Optimal (type 3, 52 leaves, 12 steps) :

$$\text{ArcTan}[\sqrt{x}] + \frac{\sqrt{x^3} \text{ArcTan}[\sqrt{x}]}{x^{3/2}} + \text{ArcTanh}[\sqrt{x}] - \frac{\sqrt{x^3} \text{ArcTanh}[\sqrt{x}]}{x^{3/2}}$$

Result (type 8, 27 leaves) :

$$\int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx$$

Problem 665: Unable to integrate problem.

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx$$

Optimal (type 3, 52 leaves, 13 steps) :

$$\text{ArcTan}[\sqrt{x}] + \frac{\sqrt{x^3} \text{ArcTan}[\sqrt{x}]}{x^{3/2}} + \text{ArcTanh}[\sqrt{x}] - \frac{\sqrt{x^3} \text{ArcTanh}[\sqrt{x}]}{x^{3/2}}$$

Result (type 8, 17 leaves) :

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx$$

Problem 686: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x} \sqrt{5+x}} dx$$

Optimal (type 3, 12 leaves, 3 steps) :

$$-\text{ArcSin}\left[\frac{1}{4} (-1-x)\right]$$

Result (type 3, 45 leaves):

$$\frac{2 \sqrt{-3+x} \sqrt{5+x} \text{ArcSinh}\left[\frac{\sqrt{-3+x}}{2 \sqrt{2}}\right]}{\sqrt{-(-3+x) (5+x)}}$$

Problem 687: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(3-x) (5+x)}} dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-\text{ArcSin}\left[\frac{1}{4} (-1-x)\right]$$

Result (type 3, 45 leaves):

$$\frac{2 \sqrt{-3+x} \sqrt{5+x} \text{ArcSinh}\left[\frac{\sqrt{-3+x}}{2 \sqrt{2}}\right]}{\sqrt{-(-3+x) (5+x)}}$$

Problem 689: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3-x} \sqrt{5+x}} dx$$

Optimal (type 3, 4 leaves, 3 steps):

$$\text{ArcSin}[4+x]$$

Result (type 3, 42 leaves):

$$\frac{2 \sqrt{3+x} \sqrt{5+x} \text{ArcSinh}\left[\frac{\sqrt{3+x}}{\sqrt{2}}\right]}{\sqrt{- (3+x) (5+x)}}$$

Problem 690: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(-3-x) (5+x)}} dx$$

Optimal (type 3, 4 leaves, 3 steps):

$$\text{ArcSin}[4+x]$$

Result (type 3, 42 leaves):

$$\frac{2 \sqrt{3+x} \sqrt{5+x} \operatorname{ArcSinh}\left[\frac{\sqrt{3+x}}{\sqrt{2}}\right]}{\sqrt{- (3+x) (5+x)}}$$

Problem 698: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} dx$$

Optimal (type 2, 11 leaves, 2 steps) :

$$-2 \sqrt{1-x}$$

Result (type 2, 23 leaves) :

$$\frac{2 (-1+x) \sqrt{1+x}}{\sqrt{1-x^2}}$$

Problem 700: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} dx$$

Optimal (type 2, 9 leaves, 2 steps) :

$$2 \sqrt{1+x}$$

Result (type 2, 25 leaves) :

$$\frac{2 \sqrt{1-x} (1+x)}{\sqrt{1-x^2}}$$

Problem 704: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} dx$$

Optimal (type 2, 11 leaves, 2 steps) :

$$\frac{2}{3} (1+x)^{3/2}$$

Result (type 2, 27 leaves) :

$$\frac{2 (1+x) \sqrt{1-x^2}}{3 \sqrt{1-x}}$$

Problem 706: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x} \sqrt{2+3x}}{\sqrt{1-x^2}} dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$\sqrt{1+x} \sqrt{2+3x} - \frac{\text{ArcSinh}[\sqrt{2+3x}]}{\sqrt{3}}$$

Result (type 3, 79 leaves):

$$\frac{\sqrt{1-x} \left(3 (1+x) \sqrt{2+3x} + \sqrt{3} \sqrt{-1-x} \text{ArcTan}\left[\frac{\sqrt{3} \sqrt{-1-x}}{\sqrt{2+3x}}\right]\right)}{3 \sqrt{1-x^2}}$$

Problem 707: Result more than twice size of optimal antiderivative.

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2} x} dx$$

Optimal (type 3, 43 leaves, 7 steps):

$$\frac{4 \sqrt{1+x}}{\sqrt{1-x}} - \text{ArcSin}[x] - \text{ArcTanh}[\sqrt{1-x} \sqrt{1+x}]$$

Result (type 3, 101 leaves):

$$-\frac{4 \sqrt{1-x^2}}{-1+x} - 2 \text{ArcSin}\left[\frac{\sqrt{1+x}}{\sqrt{2}}\right] + \text{Log}\left[1-\sqrt{1+x}\right] - \text{Log}\left[2+\sqrt{1-x}-\sqrt{1+x}\right] - \text{Log}\left[1+\sqrt{1+x}\right] + \text{Log}\left[2+\sqrt{1-x}+\sqrt{1+x}\right]$$

Problem 709: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+a x)^{3/2}}{x (1-a x)^{3/2}} dx$$

Optimal (type 3, 51 leaves, 7 steps):

$$\frac{4 \sqrt{1+a x}}{\sqrt{1-a x}} - \text{ArcSin}[a x] - \text{ArcTanh}[\sqrt{1-a x} \sqrt{1+a x}]$$

Result (type 3, 74 leaves):

$$\frac{4 \sqrt{1-a^2 x^2}}{1-a x} + \text{Log}[x] - \text{Log}\left[1+\sqrt{1-a^2 x^2}\right] - i \text{Log}\left[2 \left(-i a x+\sqrt{1-a^2 x^2}\right)\right]$$

Problem 712: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx$$

Optimal (type 3, 2 leaves, 2 steps):

`ArcSin[x]`

Result (type 3, 32 leaves):

$$-\text{ArcTan}\left[\frac{x\sqrt{1+x^2}\sqrt{1-x^4}}{-1+x^4}\right]$$

Problem 714: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} dx$$

Optimal (type 3, 2 leaves, 2 steps):

`ArcSinh[x]`

Result (type 3, 42 leaves):

$$\text{Log}[1-x^2] - \text{Log}[-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}]$$

Problem 716: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx$$

Optimal (type 3, 23 leaves, 3 steps):

$$\frac{1}{2}x\sqrt{1-x^2} + \frac{\text{ArcSin}[x]}{2}$$

Result (type 3, 50 leaves):

$$\frac{1}{2}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1+x^2}} + \text{ArcTan}\left[\frac{x\sqrt{1+x^2}}{\sqrt{1-x^4}}\right]\right)$$

Problem 718: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$\frac{1}{2}x\sqrt{1+x^2} + \frac{\text{ArcSinh}[x]}{2}$$

Result (type 3, 70 leaves):

$$\frac{1}{2} \left(\frac{x \sqrt{1-x^4}}{\sqrt{1-x^2}} + \text{Log}[1-x^2] - \text{Log}[-x+x^3+\sqrt{1-x^2} \sqrt{1-x^4}] \right)$$

Problem 768: Unable to integrate problem.

$$\int \frac{\sqrt{bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Optimal (type 3, 47 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{b} x}{\sqrt{b x^2+\sqrt{a+b^2 x^4}}}\right]}{\sqrt{2} \sqrt{b}}$$

Result (type 8, 39 leaves):

$$\int \frac{\sqrt{bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Problem 769: Unable to integrate problem.

$$\int \frac{\sqrt{-bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2} \sqrt{b} x}{\sqrt{-b x^2+\sqrt{a+b^2 x^4}}}\right]}{\sqrt{2} \sqrt{b}}$$

Result (type 8, 40 leaves):

$$\int \frac{\sqrt{-bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Problem 770: Unable to integrate problem.

$$\int \frac{\sqrt{2x^2 + \sqrt{3+4x^4}}}{(c+dx) \sqrt{3+4x^4}} dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \operatorname{ArcTan}\left[\frac{-\sqrt{3} d + 2 i c x}{\sqrt{2 i c^2 - \sqrt{3} d^2} \sqrt{\sqrt{3} - 2 i x^2}}\right] - \left(\frac{1}{2} + \frac{i}{2}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{3} d - 2 i c x}{\sqrt{2 i c^2 + \sqrt{3} d^2} \sqrt{\sqrt{3} + 2 i x^2}}\right]}{\sqrt{2 i c^2 - \sqrt{3} d^2}}$$

Result (type 8, 42 leaves):

$$\int \frac{\sqrt{2 x^2 + \sqrt{3 + 4 x^4}}}{(c + d x) \sqrt{3 + 4 x^4}} dx$$

Problem 771: Unable to integrate problem.

$$\int \frac{\sqrt{2 x^2 + \sqrt{3 + 4 x^4}}}{(c + d x)^2 \sqrt{3 + 4 x^4}} dx$$

Optimal (type 3, 268 leaves, 7 steps):

$$\begin{aligned} & \frac{\left(\frac{1}{2} - \frac{i}{2}\right) d \sqrt{\sqrt{3} - 2 i x^2}}{(2 i c^2 - \sqrt{3} d^2) (c + d x)} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) d \sqrt{\sqrt{3} + 2 i x^2}}{(2 i c^2 + \sqrt{3} d^2) (c + d x)} + \\ & \frac{(1 + i) c \operatorname{ArcTan}\left[\frac{-\sqrt{3} d + 2 i c x}{\sqrt{2 i c^2 - \sqrt{3} d^2} \sqrt{\sqrt{3} - 2 i x^2}}\right]}{(2 i c^2 - \sqrt{3} d^2)^{3/2}} + \frac{(1 - i) c \operatorname{ArcTanh}\left[\frac{\sqrt{3} d - 2 i c x}{\sqrt{2 i c^2 + \sqrt{3} d^2} \sqrt{\sqrt{3} + 2 i x^2}}\right]}{(2 i c^2 + \sqrt{3} d^2)^{3/2}} \end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\sqrt{2 x^2 + \sqrt{3 + 4 x^4}}}{(c + d x)^2 \sqrt{3 + 4 x^4}} dx$$

Problem 775: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2 + \frac{b}{x^2}}}{b + 2 x^2} dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$-\frac{\operatorname{ArcCsch}\left[\frac{\sqrt{2} x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 54 leaves):

$$\frac{\sqrt{2 + \frac{b}{x^2}} \times \left(\operatorname{Log}[x] - \operatorname{Log}\left[b + \sqrt{b} \sqrt{b + 2 x^2}\right]\right)}{\sqrt{b} \sqrt{b + 2 x^2}}$$

Problem 776: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2 - \frac{b}{x^2}}}{-b + 2x^2} dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$-\frac{\text{ArcCsc}\left[\frac{\sqrt{2} x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 64 leaves):

$$-\frac{\frac{i}{\sqrt{b}} \sqrt{2 - \frac{b}{x^2}} x \log\left[\frac{2 \left(-i \sqrt{b} + \sqrt{-b+2 x^2}\right)}{x}\right]}{\sqrt{b} \sqrt{-b+2 x^2}}$$

Problem 783: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1+x) \sqrt{2x+x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTan}\left[\sqrt{2x+x^2}\right]$$

Result (type 3, 37 leaves):

$$\frac{2 \sqrt{x} \sqrt{2+x} \text{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{2+x}}\right]}{\sqrt{x (2+x)}}$$

Problem 784: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1+2x) \sqrt{x+x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTan}\left[2 \sqrt{x+x^2}\right]$$

Result (type 3, 37 leaves):

$$\frac{2 \sqrt{x} \sqrt{1+x} \text{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{1+x}}\right]}{\sqrt{x (1+x)}}$$

Problem 786: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x-x^2}}{1+x} dx$$

Optimal (type 3, 54 leaves, 6 steps):

$$\sqrt{x-x^2} - \frac{3}{2} \text{ArcSin}[1-2x] + \sqrt{2} \text{ArcTan}\left[\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right]$$

Result (type 3, 120 leaves):

$$\frac{1}{2\sqrt{-1+x}\sqrt{x}}\sqrt{-(-1+x)x}\left(2\sqrt{-1+x}\sqrt{x}-6\text{Log}\left[\sqrt{-1+x}+\sqrt{x}\right]+\sqrt{2}\text{Log}\left[1-2\sqrt{2}\sqrt{-1+x}\sqrt{x}-3x\right]-\sqrt{2}\text{Log}\left[1+2\sqrt{2}\sqrt{-1+x}\sqrt{x}-3x\right]\right)$$

Problem 808: Result unnecessarily involves higher level functions.

$$\int \frac{(1+\sqrt{x})^{1/3}}{x} dx$$

Optimal (type 3, 67 leaves, 6 steps):

$$6(1+\sqrt{x})^{1/3}-2\sqrt{3}\text{ArcTan}\left[\frac{1+2(1+\sqrt{x})^{1/3}}{\sqrt{3}}\right]+3\text{Log}\left[1-(1+\sqrt{x})^{1/3}\right]-\frac{\text{Log}[x]}{2}$$

Result (type 5, 51 leaves):

$$\frac{6+6\sqrt{x}-3\left(1+\frac{1}{\sqrt{x}}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{1}{\sqrt{x}}\right]}{\left(1+\sqrt{x}\right)^{2/3}}$$

Problem 813: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}} dx$$

Optimal (type 3, 65 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{b c-a d-2 b d x}{2 \sqrt{b} \sqrt{d} \sqrt{a c+(b c-a d) x-b d x^2}}\right]}{\sqrt{b} \sqrt{d}}$$

Result (type 3, 99 leaves):

$$\frac{\pm \sqrt{a+b x} \sqrt{c-d x} \text{Log}\left[2 \sqrt{a+b x} \sqrt{c-d x}-\frac{i (-b c+a d+2 b d x)}{\sqrt{b} \sqrt{d}}\right]}{\sqrt{b} \sqrt{d} \sqrt{(a+b x)(c-d x)}}$$

Problem 814: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x} (1 - x^2)} dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$\text{ArcTan}[\sqrt{x}] + \text{ArcTanh}[\sqrt{x}]$$

Result (type 3, 33 leaves):

$$\text{ArcTan}[\sqrt{x}] - \frac{1}{2} \text{Log}[1 - \sqrt{x}] + \frac{1}{2} \text{Log}[1 + \sqrt{x}]$$

Problem 815: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x}}{x - x^3} dx$$

Optimal (type 3, 13 leaves, 5 steps):

$$\text{ArcTan}[\sqrt{x}] + \text{ArcTanh}[\sqrt{x}]$$

Result (type 3, 33 leaves):

$$\text{ArcTan}[\sqrt{x}] - \frac{1}{2} \text{Log}[1 - \sqrt{x}] + \frac{1}{2} \text{Log}[1 + \sqrt{x}]$$

Problem 818: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTan}[\sqrt{2x+x^2}]$$

Result (type 3, 37 leaves):

$$\frac{2\sqrt{x}\sqrt{2+x}\text{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{2+x}}\right]}{\sqrt{x(2+x)}}$$

Problem 826: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x}{x - \sqrt{1+2x^2}} dx$$

Optimal (type 3, 31 leaves, 7 steps):

$$-x - \sqrt{1 + 2x^2} + \text{ArcTan}[x] + \text{ArcTan}\left[\sqrt{1 + 2x^2}\right]$$

Result (type 3, 101 leaves):

$$\frac{1}{4} \left(-4x - 4\sqrt{1 + 2x^2} + 4\text{ArcTan}[x] - 4\text{ArcTan}\left[\frac{1}{\sqrt{1 + 2x^2}}\right] + 2 \pm \text{Log}[1 + x^2] - \pm \text{Log}[1 + 3x^2 - 2x\sqrt{1 + 2x^2}] - \pm \text{Log}[1 + 3x^2 + 2x\sqrt{1 + 2x^2}] \right)$$

Problem 838: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(1-x)x}} dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\text{ArcSin}[1 - 2x]$$

Result (type 3, 38 leaves):

$$\frac{2\sqrt{-1+x}\sqrt{x}\text{Log}\left[\sqrt{-1+x}+\sqrt{x}\right]}{\sqrt{-(-1+x)x}}$$

Problem 841: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1 + \sqrt{5} - x^2 + \sqrt{5}x^2} dx$$

Optimal (type 3, 24 leaves, 2 steps):

$$\frac{1}{2}\text{ArcTan}\left[\sqrt{\frac{1}{2}(3 - \sqrt{5})}x\right]$$

Result (type 3, 39 leaves):

$$\frac{1}{4} \pm \text{Log}[1 + \sqrt{5} - 2 \pm x] - \frac{1}{4} \pm \text{Log}[1 + \sqrt{5} + 2 \pm x]$$

Problem 844: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{(1-x^2)(3+x^2)} dx$$

Optimal (type 4, 48 leaves, 6 steps):

$$\frac{1}{3}x\sqrt{3 - 2x^2 - x^4} - \frac{2\text{EllipticE}[\text{ArcSin}[x], -\frac{1}{3}]}{\sqrt{3}} + \frac{4\text{EllipticF}[\text{ArcSin}[x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 59 leaves):

$$\frac{1}{3} \left(x \sqrt{3 - 2x^2 - x^4} - 2 i \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right], -3\right] - 4 i \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right], -3\right] \right)$$

Problem 845: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} dx$$

Optimal (type 4, 12 leaves, 3 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[x], -\frac{1}{3}\right]}{\sqrt{3}}$$

Result (type 4, 18 leaves):

$$-\frac{i}{\sqrt{3}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right], -3\right]$$

Problem 856: Unable to integrate problem.

$$\int \sqrt{1 - x^2 + x \sqrt{-1 + x^2}} dx$$

Optimal (type 3, 63 leaves, ? steps):

$$\frac{1}{4} \left(3x + \sqrt{-1 + x^2} \right) \sqrt{1 - x^2 + x \sqrt{-1 + x^2}} + \frac{3 \text{ArcSin}[x - \sqrt{-1 + x^2}]}{4 \sqrt{2}}$$

Result (type 8, 24 leaves):

$$\int \sqrt{1 - x^2 + x \sqrt{-1 + x^2}} dx$$

Problem 857: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-x + \sqrt{x} \sqrt{1+x}}}{\sqrt{1+x}} dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2} \left(\sqrt{x} + 3 \sqrt{1+x} \right) \sqrt{-x + \sqrt{x} \sqrt{1+x}} - \frac{3 \text{ArcSin}[\sqrt{x} - \sqrt{1+x}]}{2 \sqrt{2}}$$

Result (type 3, 180 leaves):

$$-\left(\left((1+x) (1+2x-2\sqrt{x}\sqrt{1+x})^2 \left(2\sqrt{-x+\sqrt{x}\sqrt{1+x}} (-3-2x+2\sqrt{x}\sqrt{1+x}) + 3\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}} \operatorname{Log}[2\sqrt{-x+\sqrt{x}\sqrt{1+x}} + \sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}] \right) \right) \right) / \\ \left(4 (-\sqrt{x} + \sqrt{1+x})^3 (1+x - \sqrt{x}\sqrt{1+x})^2 \right)$$

Problem 858: Unable to integrate problem.

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\sqrt{2(1+\sqrt{5})} \operatorname{ArcTan}[\sqrt{-2+\sqrt{5}} (x+\sqrt{1+x^2})] + \\ \sqrt{2(-1+\sqrt{5})} \operatorname{ArcTanh}[\sqrt{2+\sqrt{5}} (x+\sqrt{1+x^2})]$$

Result (type 8, 34 leaves):

$$-\int \frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} dx$$

Problem 859: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$-\sqrt{\frac{1}{2}(1+\sqrt{5})} \operatorname{ArcTan}[\frac{2\sqrt{5} - (5+\sqrt{5})x}{\sqrt{10(1+\sqrt{5})}\sqrt{2+2x+x^2}}] - \\ \sqrt{\frac{1}{2}(-1+\sqrt{5})} \operatorname{ArcTanh}[\frac{2\sqrt{5} + (5-\sqrt{5})x}{\sqrt{10(-1+\sqrt{5})}\sqrt{2+2x+x^2}}]$$

Result (type 3, 433 leaves):

$$\begin{aligned}
& \frac{1}{4} \left(2 \sqrt{1+2 \text{i}} \right. \\
& \quad \text{ArcTan} \left[\left((8+8 \text{i}) - (1-4 \text{i}) x^3 + 5 \text{i} \sqrt{1+2 \text{i}} \sqrt{2+2x+x^2} + x^2 \left((-2+13 \text{i}) + 5 \sqrt{1+2 \text{i}} \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{2+2x+x^2} \right) + (1+\text{i}) x \left((9+5 \text{i}) + 5 \sqrt{1+2 \text{i}} \sqrt{2+2x+x^2} \right) \right] / \\
& \quad \left((4+14 \text{i}) + (2+2 \text{i}) x + (4-11 \text{i}) x^2 - (3+8 \text{i}) x^3 \right)] + 2 \text{i} \sqrt{1-2 \text{i}} \\
& \quad \text{ArcTanh} \left[\left((-8+8 \text{i}) + (1+4 \text{i}) x^3 + 5 \text{i} \sqrt{1-2 \text{i}} \sqrt{2+2x+x^2} + x^2 \left((2+13 \text{i}) - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \sqrt{1-2 \text{i}} \sqrt{2+2x+x^2} \right) + (1+\text{i}) x \left((5+9 \text{i}) + 5 \text{i} \sqrt{1-2 \text{i}} \sqrt{2+2x+x^2} \right) \right] / \\
& \quad \left((-14-4 \text{i}) - (2+2 \text{i}) x + (11-4 \text{i}) x^2 + (8+3 \text{i}) x^3 \right)] + \\
& \quad \text{i} \left(\left(\sqrt{1-2 \text{i}} - \sqrt{1+2 \text{i}} \right) \text{Log}[1+x^2] - \sqrt{1-2 \text{i}} \text{Log}[(7-4 \text{i}) + (8-4 \text{i}) x + (3-2 \text{i}) x^2 + \right. \\
& \quad \left. \left. 4 \sqrt{1-2 \text{i}} \sqrt{2+2x+x^2} + 2 \sqrt{1-2 \text{i}} x \sqrt{2+2x+x^2} \right] + \sqrt{1+2 \text{i}} \text{Log}[(7+4 \text{i}) + (8+4 \text{i}) x + (3+2 \text{i}) x^2 + 4 \sqrt{1+2 \text{i}} \sqrt{2+2x+x^2} + 2 \sqrt{1+2 \text{i}} x \sqrt{2+2x+x^2}] \right)
\end{aligned}$$

Problem 863: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{a+b c^4 + 4 b c^3 d x + 6 b c^2 d^2 x^2 + 4 b c d^3 x^3 + b d^4 x^4}} dx$$

Optimal (type 4, 184 leaves, 7 steps) :

$$\begin{aligned}
& \frac{\text{ArcTanh} \left[\frac{\sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{a+b d^4 \left(\frac{c}{d} + x \right)^4}} \right]}{2 \sqrt{b} d^2} - \\
& \left(c \left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right) \sqrt{\frac{a+b d^4 \left(\frac{c}{d} + x \right)^4}{\left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{b^{1/4} (c+d x)}{a^{1/4}} \right], \frac{1}{2} \right] \right) /
\end{aligned}$$

$$\left(2 a^{1/4} b^{1/4} d^2 \sqrt{a+b d^4 \left(\frac{c}{d} + x \right)^4} \right)$$

Result (type 4, 330 leaves) :

$$\begin{aligned}
& \left((-1)^{1/4} \sqrt{2} \sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} (c + d x) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c + d x)}} \left(\pm \sqrt{a} + \sqrt{b} (c + d x)^2 \right) \right. \\
& \left(\left((-1)^{1/4} a^{1/4} - b^{1/4} c \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} (c + d x) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c + d x)}} \right], -1] - \right. \\
& \left. \left. 2 (-1)^{1/4} a^{1/4} \text{EllipticPi}\left[-\frac{i}{2}, \text{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} (c + d x) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c + d x)}} \right], -1\right] \right) \right) / \\
& \left(a^{1/4} \sqrt{b} d^2 \sqrt{\frac{\frac{i}{2} \sqrt{a} + \sqrt{b} (c + d x)^2}{\left((-1)^{1/4} a^{1/4} - b^{1/4} (c + d x) \right)^2}} \sqrt{a + b (c + d x)^4} \right)
\end{aligned}$$

Problem 864: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b c^4 + 4 b c^3 d x + 6 b c^2 d^2 x^2 + 4 b c d^3 x^3 + b d^4 x^4}} dx$$

Optimal (type 4, 131 leaves, 2 steps):

$$\begin{aligned}
& \left(\left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right) \sqrt{\frac{a + b d^4 \left(\frac{c}{d} + x \right)^4}{\left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} (c + d x)}{a^{1/4}} \right], \frac{1}{2} \right] \right) / \\
& \left(2 a^{1/4} b^{1/4} d \sqrt{a + b d^4 \left(\frac{c}{d} + x \right)^4} \right)
\end{aligned}$$

Result (type 4, 90 leaves):

$$\begin{aligned}
& - \frac{\frac{i}{2} \sqrt{\frac{a+b(c+d x)^4}{a}} \text{EllipticF}\left[\frac{i}{2} \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} (c + d x) \right], -1 \right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} d \sqrt{a + b (c + d x)^4}}
\end{aligned}$$

Problem 865: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a - c x^4}{\sqrt{a + b x^2 + c x^4} (a d + a e x^2 + c d x^4)} dx$$

Optimal (type 3, 54 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{b d-a e} x}{\sqrt{d} \sqrt{a+b x^2+c x^4}}\right]}{\sqrt{d} \sqrt{b d-a e}}$$

Result (type 4, 419 leaves) :

$$\begin{aligned} & \left(\frac{i}{\sqrt{b + \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \right. \\ & \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] - \right. \\ & \quad \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4 a c}) d}{a e - \sqrt{a} \sqrt{-4 c d^2 + a e^2}}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] - \\ & \quad \left. \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4 a c}) d}{a e + \sqrt{a} \sqrt{-4 c d^2 + a e^2}}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \right. \right. \\ & \quad \left. \left. \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right]\right) \Bigg) \Bigg/ \left(\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} d \sqrt{a + b x^2 + c x^4}\right) \end{aligned}$$

Problem 866: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a - c x^4}{\sqrt{a - b x^2 + c x^4} (a d + a e x^2 + c d x^4)} dx$$

Optimal (type 3, 53 leaves, 2 steps) :

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b d+a e} x}{\sqrt{d} \sqrt{a-b x^2+c x^4}}\right]}{\sqrt{d} \sqrt{b d+a e}}$$

Result (type 4, 416 leaves) :

$$\left(\frac{i}{2} \sqrt{2 + \frac{4 c x^2}{-b + \sqrt{b^2 - 4 a c}}} \sqrt{1 - \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right. \\ \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b - \sqrt{b^2 - 4 a c}}{b + \sqrt{b^2 - 4 a c}}\right] - \text{EllipticPi}\left[\frac{(b - \sqrt{b^2 - 4 a c}) d}{-a e + \sqrt{a} \sqrt{-4 c d^2 + a e^2}}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b - \sqrt{b^2 - 4 a c}}{b + \sqrt{b^2 - 4 a c}}\right] - \text{EllipticPi}\left[\frac{(-b + \sqrt{b^2 - 4 a c}) d}{a e + \sqrt{a} \sqrt{-4 c d^2 + a e^2}}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b - \sqrt{b^2 - 4 a c}}{b + \sqrt{b^2 - 4 a c}}\right] \right) \right) / \left(2 \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} d \sqrt{a - b x^2 + c x^4} \right)$$

Problem 870: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e f - e f x^2}{(a d + b d x + a d x^2) \sqrt{a + b x + c x^2 + b x^3 + a x^4}} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{e f \text{ArcTan}\left[\frac{a b + (4 a^2 + b^2 - 2 a c) x + a b x^2}{2 a \sqrt{2 a - c} \sqrt{a + b x + c x^2 + b x^3 + a x^4}}\right]}{a \sqrt{2 a - c} d}$$

Result (type 4, 13 884 leaves):

$$-\frac{1}{d} e f \\ \left(- \left(\left(8 (x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2])^2 \left(\text{EllipticF}[\text{ArcSin}\left[\sqrt{\left((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) / ((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]))\right) \right)^2 \right. \\ \left. \left(\left(\left(\left(\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3] \right) \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4] \right) / \left(\left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3] \right) \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4] \right) \right)$$

$$\begin{aligned}
& 2 a \text{EllipticPi}\left[\left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]\right) \right. \\
& \quad \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]\right) \Big/ \\
& \quad \left(\left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]\right) \right. \\
& \quad \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]\right) \Big), \\
& \text{ArcSin}\left[\sqrt{\left((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])\right) / \right. \\
& \quad \left.\left((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])\right)\right], \\
& - \left(\left(\left(\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]\right) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])\right) / \right. \\
& \quad \left.\left(\left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]\right) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])\right)\right) \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]\right) \\
& \sqrt{\left(\left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]\right) (x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3])\right) / \right. \\
& \quad \left.\left((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3])\right)\right) \\
& \sqrt{\left(\left((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])\right) / \right. \\
& \quad \left.\left((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])\right)\right) \\
& \sqrt{\left(\left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]\right) (x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])\right) / \right. \\
& \quad \left.\left((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])\right)\right) \\
& \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]\right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(\left(-\frac{-b - \sqrt{-4 a^2 + b^2}}{2 a} + \frac{-b + \sqrt{-4 a^2 + b^2}}{2 a}\right) \sqrt{x (b + c x + b x^2) + a (1 + x^4)}\right. \\
& \quad \left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]\right) \\
& \quad \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]\right) \\
& \quad \left(-b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]\right) \\
& \quad \left(\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) / \\
& \left(a^2 \left(-\frac{-b - \sqrt{-4 a^2 + b^2}}{2 a} + \frac{-b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \right. \\
& \sqrt{x (b + c x + b x^2) + a (1 + x^4)} \\
& \left(b - \sqrt{-4 a^2 + b^2} + \right. \\
& \left. 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] \right) \\
& \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \right. \\
& \left. \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \\
& \left(-b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \\
& \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \right. \\
& \left. \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) - \\
& \left(2 b \sqrt{-4 a^2 + b^2} (x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right])^2 \right. \\
& \left(\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right]\right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right]\right)}\right] / \right. \\
& \left. \left((x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right]) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right]\right) \right) / \right. \\
& \left. \left(\left(\left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right]\right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right]\right) - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right]\right) \right) \right) / \\
& \left(-b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] \right) - \\
& 2 a \text{EllipticPi}\left[\left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right]\right) \right. \\
& \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \right. \\
& \left. \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right]\right) / \\
& \left(\left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right]\right) \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] + \right. \right. \\
& \left. \left. \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right]\right) \right), \\
& \text{ArcSin}\left[\sqrt{\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right]\right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right]\right)}\right] / \\
& \left((x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right]) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right]\right) \right) / \\
& \left(\left(\left(\left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right]\right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right]\right) - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right]\right) \right) / \\
& \left(\left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right]\right) \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] + \right. \right. \\
& \left. \left. \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right]\right) \right),
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \\
& \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \\
& \sqrt{\left(\left(\left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \right.} \\
& \quad \left. \left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) \right) / \\
& \quad \left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) \right) \\
& \sqrt{\left(\left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right) /} \\
& \quad \left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right) \\
& \sqrt{\left(\left(\left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \right.} \\
& \quad \left. \left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right) / \\
& \quad \left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right) \\
& \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) / \\
& \left(a^2 \left(-\frac{-b - \sqrt{-4 a^2 + b^2}}{2 a} + \frac{-b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \right. \\
& \quad \sqrt{x (b + c x + b x^2) + a (1 + x^4)} \\
& \quad \left(b - \sqrt{-4 a^2 + b^2} + \right. \\
& \quad \left. \left. 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] \right) \right. \\
& \quad \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \\
& \quad \left(-b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \\
& \quad \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) - \\
& \quad \left. \left(8 (x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right])^2 \right. \right. \\
& \quad \left(\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) /} \right. \right. \\
& \quad \left. \left. \left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right) \right), \right. \\
& \quad \left. \left(\left(\left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) / \left(\left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-b - \sqrt{-4 a^2 + b^2} - 2 a \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) - \\
& 2 a \operatorname{EllipticPi} \left[\left(b + \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \right. \\
& \left. (-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right] / \\
& \left(\left(b + \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) (-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right), \\
& \operatorname{ArcSin} \left[\sqrt{\left((x - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) / \left((x - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right)} \right], \\
& - \left(\left((\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) / \left((-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) \right] \\
& \left(-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \\
& \sqrt{\left(\left((-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) \right. \right. \\
& \left. \left. (x - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) \right) / \left((x - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right. \\
& \left. \left. + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) \right) \right) \\
& \sqrt{\left(\left((x - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) / \left((x - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) \right) \\
& \sqrt{\left(\left((-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) \right. \right. \\
& \left. \left. (x - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) / \left((x - \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right. \\
& \left. \left. + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) \right) \\
& \left(-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4] \right) / \\
& \left(\left(\frac{-b - \sqrt{-4 a^2 + b^2}}{2 a} - \frac{-b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \right. \\
& \left. \sqrt{x (b + c x + b x^2) + a (1 + x^4)} \right. \\
& \left(b + \sqrt{-4 a^2 + b^2} + \right. \\
& \left. 2 a \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) \\
& \left(-\operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \\
& \left(-b - \sqrt{-4 a^2 + b^2} - 2 a \operatorname{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \right. \\
& \quad \left. \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) + \\
& \left(2 b^2 (x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2])^2 \right. \\
& \quad \left(\text{EllipticF} [\text{ArcSin} [\sqrt{((x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) / ((x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]))}], \right. \\
& \quad \left. - ((\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) / ((-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])))] \right) \\
& \left(-b - \sqrt{-4 a^2 + b^2} - 2 a \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) - \\
& 2 a \text{EllipticPi} \left[\left(b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \right. \\
& \quad \left. (-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right] / \\
& \left(\left(b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) (-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right), \\
& \text{ArcSin} \left[\sqrt{((x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) / ((x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]))}], \right. \\
& \quad \left. - ((\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) / ((-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]))] \right) \\
& \left(-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \\
& \sqrt{(((-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) \\
& (x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3])) / ((x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3])))} \\
& \sqrt{((((x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) / ((x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])))} \\
& \sqrt{(((-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) \\
& (x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) / ((-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])))}
\end{aligned}$$

$$\begin{aligned}
& \left(\left(x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right)^{\wedge} \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4] \right) \right) \\
& \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4] \right) \Bigg) \\
& \left(a^2 \left(\frac{-b - \sqrt{-4 a^2 + b^2}}{2 a} - \frac{-b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \right. \\
& \sqrt{x (b + c x + b x^2)} + a (1 + x^4) \\
& \left(b + \sqrt{-4 a^2 + b^2} + \right. \\
& \left. 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) \\
& \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \right. \\
& \left. \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \\
& \left(-b - \sqrt{-4 a^2 + b^2} - 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \\
& \left(\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \right. \\
& \left. \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4] \right) + \\
& \left(2 b \sqrt{-4 a^2 + b^2} (x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2])^2 \right. \\
& \left(\text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])}] / \right. \\
& \left. ((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) \right], \\
& - \left(((\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) / \right. \\
& \left. ((-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) \right) \\
& \left(-b - \sqrt{-4 a^2 + b^2} - 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) - \\
& 2 a \text{EllipticPi} \left[\left(b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \right. \\
& \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \right. \\
& \left. \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4] \right) \Bigg) / \\
& \left(\left(b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) (-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] + \right. \\
& \left. \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right), \\
& \text{ArcSin}[\sqrt{((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])}] / \right. \\
& \left. ((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) \right], \\
& - \left(((\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) / \left(\left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right) \\
& \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \\
& \sqrt{\left(\left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) / \left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) \right)} \\
& \sqrt{\left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) / \left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right)} \\
& \sqrt{\left(\left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) / \left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right)} \\
& \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) / \\
& \left(a^2 \left(\frac{-b - \sqrt{-4 a^2 + b^2}}{2 a} - \frac{-b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \right. \\
& \sqrt{x (b + c x + b x^2) + a (1 + x^4)} \\
& \left(b + \sqrt{-4 a^2 + b^2} + \right. \\
& \left. 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] \right) \\
& \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \right. \\
& \left. \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \\
& \left(-b - \sqrt{-4 a^2 + b^2} - 2 a \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \\
& \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \right. \\
& \left. \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) + \\
& \left(2 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] \right) \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) / \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(\left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right) \left(-\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] + \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right) \right], \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(\left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) / \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(\left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1\right] - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3\right] \right) \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(\text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4\right] \right) \right) \right. \right. \right. \right. \\
& \left(x - \text{Root}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2\right] \right)^2
\end{aligned}$$

$$\begin{aligned} & \sqrt{\left(\left(\left(-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 1\right]+\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 2\right]\right.\right.} \\ & \quad \left.\left.(x-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 3\right])\right) /\right. \\ & \quad \left.\left.\left(\left(x-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 2\right]\right)-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 1\right]\right.\right.} \\ & \quad \left.\left.\left.\left.\left.+\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 3\right]\right)\right)\right)\right) \\ & \left(\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 1\right]-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 4\right]\right) \\ & \sqrt{\left(\left(\left(-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 1\right]+\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 2\right]\right.\right.\right.} \\ & \quad \left.\left.\left.(x-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 4\right])\right) /\right. \\ & \quad \left.\left.\left.\left(\left(x-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 2\right]\right)-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 1\right]\right.\right.\right.} \\ & \quad \left.\left.\left.\left.\left.+\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 4\right]\right)\right)\right)\right) \\ & \sqrt{\left(\left(\left(\left(x-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 1\right]\right)-\text{Root}\left[a+b \# 1+c \# 1^2+\right.\right.\right.\right.} \\ & \quad \left.\left.\left.\left.b \# 1^3+a \# 1^4 \&, 2\right]+\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 4\right]\right)\right) /\right. \\ & \quad \left.\left.\left.\left.\left(\left(x-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 2\right]\right)-\text{Root}\left[a+b \# 1+c \# 1^2+\right.\right.\right.\right.} \\ & \quad \left.\left.\left.\left.b \# 1^3+a \# 1^4 \&, 1\right]+\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 4\right]\right)\right)\right)\right) / \\ & \left(a \sqrt{x \left(b+c x+b x^2\right)+a \left(1+x^4\right)}\left(-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 1\right]+\right.\right. \\ & \quad \left.\left.\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 2\right]\right)\right. \\ & \quad \left.\left.\left(-\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 2\right]+\right.\right.\right. \\ & \quad \left.\left.\left.\left.\text{Root}\left[a+b \# 1+c \# 1^2+b \# 1^3+a \# 1^4 \&, 4\right]\right)\right)\right) \end{aligned}$$

Problem 871: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e f - e f x^2}{\left(-a d + b d x - a d x^2\right) \sqrt{-a + b x + c x^2 + b x^3 - a x^4}} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{e f \operatorname{ArcTanh} \left[\frac{a b - (4 a^2 + b^2 + 2 a c) x + a b x^2}{2 a \sqrt{2 a + c} \sqrt{-a + b x + c x^2 + b x^3 - a x^4}} \right]}{a \sqrt{2 a + c} d}$$

Result (type 4, 15 147 leaves) :

$$\frac{1}{d} e f$$

$$\begin{aligned} & - \left(\left(8 \left(x - \text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2 \right] \right)^2 \left(\text{EllipticF} \left[\text{ArcSin} \left[\sqrt{ \left(\left(x - \text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2 \right] \right) \left(\text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2 \right] - \text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4 \right] \right) } / \left(\left(x - \text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2 \right] \right) \left(\text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1 \right] - \text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4 \right] \right) \right) \right) \right), \\ & - \left(\left(\left(\text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2 \right] - \text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 3 \right] \right) \left(\text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1 \right] - \text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4 \right] \right) \right) / \left(\left(-\text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1 \right] + \text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 3 \right] \right) \left(\text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2 \right] - \text{Root} \left[a - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4 \right] \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left(b + \sqrt{-4 a^2 + b^2} - 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) - \\
& 2 a \operatorname{EllipticPi}\left[\left(\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \right. \right. \\
& \left. \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) / \\
& \left(\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right), \\
& \operatorname{ArcSin}\left[\sqrt{\left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) / \right. \\
& \left. \left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \right], \\
& - \left(\left(\left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) / \right. \\
& \left. \left(\left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] \right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \right], \\
& \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \\
& \sqrt{\left(\left(\left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] \right) \right) / \right.} \\
& \left. \left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] \right) \right) \right), \\
& \sqrt{\left(\left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) / \right.} \\
& \left. \left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \right), \\
& \sqrt{\left(\left(\left(\left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) / \right.} \\
& \left. \left(\left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \right), \\
& \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) / \\
& \left(\left(-\frac{b - \sqrt{-4 a^2 + b^2}}{2 a} + \frac{b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \right. \\
& \sqrt{x (b + c x + b x^2) - a (1 + x^4)} \\
& \left(-b - \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) \\
& \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \\
& \left(b + \sqrt{-4 a^2 + b^2} - 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \right. \\
& \quad \left. \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \Bigg) + \\
& \left(2 b^2 (x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2])^2 \right. \\
& \quad \left(\text{EllipticF} [\text{ArcSin} [\sqrt{((x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1])} \\
& \quad (\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]) / ((x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (\text{Root} [a - b \#1 - \\
& \quad c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]))], \\
& \quad - ((\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + \\
& \quad a \#1^4 \&, 3]) (\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - \\
& \quad c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / ((-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \\
& \quad \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) (\text{Root} [a - b \#1 - c \#1^2 - \\
& \quad b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])))] \\
& \quad \left(b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) - \\
& \quad 2 a \text{EllipticPi} \left[\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \right. \\
& \quad \left. (-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]) \right] / \\
& \quad \left(\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) (-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]) \right), \\
& \text{ArcSin} [\sqrt{((x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]) (\text{Root} [a - b \#1 - \\
& \quad c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]) / \\
& \quad ((x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (\text{Root} [a - b \#1 - c \#1^2 - \\
& \quad b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]))}], \\
& \quad - ((\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + \\
& \quad a \#1^4 \&, 3]) (\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - \\
& \quad c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / ((-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \\
& \quad \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) (\text{Root} [a - b \#1 - c \#1^2 - \\
& \quad b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])))] \\
& \quad (-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \Big) \\
& \quad \sqrt{(((-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \\
& \quad (x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3])) / \\
& \quad ((x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, \\
& \quad 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3])))} \\
& \quad \sqrt{((((x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]) (\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + \\
& \quad a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / \\
& \quad ((x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (\text{Root} [a - b \#1 - c \#1^2 - \\
& \quad b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]))}) \\
& \quad \sqrt{(((-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \\
& \quad (x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / \\
& \quad ((x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (\text{Root} [a - b \#1 - c \#1^2 - \\
& \quad b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]))}) \\
& \quad \sqrt{(((-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \\
& \quad (x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / \\
& \quad ((x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (\text{Root} [a - b \#1 - c \#1^2 - \\
& \quad b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]))})
\end{aligned}$$

$$\begin{aligned}
& \left(\left(x - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] \right) \left(-\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] + \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4] \right) \right) \\
& \left(-\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] + \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4] \right) \Big) / \\
& \left(a^2 \left(-\frac{b - \sqrt{-4 a^2 + b^2}}{2 a} + \frac{b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \right. \\
& \sqrt{x (b + c x + b x^2) - a (1 + x^4)} \\
& \left(-b - \sqrt{-4 a^2 + b^2} + \right. \\
& 2 a \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] \Big) \\
& \left(-\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] + \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] \right) \\
& \left(b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] \right) \\
& \left(\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4] \right) \Big) + \\
& \left(2 b \sqrt{-4 a^2 + b^2} (x - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2])^2 \right. \\
& \left(\text{EllipticF}[\text{ArcSin}[\sqrt{\left((x - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1]) \right.} \right. \\
& \left. \left(\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4] \right) / \left((x - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2]) (\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4]) \right) \right), \right. \\
& \left. - \left(\left(\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 3] \right) (\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4]) / \left((-\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] + \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 3]) (\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4]) \right) \right) \right] \\
& \left(b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] \right) - \\
& 2 a \text{EllipticPi}[\left(\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] \right) \right. \\
& \left(-\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] + \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4] \right) \Big) / \\
& \left(\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] \right) (-\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] + \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4]) \right), \\
& \text{ArcSin}[\sqrt{\left((x - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1]) (\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2] - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4]) / \right.} \\
& \left. \left((x - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 2]) (\text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 1] - \text{Root}[\text{a} - b \# 1 - c \# 1^2 - b \# 1^3 + a \# 1^4 \&, 4]) \right) \right],
\end{aligned}$$

$$\begin{aligned}
& - \left(\left(\left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] \right) \left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) / \left(\left(-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] \right) \left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \right) \\
& \left(-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \Big) \\
& \sqrt{\left(\left(\left(-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \right. \\
& \quad \left. \left(x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] \right) \right) / \\
& \quad \left(\left(x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] \right) \right) \\
& \sqrt{\left(\left(x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) \left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right.} \\
& \quad \left. \left(\left(x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \\
& \sqrt{\left(\left(-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \right.} \\
& \quad \left. \left(\left(x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) / \\
& \quad \left(\left(x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \\
& \left(-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \Big) \Big) / \\
& \left(a^2 \left(-\frac{b - \sqrt{-4 a^2 + b^2}}{2 a} + \frac{b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \right. \\
& \quad \sqrt{x (b + c x + b x^2) - a (1 + x^4)} \\
& \quad \left(-b - \sqrt{-4 a^2 + b^2} + \right. \\
& \quad \left. \left. 2 a \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) \right. \\
& \quad \left(-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \\
& \quad \left(b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \\
& \quad \left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \Big) - \\
& \left(8 (x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2])^2 \right. \\
& \quad \left(\text{EllipticF} [\text{ArcSin} [\sqrt{\left(\left(x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) \right.} \right. \\
& \quad \left. \left. (\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]) / \left(\left(x - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \right) \right), \\
& \quad \left(\left(\left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] \right) \left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) / \left(\left(-\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3] \right) \left(\text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(b - \sqrt{-4 a^2 + b^2} - 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \\
& \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \right. \\
& \quad \left. \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) + \\
& \left(2 b^2 (x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2])^2 \right. \\
& \quad \left(\operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1])} \right. \\
& \quad \quad \left. (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]) / ((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]))], \right. \\
& \quad \quad \left. - ((\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / ((-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]))]) \right. \\
& \quad \quad \left. \left(b - \sqrt{-4 a^2 + b^2} - 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) - \right. \\
& \quad \quad \left. 2 a \operatorname{EllipticPi}\left[\left(-b + \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \right. \right. \\
& \quad \quad \left. \left. (-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]) \right) / \right. \\
& \quad \quad \left. \left(-b + \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) (-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]) \right), \\
& \quad \quad \operatorname{ArcSin}[\sqrt{((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / ((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]))], \right. \\
& \quad \quad \left. - ((\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / ((-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4]))]) \right. \\
& \quad \quad \left. (-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \right) \\
& \quad \vee (((-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \\
& \quad \quad (x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3])) / ((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]))) \\
& \quad \vee (((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / ((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])))
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \right.} \\
& \quad \left. \left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) / \\
& \quad \left(\left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) \\
& \quad \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \Big) / \\
& \left(a^2 \left(\frac{b - \sqrt{-4 a^2 + b^2}}{2 a} - \frac{b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \right. \\
& \quad \sqrt{x (b + c x + b x^2) - a (1 + x^4)} \\
& \quad \left(-b + \sqrt{-4 a^2 + b^2} + \right. \\
& \quad \left. 2 a \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] \right) \\
& \quad \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \\
& \quad \left(b - \sqrt{-4 a^2 + b^2} - 2 a \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \\
& \quad \left(\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] - \right. \\
& \quad \left. \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) - \\
& \left(2 b \sqrt{-4 a^2 + b^2} (x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right])^2 \right. \\
& \quad \left(\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] \right) \right.} \right. \right. \right. \\
& \quad \left. \left. \left. \left(\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) / \left(\left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) \right], \right. \\
& \quad \left. \left(\left(\left(\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3 \right] \right) \left(\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) / \left(\left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3 \right] \right) \left(\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) \right) \Big) \\
& \quad \left(b - \sqrt{-4 a^2 + b^2} - 2 a \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] \right) - \\
& \quad 2 a \text{EllipticPi}\left[\left(\left(-b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \right. \right. \\
& \quad \left. \left. \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) / \right. \\
& \quad \left(\left(-b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] \right) \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right), \right. \\
& \quad \left. \text{ArcSin}\left[\sqrt{\left(\left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] \right) \left(\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) / \right. \right. \right. \\
& \quad \left. \left. \left. \left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left(-b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\left(\left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \right.} \\
& \quad \left. \left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3 \right] \right) \right) / \\
& \quad \left(\left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3 \right] \right) \right) \\
& \quad \left(\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \\
& \sqrt{\left(\left(\left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \right.} \\
& \quad \left. \left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) / \\
& \quad \left(\left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) \\
& \sqrt{\left(\left(\left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] \right) \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) + \right.} \\
& \quad \left. \left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) / \\
& \quad \left(\left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] \right) \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) + \right. \\
& \quad \left. \left(x - \text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(-\text{Root}\left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) \Bigg)
\end{aligned}$$

Problem 872: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{ax^2 + bx \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}}{x \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{\sqrt{2} b \operatorname{ArcSinh}\left[\frac{a x+b \sqrt{-\frac{a}{b^2}+\frac{a^2 x^2}{b^2}}}{\sqrt{a}}\right]}{\sqrt{a}}$$

Result (type 3, 199 leaves):

$$\begin{aligned}
& - \left(\left(x \sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)} \left(-1 + a x^2 + b x \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right) \right. \right. \\
& \left. \left. \left(\log \left[1 - \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] - \log \left[1 + \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] \right) \right) / \right. \\
& \left. \left(\sqrt{2} \sqrt{\frac{a (-1 + a x^2)}{b^2}} \left(x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)^{3/2} \right) \right) \right)
\end{aligned}$$

Problem 873: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-a x^2 + b x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{\sqrt{2} b \operatorname{ArcSin} \left[\frac{a x - b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 213 leaves):

$$\begin{aligned}
& \left(b^2 \sqrt{\frac{a(1+ax^2)}{b^2}} \sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \sqrt{x \left(-ax + b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \right. \\
& \left. \left(\text{Log} \left[1 - \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right] - \text{Log} \left[1 + \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right] \right) \right) / \\
& \left(\sqrt{2} a^2 x \left(-1 - ax^2 + bx \sqrt{\frac{a(1+ax^2)}{b^2}} \right) \right)
\end{aligned}$$

Problem 874: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x \left(ax + b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}}{x \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\sqrt{2} b \text{ArcSinh} \left[\frac{ax+b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 199 leaves):

$$\begin{aligned}
& - \left(\left(x \sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)} \left(-1 + a x^2 + b x \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right) \right. \right. \\
& \left. \left. \left(\log \left[1 - \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] - \log \left[1 + \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] \right) \right) / \right. \\
& \left. \left(\sqrt{2} \sqrt{\frac{a (-1 + a x^2)}{b^2}} \left(x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)^{3/2} \right) \right) \right)
\end{aligned}$$

Problem 875: Result more than twice size of optimal antiderivative.

$$\int \frac{x \left(-a x + b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\sqrt{2} b \operatorname{ArcSin} \left[\frac{a x - b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 213 leaves):

$$\left(b^2 \sqrt{\frac{a (1 + a x^2)}{b^2}} \sqrt{a x \left(a x - b \sqrt{\frac{a (1 + a x^2)}{b^2}} \right)} \sqrt{x \left(-a x + b \sqrt{\frac{a (1 + a x^2)}{b^2}} \right)} \right.$$

$$\left. \left(\text{Log} \left[1 - \frac{\sqrt{a x \left(a x - b \sqrt{\frac{a (1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] - \text{Log} \left[1 + \frac{\sqrt{a x \left(a x - b \sqrt{\frac{a (1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] \right) \right) /$$

$$\left(\sqrt{2} a^2 x \left(-1 - a x^2 + b x \sqrt{\frac{a (1 + a x^2)}{b^2}} \right) \right)$$

Problem 876: Result more than twice size of optimal antiderivative.

$$\int \frac{-\sqrt{-4+x} - 4 \sqrt{-1+x} + \sqrt{-4+x} x + \sqrt{-1+x} x}{(1 + \sqrt{-4+x} + \sqrt{-1+x}) (4 - 5 x + x^2)} dx$$

Optimal (type 3, 19 leaves, 3 steps):

$$2 \text{Log} [1 + \sqrt{-4+x} + \sqrt{-1+x}]$$

Result (type 3, 75 leaves):

$$-\text{ArcTanh} [\sqrt{-4+x}] + \text{ArcTanh} \left[\frac{\sqrt{-1+x}}{2} \right] +$$

$$\frac{1}{2} \text{Log} [17 - 4 \sqrt{-4+x} \sqrt{-1+x} - 5 x] + \frac{1}{2} \text{Log} [5 - 2 \sqrt{-4+x} \sqrt{-1+x} - 2 x]$$

Problem 877: Unable to integrate problem.

$$\int \frac{1}{x (3 + 3 x + x^2) (3 + 3 x + 3 x^2 + x^3)^{1/3}} dx$$

Optimal (type 3, 123 leaves, 9 steps):

$$-\frac{\text{ArcTan} \left[\frac{1}{\sqrt{3}} + \frac{2 (1+x)}{3^{1/6} (2+(1+x)^3)^{1/3}} \right]}{3^{5/6}} + \frac{\text{Log} \left[1 - \frac{3^{1/3} (1+x)}{(2+(1+x)^3)^{1/3}} \right]}{3 \times 3^{1/3}} - \frac{\text{Log} \left[1 + \frac{3^{2/3} (1+x)^2}{(2+(1+x)^3)^{2/3}} + \frac{3^{1/3} (1+x)}{(2+(1+x)^3)^{1/3}} \right]}{6 \times 3^{1/3}}$$

Result (type 8, 33 leaves):

$$\int \frac{1}{x (3 + 3 x + x^2) (3 + 3 x + 3 x^2 + x^3)^{1/3}} dx$$

Problem 878: Unable to integrate problem.

$$\int \frac{1 - x^2}{(1 - x + x^2) (1 - x^3)^{2/3}} dx$$

Optimal (type 3, 103 leaves, ? steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}} - \frac{\operatorname{Log}\left[1+2 (1-x)^3-x^3\right]}{2 \times 2^{2/3}} + \frac{3 \operatorname{Log}\left[2^{1/3} (1-x)+(1-x^3)^{1/3}\right]}{2 \times 2^{2/3}}$$

Result (type 8, 31 leaves):

$$\int \frac{1 - x^2}{(1 - x + x^2) (1 - x^3)^{2/3}} dx$$

Problem 879: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\sqrt{-1+x^4} (1+x^4)} dx$$

Optimal (type 3, 49 leaves, ? steps):

$$-\frac{1}{4} \operatorname{ArcTan}\left[\frac{1+x^2}{x \sqrt{-1+x^4}}\right] - \frac{1}{4} \operatorname{ArcTanh}\left[\frac{1-x^2}{x \sqrt{-1+x^4}}\right]$$

Result (type 6, 114 leaves):

$$-\left(\left(7 x^3 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, x^4, -x^4\right]\right) / \left(3 \sqrt{-1+x^4} (1+x^4) \left(-7 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, x^4, -x^4\right] + 2 x^4 \left(2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, x^4, -x^4\right] - \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, x^4, -x^4\right]\right)\right)\right)$$

Problem 880: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a - c x^4}{(a e + c d x^2) (d + e x^2) \sqrt{a + b x^2 + c x^4}} dx$$

Optimal (type 3, 80 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{c d^2 - b d e + a e^2} x}{\sqrt{d} \sqrt{e} \sqrt{a + b x^2 + c x^4}}\right]}{\sqrt{d} \sqrt{e} \sqrt{c d^2 - b d e + a e^2}}$$

Result (type 4, 383 leaves) :

$$\begin{aligned} & \left(\frac{i}{\sqrt{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \right. \\ & \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] - \right. \\ & \quad \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4 a c}) d}{2 a e}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] - \\ & \quad \left. \left. \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4 a c}) e}{2 c d}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right]\right) \right) / \\ & \left(\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} d e \sqrt{a + b x^2 + c x^4} \right) \end{aligned}$$

Problem 882: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\frac{1}{x} + \sqrt{1 - x^2}} dx$$

Optimal (type 3, 122 leaves, 12 steps) :

$$\text{ArcSin}[x] - \frac{\text{ArcTan}\left[\frac{1-2 x^2}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}} \sqrt{1-x^2}}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{-\frac{i-\sqrt{3}}{i+\sqrt{3}} x}{\sqrt{1-x^2}}\right]}{\sqrt{3}}$$

Result (type 3, 2681 leaves) :

$$\begin{aligned} & \frac{\left(1 + x \sqrt{1 - x^2}\right) \text{ArcSin}[x]}{x \left(\frac{1}{x} + \sqrt{1 - x^2}\right)} + \\ & \left(\left(-\frac{i}{2} + \sqrt{3}\right) \left(1 + x \sqrt{1 - x^2}\right) \text{ArcTan}\left(x \left(7 \frac{i}{2} - \sqrt{3} + 8 \frac{i}{2} \sqrt{3} x + 7 \frac{i}{2} x^2 + \sqrt{3} x^2\right)\right) \right. \\ & \left. \left(-6 \frac{i}{2} + 2 \sqrt{3} + 3 x - 11 \frac{i}{2} \sqrt{3} x - 18 \frac{i}{2} x^2 - 2 \sqrt{3} x^2 - 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 - 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} \right. \right. \\ & \left. \left. \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} x^2 \sqrt{1 - x^2}\right) \right) \end{aligned}$$

$$\begin{aligned}
& \left(2 \sqrt{6 (1 - \frac{1}{x} \sqrt{3})} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) - \left((-\frac{1}{x} + \sqrt{3}) (1 + x \sqrt{1 - x^2}) \right) \\
& \text{ArcTan} \left[\left(x (7 \frac{1}{x} - \sqrt{3} - 8 \frac{1}{x} \sqrt{3} x + 7 \frac{1}{x} x^2 + \sqrt{3} x^2) \right) \right] / \left(6 \frac{1}{x} - 2 \sqrt{3} + 3 x - 11 \frac{1}{x} \sqrt{3} x + 18 \frac{1}{x} x^2 + \right. \\
& \quad \left. 2 \sqrt{3} x^2 - 3 x^3 - 3 \frac{1}{x} \sqrt{3} x^3 + 2 \frac{1}{x} \sqrt{2 (1 - \frac{1}{x} \sqrt{3})} \sqrt{1 - x^2} - 2 \frac{1}{x} \sqrt{6 (1 - \frac{1}{x} \sqrt{3})} x \sqrt{1 - x^2} + \right. \\
& \quad \left. 2 \frac{1}{x} \sqrt{2 (1 - \frac{1}{x} \sqrt{3})} x^2 \sqrt{1 - x^2} \right] \Bigg) / \left(2 \sqrt{6 (1 - \frac{1}{x} \sqrt{3})} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) - \\
& \left((\frac{1}{x} + \sqrt{3}) (1 + x \sqrt{1 - x^2}) \text{ArcTan} \left[\left(x (-7 \frac{1}{x} - \sqrt{3} - 8 \frac{1}{x} \sqrt{3} x - 7 \frac{1}{x} x^2 + \sqrt{3} x^2) \right) \right] \right. \\
& \quad \left. \left(-6 \frac{1}{x} - 2 \sqrt{3} - 3 x - 11 \frac{1}{x} \sqrt{3} x - 18 \frac{1}{x} x^2 + 2 \sqrt{3} x^2 + 3 x^3 - 3 \frac{1}{x} \sqrt{3} x^3 - 2 \frac{1}{x} \sqrt{2 (1 + \frac{1}{x} \sqrt{3})} \right. \right. \\
& \quad \left. \left. \sqrt{1 - x^2} - 2 \frac{1}{x} \sqrt{6 (1 + \frac{1}{x} \sqrt{3})} x \sqrt{1 - x^2} - 2 \frac{1}{x} \sqrt{2 (1 + \frac{1}{x} \sqrt{3})} x^2 \sqrt{1 - x^2} \right] \right) / \\
& \left(2 \sqrt{6 (1 + \frac{1}{x} \sqrt{3})} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) + \left((\frac{1}{x} + \sqrt{3}) (1 + x \sqrt{1 - x^2}) \right. \\
& \quad \left. \text{ArcTan} \left[\left(x (-7 \frac{1}{x} - \sqrt{3} + 8 \frac{1}{x} \sqrt{3} x - 7 \frac{1}{x} x^2 + \sqrt{3} x^2) \right) \right] \right) / \left(6 \frac{1}{x} + 2 \sqrt{3} - 3 x - 11 \frac{1}{x} \sqrt{3} x + 18 \frac{1}{x} x^2 - \right. \\
& \quad \left. 2 \sqrt{3} x^2 + 3 x^3 - 3 \frac{1}{x} \sqrt{3} x^3 + 2 \frac{1}{x} \sqrt{2 (1 + \frac{1}{x} \sqrt{3})} \sqrt{1 - x^2} - 2 \frac{1}{x} \sqrt{6 (1 + \frac{1}{x} \sqrt{3})} x \sqrt{1 - x^2} + \right. \\
& \quad \left. 2 \frac{1}{x} \sqrt{2 (1 + \frac{1}{x} \sqrt{3})} x^2 \sqrt{1 - x^2} \right] \Bigg) / \left(2 \sqrt{6 (1 + \frac{1}{x} \sqrt{3})} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) + \\
& \frac{\frac{1}{x} (-\frac{1}{x} + \sqrt{3}) (1 + x \sqrt{1 - x^2}) \text{Log} \left[\left(-\frac{1}{x} + \sqrt{3} - 2 x \right)^2 \left(\frac{1}{x} + \sqrt{3} - 2 x \right)^2 \right]}{4 \sqrt{6 (1 - \frac{1}{x} \sqrt{3})} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} - \\
& \frac{\frac{1}{x} (\frac{1}{x} + \sqrt{3}) (1 + x \sqrt{1 - x^2}) \text{Log} \left[\left(-\frac{1}{x} + \sqrt{3} - 2 x \right)^2 \left(\frac{1}{x} + \sqrt{3} - 2 x \right)^2 \right]}{4 \sqrt{6 (1 + \frac{1}{x} \sqrt{3})} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} - \\
& \frac{\frac{1}{x} (-\frac{1}{x} + \sqrt{3}) (1 + x \sqrt{1 - x^2}) \text{Log} \left[\left(-\frac{1}{x} + \sqrt{3} + 2 x \right)^2 \left(\frac{1}{x} + \sqrt{3} + 2 x \right)^2 \right]}{4 \sqrt{6 (1 - \frac{1}{x} \sqrt{3})} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} + \\
& \frac{\frac{1}{x} (\frac{1}{x} + \sqrt{3}) (1 + x \sqrt{1 - x^2}) \text{Log} \left[\left(-\frac{1}{x} + \sqrt{3} + 2 x \right)^2 \left(\frac{1}{x} + \sqrt{3} + 2 x \right)^2 \right]}{4 \sqrt{6 (1 + \frac{1}{x} \sqrt{3})} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)}
\end{aligned}$$

$$\begin{aligned}
& \frac{\frac{1}{2} \left(\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[\left(-\frac{1}{2} + \sqrt{3} + 2x \right)^2 \left(\frac{1}{2} + \sqrt{3} + 2x \right)^2 \right]}{4 \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} - \\
& \frac{\frac{1}{2} \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[-\frac{1}{2} - \frac{\frac{1}{2} \sqrt{3}}{2} + x^2 \right]}{2 \sqrt{3} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} + \\
& \frac{\frac{1}{2} \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[-\frac{1}{2} + \frac{\frac{1}{2} \sqrt{3}}{2} + x^2 \right]}{2 \sqrt{3} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} - \\
& \left(\frac{1}{2} \left(-\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[3 \frac{1}{2} + \sqrt{3} - 3x - 5 \frac{1}{2} \sqrt{3} x + 10 \frac{1}{2} x^2 + 3x^3 - 3 \frac{1}{2} \sqrt{3} x^3 + \right. \right. \\
& \left. \left. \frac{1}{2} x^4 - \sqrt{3} x^4 + 2 \frac{1}{2} \sqrt{2 \left(1 - \frac{1}{2} \sqrt{3} \right)} \sqrt{1 - x^2} - 3 \frac{1}{2} \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x \sqrt{1 - x^2} + \right. \right. \\
& \left. \left. 5 \frac{1}{2} \sqrt{2 \left(1 - \frac{1}{2} \sqrt{3} \right)} x^2 \sqrt{1 - x^2} - \frac{1}{2} \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x^3 \sqrt{1 - x^2} \right] \right) / \\
& \left(4 \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) + \left(\frac{1}{2} \left(-\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \right. \\
& \left. \operatorname{Log} \left[3 \frac{1}{2} + \sqrt{3} + 3x + 5 \frac{1}{2} \sqrt{3} x + 10 \frac{1}{2} x^2 - 3x^3 + 3 \frac{1}{2} \sqrt{3} x^3 + \frac{1}{2} x^4 - \sqrt{3} x^4 + \right. \right. \\
& \left. \left. 2 \frac{1}{2} \sqrt{2 \left(1 - \frac{1}{2} \sqrt{3} \right)} \sqrt{1 - x^2} + 3 \frac{1}{2} \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x \sqrt{1 - x^2} + 5 \frac{1}{2} \sqrt{2 \left(1 - \frac{1}{2} \sqrt{3} \right)} x^2 \sqrt{1 - x^2} + \right. \right. \\
& \left. \left. \frac{1}{2} \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x^3 \sqrt{1 - x^2} \right] \right) / \left(4 \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) - \\
& \left(\frac{1}{2} \left(\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[-3 \frac{1}{2} + \sqrt{3} + 3x - 5 \frac{1}{2} \sqrt{3} x - 10 \frac{1}{2} x^2 - 3x^3 - 3 \frac{1}{2} \sqrt{3} x^3 - \right. \right. \\
& \left. \left. \frac{1}{2} x^4 - \sqrt{3} x^4 - 2 \frac{1}{2} \sqrt{2 \left(1 + \frac{1}{2} \sqrt{3} \right)} \sqrt{1 - x^2} - 3 \frac{1}{2} \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \sqrt{1 - x^2} - \right. \right. \\
& \left. \left. 5 \frac{1}{2} \sqrt{2 \left(1 + \frac{1}{2} \sqrt{3} \right)} x^2 \sqrt{1 - x^2} - \frac{1}{2} \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x^3 \sqrt{1 - x^2} \right] \right) / \\
& \left(4 \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) + \left(\frac{1}{2} \left(\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \right. \\
& \left. \operatorname{Log} \left[-3 \frac{1}{2} + \sqrt{3} - 3x + 5 \frac{1}{2} \sqrt{3} x - 10 \frac{1}{2} x^2 + 3x^3 + 3 \frac{1}{2} \sqrt{3} x^3 - \frac{1}{2} x^4 - \sqrt{3} x^4 - \right. \right. \\
& \left. \left. 2 \frac{1}{2} \sqrt{2 \left(1 + \frac{1}{2} \sqrt{3} \right)} \sqrt{1 - x^2} + 3 \frac{1}{2} \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \sqrt{1 - x^2} - 5 \frac{1}{2} \sqrt{2 \left(1 + \frac{1}{2} \sqrt{3} \right)} x^2 \sqrt{1 - x^2} + \right. \right. \\
& \left. \left. \frac{1}{2} \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x^3 \sqrt{1 - x^2} \right] \right) / \left(4 \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right)
\end{aligned}$$

Problem 883: Result more than twice size of optimal antiderivative.

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}} dx$$

Optimal (type 3, 122 leaves, 13 steps):

$$\text{ArcSin}[x] - \frac{\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{x}{\sqrt{\frac{-\frac{i}{2}-\sqrt{3}}{\frac{i}{2}+\sqrt{3}}}\sqrt{1-x^2}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{-\frac{i}{2}-\sqrt{3}}{\frac{i}{2}+\sqrt{3}}}x}{\sqrt{1-x^2}}\right]}{\sqrt{3}}$$

Result (type 3, 2155 leaves):

$$\begin{aligned} & \text{ArcSin}[x] + \\ & \left(\left(-\frac{i}{2} + \sqrt{3} \right) \text{ArcTan}\left[\left(x \left(7 \frac{i}{2} - \sqrt{3} + 8 \frac{i}{2} \sqrt{3} x + 7 \frac{i}{2} x^2 + \sqrt{3} x^2 \right) \right) \right] \Big/ \left(-6 \frac{i}{2} + 2 \sqrt{3} + 3 x - 11 \frac{i}{2} \sqrt{3} x - \right. \right. \\ & \quad \left. \left. 18 \frac{i}{2} x^2 - 2 \sqrt{3} x^2 - 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 - 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3} \right)} \sqrt{1-x^2} - \right. \right. \\ & \quad \left. \left. 2 \frac{i}{2} \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3} \right)} x \sqrt{1-x^2} - 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3} \right)} x^2 \sqrt{1-x^2} \right] \right) \Big/ \left(2 \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3} \right)} \right) - \\ & \left(\left(-\frac{i}{2} + \sqrt{3} \right) \text{ArcTan}\left[\left(x \left(7 \frac{i}{2} - \sqrt{3} - 8 \frac{i}{2} \sqrt{3} x + 7 \frac{i}{2} x^2 + \sqrt{3} x^2 \right) \right) \right] \right. \\ & \quad \left. \left(6 \frac{i}{2} - 2 \sqrt{3} + 3 x - 11 \frac{i}{2} \sqrt{3} x + 18 \frac{i}{2} x^2 + 2 \sqrt{3} x^2 - 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 + 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3} \right)} \right. \right. \\ & \quad \left. \left. \sqrt{1-x^2} - 2 \frac{i}{2} \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3} \right)} x \sqrt{1-x^2} + 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3} \right)} x^2 \sqrt{1-x^2} \right] \right) \Big/ \\ & \left(2 \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3} \right)} \right) - \left(\left(\frac{i}{2} + \sqrt{3} \right) \text{ArcTan}\left[\left(x \left(-7 \frac{i}{2} - \sqrt{3} - 8 \frac{i}{2} \sqrt{3} x - 7 \frac{i}{2} x^2 + \sqrt{3} x^2 \right) \right) \right] \right. \\ & \quad \left. \left(-6 \frac{i}{2} - 2 \sqrt{3} - 3 x - 11 \frac{i}{2} \sqrt{3} x - 18 \frac{i}{2} x^2 + 2 \sqrt{3} x^2 + 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 - 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3} \right)} \right. \right. \\ & \quad \left. \left. \sqrt{1-x^2} - 2 \frac{i}{2} \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3} \right)} x \sqrt{1-x^2} - 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3} \right)} x^2 \sqrt{1-x^2} \right] \right) \Big/ \\ & \left(2 \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3} \right)} \right) + \left(\left(\frac{i}{2} + \sqrt{3} \right) \text{ArcTan}\left[\left(x \left(-7 \frac{i}{2} - \sqrt{3} + 8 \frac{i}{2} \sqrt{3} x - 7 \frac{i}{2} x^2 + \sqrt{3} x^2 \right) \right) \right] \right. \\ & \quad \left. \left(6 \frac{i}{2} + 2 \sqrt{3} - 3 x - 11 \frac{i}{2} \sqrt{3} x + 18 \frac{i}{2} x^2 - 2 \sqrt{3} x^2 + 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 + 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3} \right)} \right. \right. \\ & \quad \left. \left. \sqrt{1-x^2} - 2 \frac{i}{2} \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3} \right)} x \sqrt{1-x^2} + 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3} \right)} x^2 \sqrt{1-x^2} \right] \right) \Big/ \end{aligned}$$

$$\begin{aligned}
& \left(2 \sqrt{6 (1 + \frac{i}{2} \sqrt{3})} \right) + \frac{\frac{i}{2} (-\frac{i}{2} + \sqrt{3}) \operatorname{Log}[(-\frac{i}{2} + \sqrt{3} - 2x)^2 (\frac{i}{2} + \sqrt{3} - 2x)^2]}{4 \sqrt{6 (1 - \frac{i}{2} \sqrt{3})}} - \\
& \frac{\frac{i}{2} (\frac{i}{2} + \sqrt{3}) \operatorname{Log}[(-\frac{i}{2} + \sqrt{3} - 2x)^2 (\frac{i}{2} + \sqrt{3} - 2x)^2]}{4 \sqrt{6 (1 + \frac{i}{2} \sqrt{3})}} - \\
& \frac{\frac{i}{2} (-\frac{i}{2} + \sqrt{3}) \operatorname{Log}[(-\frac{i}{2} + \sqrt{3} + 2x)^2 (\frac{i}{2} + \sqrt{3} + 2x)^2]}{4 \sqrt{6 (1 - \frac{i}{2} \sqrt{3})}} + \\
& \frac{\frac{i}{2} (\frac{i}{2} + \sqrt{3}) \operatorname{Log}[(-\frac{i}{2} + \sqrt{3} + 2x)^2 (\frac{i}{2} + \sqrt{3} + 2x)^2]}{4 \sqrt{6 (1 + \frac{i}{2} \sqrt{3})}} - \\
& \frac{\frac{i}{2} \operatorname{Log}\left[-\frac{1}{2} - \frac{\frac{i}{2} \sqrt{3}}{2} + x^2\right]}{2 \sqrt{3}} + \\
& \frac{\frac{i}{2} \operatorname{Log}\left[-\frac{1}{2} + \frac{\frac{i}{2} \sqrt{3}}{2} + x^2\right]}{2 \sqrt{3}} - \\
& \left(\frac{i}{2} (-\frac{i}{2} + \sqrt{3}) \operatorname{Log}[3 \frac{i}{2} + \sqrt{3} - 3x - 5 \frac{i}{2} \sqrt{3} x + 10 \frac{i}{2} x^2 + 3x^3 - 3 \frac{i}{2} \sqrt{3} x^3 + \right. \\
& \quad \left. \frac{i}{2} x^4 - \sqrt{3} x^4 + 2 \frac{i}{2} \sqrt{2 (1 - \frac{i}{2} \sqrt{3})} \sqrt{1 - x^2} - 3 \frac{i}{2} \sqrt{6 (1 - \frac{i}{2} \sqrt{3})} x \sqrt{1 - x^2} + \right. \\
& \quad \left. 5 \frac{i}{2} \sqrt{2 (1 - \frac{i}{2} \sqrt{3})} x^2 \sqrt{1 - x^2} - \frac{i}{2} \sqrt{6 (1 - \frac{i}{2} \sqrt{3})} x^3 \sqrt{1 - x^2}] \right) / \left(4 \sqrt{6 (1 - \frac{i}{2} \sqrt{3})} \right) + \\
& \left(\frac{i}{2} (-\frac{i}{2} + \sqrt{3}) \operatorname{Log}[3 \frac{i}{2} + \sqrt{3} + 3x + 5 \frac{i}{2} \sqrt{3} x + 10 \frac{i}{2} x^2 - 3x^3 + 3 \frac{i}{2} \sqrt{3} x^3 + \frac{i}{2} x^4 - \right. \\
& \quad \left. \sqrt{3} x^4 + 2 \frac{i}{2} \sqrt{2 (1 - \frac{i}{2} \sqrt{3})} \sqrt{1 - x^2} + 3 \frac{i}{2} \sqrt{6 (1 - \frac{i}{2} \sqrt{3})} x \sqrt{1 - x^2} + \right. \\
& \quad \left. 5 \frac{i}{2} \sqrt{2 (1 - \frac{i}{2} \sqrt{3})} x^2 \sqrt{1 - x^2} + \frac{i}{2} \sqrt{6 (1 - \frac{i}{2} \sqrt{3})} x^3 \sqrt{1 - x^2}] \right) / \left(4 \sqrt{6 (1 - \frac{i}{2} \sqrt{3})} \right) - \\
& \left(\frac{i}{2} (\frac{i}{2} + \sqrt{3}) \operatorname{Log}[-3 \frac{i}{2} + \sqrt{3} + 3x - 5 \frac{i}{2} \sqrt{3} x - 10 \frac{i}{2} x^2 - 3x^3 - 3 \frac{i}{2} \sqrt{3} x^3 - \frac{i}{2} x^4 - \right. \\
& \quad \left. \sqrt{3} x^4 - 2 \frac{i}{2} \sqrt{2 (1 + \frac{i}{2} \sqrt{3})} \sqrt{1 - x^2} - 3 \frac{i}{2} \sqrt{6 (1 + \frac{i}{2} \sqrt{3})} x \sqrt{1 - x^2} - \right.
\end{aligned}$$

$$\left(\frac{5 \sqrt{2} \left(1 + \sqrt{3}\right) x^2 \sqrt{1-x^2} - \sqrt{6} \left(1 + \sqrt{3}\right) x^3 \sqrt{1-x^2}}{\left(4 \sqrt{6} \left(1 + \sqrt{3}\right)\right)} + \right. \\ \left(\frac{\sqrt{3} x^4 - 2 \sqrt{2} \left(1 + \sqrt{3}\right) \sqrt{1-x^2} + 3 \sqrt{6} \left(1 + \sqrt{3}\right) x \sqrt{1-x^2}}{\left(4 \sqrt{6} \left(1 + \sqrt{3}\right)\right)} - \right. \\ \left. \left. \frac{5 \sqrt{2} \left(1 + \sqrt{3}\right) x^2 \sqrt{1-x^2} + \sqrt{6} \left(1 + \sqrt{3}\right) x^3 \sqrt{1-x^2}}{\left(4 \sqrt{6} \left(1 + \sqrt{3}\right)\right)} \right) \right)$$

Problem 885: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{-44375 b^4 + 576000 b^3 c x + 576000 b^2 c^2 x^2 + 5308416 c^4 x^4}} dx$$

Optimal (type 3, 177 leaves, 1 step):

$$\frac{1}{18432 c^2} \text{Log} \left[20738073600000000 b^8 c^4 + 597005697024000000 b^6 c^6 x^2 + 2583100705996800000 b^5 c^7 x^3 + \right. \\ 951050714480640000 b^4 c^8 x^4 + 21641687369515008000 b^3 c^9 x^5 + \\ 32462531054272512000 b^2 c^{10} x^6 + 149587343098087735296 c^{12} x^8 + \\ 5308416 \sqrt{-44375 b^4 + 576000 b^3 c x + 576000 b^2 c^2 x^2 + 5308416 c^4 x^4} \\ \left. \left(12203125 b^6 c^4 + 79200000 b^5 c^5 x + 38880000 b^4 c^6 x^2 + \right. \right. \\ \left. \left. 1105920000 b^3 c^7 x^3 + 1990656000 b^2 c^8 x^4 + 12230590464 c^{10} x^6 \right) \right]$$

Result (type 4, 1671 leaves):

$$\left(2 \left(x - \frac{b \text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2 \right]}{c} \right)^2 \right. \\ \left(-\frac{1}{c} b \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{((c x - b \text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2 \right] - \right. \right. \right. \\ \left. \left. \left. \text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4 \right])} \right) / \right. \\ \left. \left((c x - b \text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2 \right]) \right. \right. \\ \left. \left. \left(\text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1 \right] - \right. \right. \\ \left. \left. \left. \text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4 \right]) \right) \right), \right. \\ \left. - \left(((\text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2 \right] - \right. \right. \\ \left. \left. \text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 3 \right]) \right. \right. \\ \left. \left. (\text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1 \right] - \right. \right. \\ \left. \left. \text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4 \right]) \right) / \right. \\ \left. \left((-\text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1 \right] + \right. \right. \\ \left. \left. \text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 3 \right]) \right. \right. \\ \left. \left. (\text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2 \right] - \right. \right. \\ \left. \left. \text{Root} \left[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4 \right]) \right) \right)$$

$$\begin{aligned}
& \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] + \\
& \frac{1}{c} \text{EllipticPi}\left[\left(-\frac{1}{c} b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1]\right.\right. + \\
& \quad \left.\left.\frac{1}{c} b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]\right)\right] / \\
& \quad \left(-\frac{1}{c} b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] + \frac{1}{c}\right. \\
& \quad \left.b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]\right), \\
& \text{ArcSin}\left[\sqrt{\left(\left(c x - b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1]\right)\right.\right.\right. \\
& \quad \left.\left.\left(\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] -\right.\right.\right. \\
& \quad \left.\left.\left.\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]\right)\right)\right) / \\
& \quad \left(\left(c x - b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]\right)\right. \\
& \quad \left.\left(\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] -\right.\right. \\
& \quad \left.\left.\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]\right)\right)\right], \\
& - \left(\left(\left(\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] - \text{Root}[\right.\right.\right. \\
& \quad \left.\left.\left.-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 3]\right)\right.\right. \\
& \quad \left.\left.\left(\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] - \text{Root}[\right.\right.\right. \\
& \quad \left.\left.\left.-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]\right)\right)\right) / \\
& \quad \left(\left(-\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] + \text{Root}[\right.\right.\right. \\
& \quad \left.\left.\left.-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 3]\right)\right.\right. \\
& \quad \left.\left.\left(\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] - \text{Root}[\right.\right.\right. \\
& \quad \left.\left.\left.-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]\right)\right)\right) \\
& \left(-b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] +\right. \\
& \quad \left.b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]\right)\Big) \\
& \sqrt{\left(\left(\left(-b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] +\right.\right.\right. \\
& \quad \left.\left.\left.b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]\right)\right.\right. \\
& \quad \left.\left.\left(x - \frac{1}{c} b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 3]\right)\right)\right) / \\
& \quad \left(c \left(x - \frac{1}{c} b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]\right)\right. \\
& \quad \left.\left(-\frac{1}{c} b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] + \frac{1}{c}\right.\right. \\
& \quad \left.\left.b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 3]\right)\right)\Big) \\
& \sqrt{\left(\left(\left(c x - b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1]\right)\right.\right. \\
& \quad \left.\left(\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] -\right.\right. \\
& \quad \left.\left.\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]\right)\right)\right) / \\
& \quad \left(\left(c x - b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]\right)\right. \\
& \quad \left.\left(\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] -\right.\right. \\
& \quad \left.\left.\text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]\right)\right)\Big) \\
& \left(\frac{b \text{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1]}{c} -\right.
\end{aligned}$$

$$\begin{aligned}
& \frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]}{c} \\
& \sqrt{\left(\left(\left(-b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] + \right. \right. \right.} \\
& \quad b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]) \\
& \quad \left. \left. \left. \left(x - \frac{1}{c} b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4] \right) \right) \right) / \\
& \quad \left(c \left(x - \frac{1}{c} b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] \right) \right. \\
& \quad \left. \left. \left. \left(-\frac{1}{c} b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] + \frac{1}{c} \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4] \right) \right) \right) \right) / \\
& \left(\sqrt{-44375 b^4 + 576000 b^3 c + 576000 b^2 c^2 x^2 + 5308416 c^4 x^4} \right. \\
& \left. \left(-\frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1]}{c} + \right. \right. \right. \\
& \quad \left. \left. \left. b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] \right) \right) \\
& \left(\frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]}{c} - \right. \\
& \quad \left. \left. \left. b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4] \right) \right) \right)
\end{aligned}$$

Problem 886: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}} dx$$

Optimal (type 3, 100 leaves, 2 steps):

$$\begin{aligned}
& \frac{1}{16} \operatorname{Log}[921 + 2864 x + 9280 x^2 + 13440 x^3 + 17024 x^4 + 19456 x^5 + 12288 x^6 + 8192 x^7 + 4096 x^8 + \\
& \sqrt{9 + 120 x + 64 x^2 + 64 x^3 + 64 x^4} (179 + 444 x + 744 x^2 + 1280 x^3 + 960 x^4 + 768 x^5 + 512 x^6)]
\end{aligned}$$

Result (type 4, 2787 leaves):

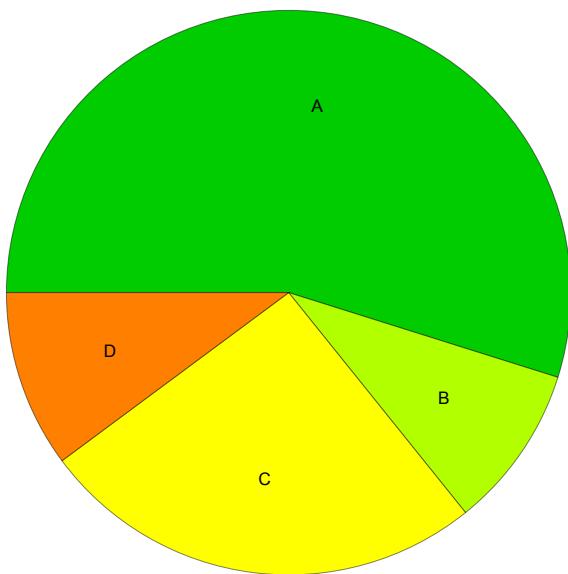
$$\begin{aligned}
& \left(8 \left(x - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] \right)^2 \right. \\
& \quad \left(-\operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\left(x - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] \right)} \right. \right. \\
& \quad \left. \left. \left(\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) / \\
& \quad \left(\left(x - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] \right) \left(\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + \right. \right. \\
& \quad \left. \left. 64 \#1^3 + 64 \#1^4 \&, 1] - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) \right),
\end{aligned}$$

$$\begin{aligned}
& - \left(\left(\left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3] \right) \left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) / \right. \\
& \quad \left. \left(\left(-\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] + \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) \right] \\
& \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] + \text{EllipticPi} [\\
& \quad \left(-\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] + \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) / \\
& \quad \left(-\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] + \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right), \\
& \text{ArcSin} [\sqrt{\left(\left(x - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] \right) \left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) / \\
& \quad \left(\left(x - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] \right) \left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right)], \\
& - \left(\left(\left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3] \right) \left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) / \right. \\
& \quad \left. \left(\left(-\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] + \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) \right] \\
& \left(-\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] + \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] \right) \\
& \sqrt{\left(\left(x - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3] \right) / \right.} \\
& \quad \left. \left(\left(x - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] \right) \right. \right. \\
& \quad \left. \left. \left(-\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] + \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3] \right) \right) \right) \\
& \left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] - \text{Root} [\right. \\
& \quad \left. \left. \left. 9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) \\
& \sqrt{\left(\left(x - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] \right) \right.} \\
& \quad \left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) / \\
& \quad \left(\left(x - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] \right) \right. \\
& \quad \left. \left. \left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) \right) \\
& \sqrt{\left(\left(x - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) / \right.} \\
& \quad \left(\left(x - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] \right) \right. \\
& \quad \left. \left. \left(-\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] + \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) \right) \Bigg) / \\
& \left(\sqrt{9 + 120 x + 64 x^2 + 64 x^3 + 64 x^4} \left(\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \right. \right. \\
& \quad \left. \left. \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) + \\
& \left(2 \text{EllipticF} [\text{ArcSin} [\sqrt{\left(\left(x - \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] \right) \right.} \right. \\
& \quad \left. \left. \left(-\text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] + \text{Root} [9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4] \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] + \right. \\
& \quad \left. \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) / \\
& \left(\left(x - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] \right) \left(-\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + \right. \right. \right. \\
& \quad \left. \left. \left. 64 \#1^3 + 64 \#1^4 \&, 1 \right] + \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) \right) , \\
& \left(\left(\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3 \right] \right) \left(\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1 \right] - \right. \right. \\
& \quad \left. \left. \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) \right) / \\
& \left(\left(\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1 \right] - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3 \right] \right) \left(\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] - \right. \right. \\
& \quad \left. \left. \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) \right] \\
& \left(x - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] \right)^2 \\
& \sqrt{\left(\left(x - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3 \right] \right) / \right.} \\
& \quad \left(\left(x - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] \right) \right. \\
& \quad \left. \left(-\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1 \right] + \right. \right. \\
& \quad \left. \left. \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3 \right] \right) \right) \\
& \left(\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1 \right] - \text{Root}\left[\right. \right. \\
& \quad \left. \left. 9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) \\
& \sqrt{\left(\left(x - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) / \right.} \\
& \quad \left(\left(x - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] \right) \right. \\
& \quad \left. \left(-\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1 \right] + \right. \right. \\
& \quad \left. \left. \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) \right) \\
& \sqrt{\left(\left(\left(x - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1 \right] \right) \right. \right.} \\
& \quad \left. \left(-\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] + \right. \right. \\
& \quad \left. \left. \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) \right) /} \\
& \left(\left(x - \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] \right) \right. \\
& \quad \left. \left(-\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1 \right] + \right. \right. \\
& \quad \left. \left. \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) \right)) / \\
& \left(\sqrt{9 + 120 x + 64 x^2 + 64 x^3 + 64 x^4} \left(-\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2 \right] + \right. \right. \\
& \quad \left. \left. \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4 \right] \right) \right)
\end{aligned}$$

Summary of Integration Test Results

886 integration problems



A - 486 optimal antiderivatives

B - 83 more than twice size of optimal antiderivatives

C - 227 unnecessarily complex antiderivatives

D - 90 unable to integrate problems

E - 0 integration timeouts